THE PRINCIPLES OF
DYNAMO ELECTRIC MACHINERY

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BY

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PREFACE

The underlying purpose which the writer has had in mind throughout the preparation of this book is to present a clear physical conception of the phenomena which take place in electrical machinery. He has made but sparing use of mathematical demonstration, not because he does not believe in the value of mathematics to the engineer, but because he is firmly convinced that a clear physical idea of the actions which take place should be obtained before an attempt is made to apply mathematical analysis. This method of studying the subject is by no means easy either for the student or for the teacher. It is not easy for the student because it requires the development of his ability to think clearly and to express his ideas in logical language. It is not easy for the teacher, because while it is a simple matter to glance through a mathematical demonstration and pick out the errors, real teaching ability is required to guide the student so that he will form correct habits of thought, and to make some effort to determine whether or not he has a real understanding of the subject. Thus it is very easy to tell the student that two harmonic motions at 90 degrees in both time and space, combine to form a rotary motion. The student sees no objection to the statement, accepts it as true and very readily learns to repeat it. It is doubtful, however, whether he has materially increased his reasoning power. In fact, in the writer's opinion, he has decreased it and instead has taken a step toward acquiring the bad habit of accepting a statement as true "because the book says so." If, on the other hand, he has carefully studied the way in which the currents rise and fall in the conductors of a polyphase induction motor, and has mastered the way in which the current sheet and consequently the flux revolve around the stator, he has made a start toward acquiring a real power of analysis. This will stand him in good stead when in later years he is confronted with problems whose answer is not "in the back of the book."

It is believed that the material presented is sufficient to satisfy the needs of students who do not intend to follow electrical

engineering as a profession. It is also thought that it is well adapted as a first text in electrical engineering for students who do expect to go further with the subject. In this case, it should be followed by a text going into the mathematical relations. The student who has mastered the contents of this book should be well prepared to take such a course with profit and understanding.

As an illustration of what may happen if the mathematics of electrical engineering is taught before a physical conception of the actions which take place has been acquired, the writer may cite the case of a student who recently came to him for an examination. The young man reproduced very well several pages of mathematical demonstrations relating to transformers, and all would have been well had not a chance question revealed the fact that he believed all the time that the function of the transformer was to step up or step down the voltage of a *continuous current* circuit. This is by no means an isolated case. There are many others nearly as bad.

The scope of the book is such as to teach the student two things about each machine studied: first, what the machine will do; second, why it does it. No attempt is made to take up questions relating to the design of electrical machinery.

The writer desires to acknowledge with gratitude the assistance given him by his associates on the faculty of the University of Michigan in the preparation of this work.

B. F. B.

ANN ARBOR, MICH.,
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CHAPTER I

GENERAL PRINCIPLES

Before reading this book, the student is supposed to have some knowledge of elementary electrical theory. Therefore only a brief review of electrical principles will be given here.

1. Effects of Electricity.—The ultimate nature of electricity is unknown to us. We are, however, able to recognize the presence of an electric current by means of many well-known effects. For example, if we were required to determine whether or not a certain wire were carrying a current of electricity, we could solve the problem in many ways. Thus, the wire would be somewhat hotter than the surrounding air, and if the current were strong enough the wire might become red hot and finally fuse. If the wire were placed in approximately a north and south direction, and a compass needle were brought near the wire, it would exhibit a tendency to set itself in an east and west direction across the wire. If the current were alternating, and the needle were light enough to follow the alternations, it would be set in vibration. If the wire were cut and the ends placed in a conducting solution, there would in general, if the current were direct, be an evolution of gases or products of the decomposition of the substance in solution at the ends of the wire. This result would also follow in many cases if the current were alternating. Currents have also a physiological effect, and hence can be perceived by the senses. Thus many can testify that a current can be felt, and although we can not see a current of electricity, it is easy to produce the effect of flashes of light by means of currents passed through the head.

It is a common belief that we know little or nothing about electricity, and that when its real nature is discovered we shall see a tremendous advance in its applications. But we know just

as little about the nature of gravity as we do about that of electricity. This ignorance, however, does not prevent us from making reasonably good derricks, and it is doubtful if the discovery of the exact nature of gravity would enable us to improve the common derrick materially. This slight digression is made merely to remove the impression that electricity is very mysterious and subtle, and that we know very little about it. In the future, undoubtedly, we shall learn many things now undreamed of in regard to the phenomena of electricity. There is, however, no probability that this added knowledge will make useless any of the laws already discovered.

2. Ohm's Law.—The simplest and the most useful of the laws of continuous electricity is known as Ohm's Law, after its discoverer. It may be expressed in the form

$$I = \frac{E}{R}$$

which may readily be changed to the forms

$$E = RI$$

$$R = \frac{E}{I}$$

The equations in the forms given apply only to continuous currents, that is, to currents whose direction and magnitude do not vary.

In this equation, E is the electromotive force of the circuit, and is generally designated for the sake of brevity as the e.m.f., R is the resistance and I is the current. The e.m.f. may be produced in various ways, as by means of a voltaic cell, by heating the junction of two dissimilar metals, by revolving coils of wire in a magnetic field, and in other ways. The e.m.f. is expressed in volts, the resistance in ohms, and the current in amperes.

3. Resistance.—The resistance of a conductor varies with the nature of the material. It is directly proportional to its length, and inversely proportional to its cross section. These relations are self-evident, and may be expressed in the formula:

$$R = K\frac{l}{a}$$

For copper, the most commonly used conductor, the value of K is 8.145×10^{-6} at a temperature of 20°C., if l is in feet and a in square inches.

However, other factors than those mentioned in the foregoing equation also affect resistance. The most important of these is the temperature. In the case of the common metals, the resistance increases with the temperature. On the other hand, the resistance of electrolytes decreases as the temperature rises, and some alloys have been produced whose resistance changes but little with the temperature. Of these latter, the most useful, perhaps, is manganin, an alloy of copper and manganese. This metal has a nearly negligible temperature coefficient.

Referring again to the metals the variation with temperature may be expressed approximately by the formula:

$$R = R_0 (1 + \alpha t)$$

in which R is the resistance at any temperature, R_0 the resistance at 0°C., and t is the temperature in degrees C. above zero. This formula is merely approximate, and for a more exact expression it would be necessary to employ terms of the second and higher degrees. For most of the pure metals, including copper, the value of α is approximately 0.00427; that is, the resistance of copper increases 0.427 per cent. per degree rise in temperature, or a rise of $2\frac{1}{2}°$ causes an increase in resistance of approximately 1 per cent.

It should be carefully noted that the resistance does *not* change with the current, as is frequently the case with similar phenomena. Thus the magnetic resistance (called reluctance) of a circuit containing iron increases with the magnetic flux.

4. Resistors in Series and in Parallel.—When a circuit contains a number of resistors connected in series, the total resistance of the circuit is the sum of the individual resistances. Thus if R_1, R_2, R_3, etc., are the different resistances and if R is the resistance of the whole circuit,

$$R = R_1 + R_2 + R_3 + \ldots \quad . \ .$$

When a circuit contains a number of resistors in parallel, the total current flowing in the circuit will be the sum of the currents in the different resistors. Thus in Fig. 29, the current in each lamp is 1 amp. and the total current flowing from the generator is 4 amp. It is supposed that the conductor connecting the different resistors has itself negligible resistance. If this is the case, it is evident that the difference of potential between the ends of the different resistors will be the same. Using the same nomenclature as before, the total current will evidently be

$E \div R$, where R is the resistance of the whole circuit, and the individual currents will be $E \div R_1$, $E \div R_2$, etc., where R_1 R_2, etc., are the resistances of the various paths. We may then write,

$$I = \frac{E}{R} = \frac{E}{R_1} + \frac{E}{R_2} + \frac{E}{R_3} + \cdot \cdot \cdot \cdot \cdot$$

And dividing by E

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdot \cdot \quad \cdot \cdot$$

or in words, the reciprocal of the resistance (the conductance) of a number of resistors in parallel is equal to the sum of the reciprocals of the resistances of the individual resistors.

Knowing the value of R_1, R_2, etc., we can readily compute the value of R from the above.

5. The Wire Table.—The diameter of commercial copper wire is measured in mils, a mil being 0.001 in. Thus a wire 0.1 in. in diameter would have a diameter of 100 mils. The cross section of wires is measured in circular mils. A circular mil is defined as the area of a circle 0.001 in. in diameter. So if a wire has a diameter of 100 mils, its area is 10,000 circular mils. To obtain the cross section of a wire in circular mils, we merely square its diameter in mils.

The sizes of the different wires are so chosen that when we pass from one wire to the one having the next smaller number, the cross section increases approximately 26 per cent.* If we pass to a wire whose number is three less the cross section is doubled. Thus No. 7 wire is double the area of No. 10. No. 4 wire has four times the cross section or double the diameter of No. 10.

It is a simple matter to reproduce approximately the whole wire table. It so happens that the resistance of a copper wire, 1 mil in diameter, and 1 ft. long is 10 ohms at a temperature of 50°F. (10°C.). No. 10 wire is nearly 100 mils in diameter (the exact diameter is 101.9 mils) and consequently has a cross section of 10,000 circular mils. Since the resistance of a wire varies directly as its length and inversely as its cross section, it follows that 1000 ft. of No. 10 has a resistance of approximately 1 ohm. The resistance of No. 11 is 26 per cent. greater, or 1.26 ohms per 1000 ft. That of No. 12 is 1.59 and that of No. 13, 2 ohms per 1000 ft. In a similar way we could readily compute the resistance of any length of any size of wire. The

foregoing applies only to the Brown & Sharp and to the American wire gages. These are the same and are the only ones used to any extent in this country.

6. Magnetism.—Magnets may be natural or artificial. Thus there is a magnetic field surrounding the earth, the lines of magnetic force extending from the region surrounding the south pole of the earth to the region surrounding the north pole. Natural magnets or loadstones, consisting of an oxide of iron, are found in nature. These loadstones are comparatively weak, so for commercial purposes use is made of pieces of hardened steel which have been made magnetic by passing current through an insulated wire wrapped around them. If the magnet must be still stronger, soft iron is substituted for the steel. The iron magnet, however, loses almost all of its magnetism as soon as the current is interrupted.

A magnet exhibits an attraction for iron, and to a small extent, for allied metals like nickel, cobalt, etc. This attractive force is somewhat concentrated in two or more regions called poles.

If the action of two magnets on one another be tried, we find that if we present in succession each pole of one magnet to one pole of the other, there is a strong attraction in one case and a repulsion in the other case. By using a third magnet, we find that two poles, both of which are attracted by one pole of the third magnet, repel one another. We then draw the conclusion that like poles repel one another and unlike poles attract.

To define strength of pole, we assume that we can have one pole of a magnet so far removed from the other that the influence of the latter is negligible, and that the pole is concentrated at a point. We say two such like poles have unit strength if they repel one another with a force of 1 dyne, if placed 1 cm. apart in air. If the poles had been unlike, they would have attracted one another with a force of 1 dyne. We can prove by experiment that the force exerted is proportional to the strength of each pole and inversely proportional to the square of the distance between them. We then have the fundamental law of magnetic attraction or repulsion,

$$F = \frac{m_1 m_2}{r^2}$$

where F is the force in dynes, r the distance apart of the poles in centimeters, and m_1 and m_2 are the strengths of the respective poles.

7. Strength of Field.—The space surrounding a magnet is called the field of force. We *define* the strength of such a field at a given point by saying it is equal to the force exerted by it on a unit pole placed at the point, and that its direction is that in which a north pole is urged. The strength of field is usually designated by the letter $\mathcal{H}$.

Flux from a Unit Pole.—Since the force at a distance of 1 cm. from a unit pole is 1 dyne, there must be a strength of field of 1 line per square centimeter at this distance. Since the area of a sphere of 1 cm. radius is 4π, it follows that the total flux of lines of force from a unit pole is 4π.

Lines of Force.—If a north magnetic pole is placed in the field of force produced by another magnet, it will tend to move in a certain direction. This direction is the direction of the field of force at this point. To a certain extent the field of force can be mapped out by drawing lines, each line indicating the direction of the field, or the direction in which a free north pole would move if subject to the influence of the field only.

8. Permeability.—If in place of air we substitute a magnetic substance such as iron, the number of lines of flux will be far greater than would be present with air. The ratio of the number of lines present with the iron, to the number present with air, is called the permeability of the iron. If we designate the two quantities respectively by $\mathcal{B}$ and $\mathcal{H}$ we have

$$\mu = \frac{\mathcal{B}}{\mathcal{H}}$$

in which μ is the permeability.

9. Generation of Electromotive Force.—It was discovered early in the history of electricity that an e.m.f. and consequently a current can be generated by the relative motion of a magnet and an electric circuit. Thus in Fig. 1, let the conductor AB be capable of sliding upon the circuit, CDE, and let there be a magnetic flux of strength $\mathcal{B}$ perpendicular to the paper. If the wire AB be moved either to the right or to the left, there will be generated in it an e.m.f. Experiment shows that the value of this e.m.f. is given by the very simple rule that it is equal in absolute units, to the number of lines cut per second. In volts it is this value divided by 100,000,000 or 10^8. If several conductors are connected in series, the e.m.f. generated will be proportionally increased. If l is the length of the conductor in

centimeters, and the average value of the flux per square centimeter is $\mathfrak{B}$, then N conductors moving with a velocity of v centimeters per second will generate an e.m.f. in volts equal to

$$E = Nl\mathfrak{B}v \div 10^8 = N\Phi_s \div 10^8 = \frac{N\Phi}{10^8 t}$$

in which Φ_s is the flux cut in 1 sec. and Φ is the total flux cut in time t. In the foregoing equation we assume that the motion is uniform. If the motion of the conductors is variable, or if the flux varies from point to point, we must find the instantaneous rate of cutting. We must then consider the very small flux cut $d\varphi$ during the very short interval of time dt and we obtain the expression

$$e = N\frac{d\varphi}{dt} \div 10^8$$

This holds true no matter how the flux or the motion varies. It is then the universal equation of the generation of e.m.f. by the cutting of lines of induction by conductors, or of conductors by lines of induction. It is entirely independent of the current flowing, and in fact it is not even necessary that the circuit be closed.

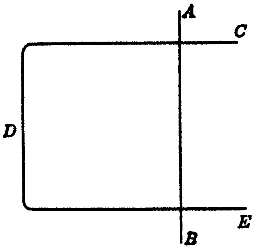

FIG. 1.

The direction of the induced e.m.f. is readily found by placing the thumb, index-finger and middle finger of the right hand, approximately at right angles to one another. If we point the index-finger in the direction of the flux, and the thumb in the direction of the motion, the middle finger will indicate the direction of the induced e.m.f.

10. Force on a Conductor in a Magnetic Field.—It was also noticed early that a conductor like that of Fig. 1 lying in a magnetic field and carrying current was subject to a force tending to move it *across* the lines of induction. In the figure as shown, this would be to the right or left. This force is proportional to the current flowing, to the number of conductors involved, to the strength of the field and to the length of the wire. We may then write,

$$F = Nl\mathfrak{B}I$$

In which l is in centimeters, I is in absolute units of current, and F is in dynes. N is the number of conductors. If I be expressed

in amperes, the force will be one-tenth as great. This force is not influenced by the material of the wire, the velocity at which it moves, etc. Indirectly, it may be influenced by these factors, since they may act to change the current or the magnetic field.

The above expression may be readily reduced to

$$F = \frac{8.85}{10^8} Nl\mathcal{B}I$$

in which F is in pounds, l is in inches, I is in amperes and $\mathcal{B}$ is in lines per square inch.

The direction in which the force is exerted may be determined by placing the thumb, index-finger and middle finger of the *left* hand, approximately at right angles to one another. If we point the index-finger in the direction of the flux, and the middle finger in the direction of the current, the thumb will indicate the direction of the motion.

11. Work.—The work done is the product of the force and the distance the conductor moves. Thus, if the conductor in the foregoing case is moved a distance, d, the work done expressed in c.g.s. units (ergs) will be (considering only one conductor)

$W = Fd = Il\mathcal{B}d = I\Phi$ or expressing I in amperes and W in joules

$$W = I\Phi \div 10^8$$

or the work done is the product of the current and the flux cut.

Power.—The power is the rate of doing work, or it is the work divided by the time, and is expressed in watts. Thus

$$P = \frac{W}{t} = \frac{I\Phi}{10^8 t} = EI$$

or the power in an electric circuit is the product of the current and the e.m.f.

12. Solenoids.—The word solenoid is derived from a Greek word signifying a pipe. A solenoid is shown in Fig. 2. If a current be passed through the turns, a magnetic flux will be set up with approximately the distribution shown. This solenoid will act much like a bar magnet. One end will attract the north pole of a permanent magnet and repel the south pole. If mounted in such a manner that it is free to move about a vertical axis, it will tend to set itself in a north and south direction in the same manner as a compass needle does. If a core of iron be substituted for the air, the magnetic effects will be greatly intensified,

and if the core be made continuous so as to surround the coils, the flux will have a continuous iron path, and it will become still greater.

The direction of the magnetic force may be determined by grasping the solenoid with the right hand, the fingers pointing in the direction of the current. The thumb will then indicate

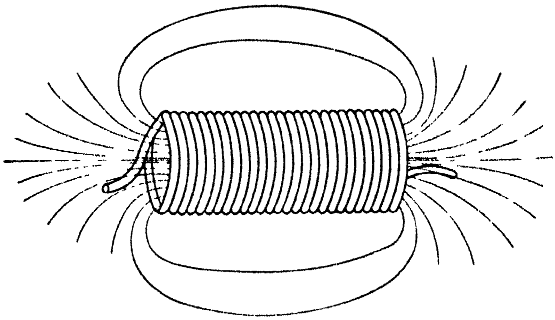

FIG. 2

the direction of the magnetic force and will point toward the north pole.

13. Force in a Solenoid.—To determine the magnetic force at any point in a solenoid, let us suppose that the solenoid is of infinite length. In Fig. 3 let a unit magnetic pole be located inside the solenoid at the end of the dotted line A. As we have previously shown, 4π lines will emanate from the unit mag-

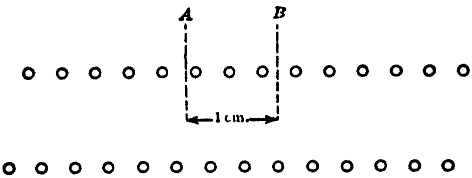

FIG. 3.

netic pole. If we have a current, I, in the wires of the solenoid, and move the pole 1 cm. along the axis of the solenoid, the work done will be the product of the lines cut, times the current, times the number of wires carrying the current or

$$W = 4\pi nI$$

in which n is the number of conductors per centimeter length of the solenoid. Since work is the product of force and distance,

and since the distance was assumed to be 1 cm., the force and the work will be the same, or $F = 4\pi nI$ or if I is in amperes, $F = 4\pi nI \div 10$.

14. Magnetomotive Force.—The magnetomotive force (in a magnetic circuit), is the line integral of the force taken around the circuit, or it is the work done in moving a unit magnetic pole around the circuit. The simplest way of deriving an expression for this is to imagine the solenoid bent around into the form of a ring as shown in Fig. 4. The mean magnetic path around the ring is l cm. The force at any point will be given by the same expression as before. The work done will be equal to the force times the length of the ring. The turns per centimeter times the length of the ring is, however, the total number of turns in the solenoid. Calling this value N we have

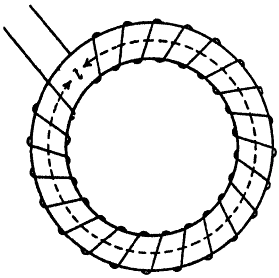

FIG. 4

$$\text{m.m.f.} = 4\pi nIl \div 10 = 4\pi NI \div 10$$
$$= 1.257NI$$

The quantity NI is called the ampere turns. The magneto-motive force is the same for a large current and a small number of turns or for a small current and a large number of turns, provided the product is the same.

15. Magnetic Circuit.—A magnetic circuit, like an electric circuit, is always closed, that is, each line of magnetic flux must return to its starting point. Thus, the *total flux* of induction across any section of the circuit must be the same, although the *flux density* in the different parts may vary widely. By the flux density we mean the number of lines of flux crossing a square centimeter, the section being taken perpendicular to the flux. We usually designate the flux density by the letter $\mathfrak{B}$, and have the relation

$$\mathfrak{B} = \frac{\Phi}{a}$$

The general law of the magnetic circuit is like that of the electric circuit and is expressed by the relation

$$\text{Flux} = \frac{\text{Magnetomotive force}}{\text{Reluctance}}$$

The method of determining the magnetotomotive force has already been explained. The reluctance is similar to the electric resistance of a circuit, and is determined by a formula of the same nature. If several different materials in series compose the magnetic circuit, we have

$$\text{Reluctance} = \frac{l_1}{\mu_1 a_1} + \frac{l_2}{\mu_2 a_2} + \ . \ . \ . \ .$$

where l_1, l_2, etc., are the lengths in centimeters of the various parts of the circuit; a_1, a_2, etc., are the areas in square centimeters; and μ_1, μ_2, are the permeabilities of the various materials. If one of these is air; μ becomes unity. We may then write the equation for the total flux in a circuit as follows:

$$\Phi = \frac{1.257NI}{\frac{l_1}{\mu_1 a_1} + \frac{l_2}{\mu_2 a_2} + \ . \ . \ .}$$

It will be noted that this is essentially the same equation used in determining the total current in an electric circuit. The more complicated form in which it is written is due to the fact that we have no convenient means of measuring directly the m.m.f. or the reluctance of a circuit, whereas we can readily measure the e.m.f. or the resistance of an electric circuit. In the magnetic circuit it is, therefore, usually necessary to *compute* the values of the m.m.f. and the reluctance.

It should also be noted that while in the electric circuit we have a great range of specific resistance, varying from that of almost perfect insulators to the low specific resistance of copper and silver, there is no such range in the magnetic reluctance of various materials. The great mass of matter falls in the one class and has a permeability the same as air, namely, unity. We have a small class of materials of which the permeability is slightly less than unity, and another small group with a permeability of more than unity. Of these, the only ones of commercial importance are iron and its alloys. These may have a permeability as high as 3000 or more, or at excessive densities nearly as low as unity.

16. Variation of Permeability.—The last sentence will serve to indicate another striking difference between magnetic reluctance and electric resistance. The latter is not at all affected by the value of the current strength. The former is decidedly so affected. In general, the permeability increases somewhat as

the flux density is gradually increased from zero to a small value. A further increase in the flux density results in a decrease in the permeability. This is well shown for several materials in the curves of Fig. 5. These also serve to indicate roughly the usual limits of the flux density per square inch. The term kilomaxwell means 1000 lines of induction.

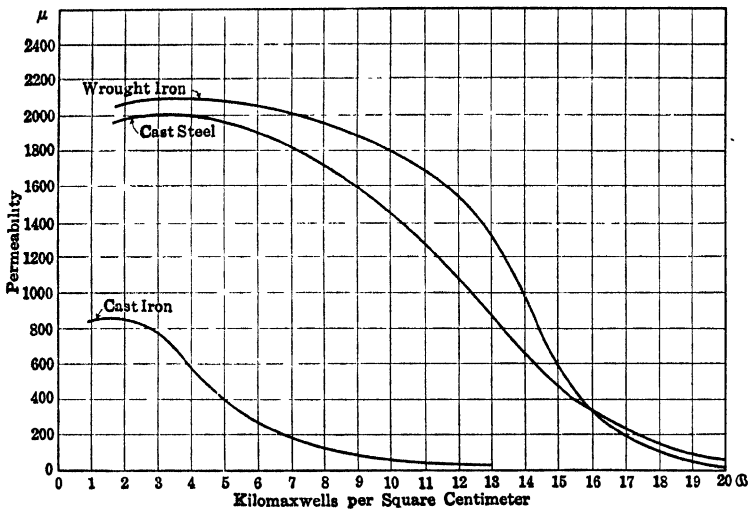

Fig. 5.

17. Magnetization Curves.—If we consider a magnetic circuit of *uniform cross section and composed of the same material throughout,* we may write the equation for the total flux in the form

$$\Phi = \frac{1.257NIa\mu}{l}$$

If we divide both sides of this equation by the area a and consider a length of only one centimeter of the circuit, we have

$$\mathcal{B} = \frac{\Phi}{a} = 1.257NI\mu$$

in which $\mathcal{B}$ is the flux density and NI is the ampere turns required to produce a flux density of $\mathcal{B}$ in 1 cm. of the material.

The most convenient way of expressing the magnetic properties of a given material is to give the ampere turns required per centimeter (or per inch) for a given number of lines per square centimeter (or per square inch).

The curves of Fig. 6 show the magnetic properties of sheet steel, cast steel, and cast iron, the materials most used in the construction of magnetic circuits. The flux is shown in lines per square inch and the magnetizing force in ampere turns per inch length.

The usual flux densities employed are about 85,000 lines per square inch in cast steel, from 90,000 to as high as 120,000 in sheet steel, and about 40,000 in cast iron. Usually it does not pay to go higher on account of the great number of ampere turns required.

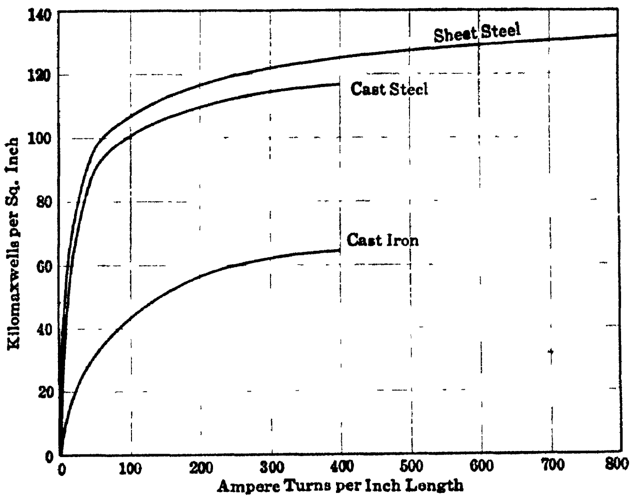

FIG. 6.

Residual Magnetism.—In Fig. 6, with zero ampere turns the flux is also zero. If, however, a piece of iron be magnetized and the exciting current then interrupted it will be found that a certain portion of the magnetism remains. In general, the amount will be greater the harder the iron. With magnet steel, prepared for this purpose, a large amount remains, so that we have a permanent magnet. All steel and iron retain some magnetism. The flux which remains in the iron is called the residual magnetism.

PROBLEMS

1. What current will an e.m.f. of 110 volts force through a resistance of 125 ohms?

2. A certain incandescent lamp connected to a 220-volt circuit takes a current of 0.333 amp. What is the resistance of the lamp?

3. Three resistors of 10, 30 and 57 ohms are connected in series. With a current of 2 amp. passing, what is the e.m.f. applied to the circuit? What is the drop over each of the resistances?

4. The armature of a certain dynamo has a resistance of 0.04 ohm at 0°C. What is its resistance at 75°C.? With a current of 50 amp. flowing, what is the power loss at 0°? At 75°?

5. In the above, what e.m.f. is required to force the current through the armature at 0°? At 75°?

6. A certain shunt-field winding has a resistance of 100 ohms at a room temperature of 30°C. With the machine hot the resistance of the field is 110 ohms. What is the temperature of the machine? With an applied e.m.f. of 220 volts, what is the power loss at 30°C. and at the operating temperature just found?

7. Two poles of strengths 10 and 15 are at a distance of 20 cm. in air. What is the attraction or repulsion between them in dynes?

8. A certain solenoid is 100 cm. long and is uniformly wound with 1000 turns of wire carrying 5 amp. What is the magnetic force in the solenoid? What is the m.m f.?

9. A ring of iron of a cross section of 2 sq. in. has a mean length of 25 in. It is uniformly wound with 2000 turns of wire carrying a current of 0 85 amp. What is the magnetic flux in the iron if its permeability is 400? What is the flux density?

10. What will be the flux in the above ring if the iron is replaced by air? By brass? By wood?

11. The foregoing ring is cut across at right angles to its axis and the ends spread so that an air gap of 0.1 in. is interposed, the length of iron remaining the same. What will be the flux, the permeability being 1000 and all other conditions remaining the same? What current will be required in order that the flux may be the same as in Example 9, the permeability being 400?

CHAPTER II

ELECTRIC MOTORS

18. Elementary Form of Electric Motor.—One of the simplest possible forms of the electric motor is illustrated in Fig. 7. It is hardly suitable for commercial use, but serves well to illustrate some of the principles already studied. Motors of this type are frequently sold as toys. The rotating part consists of a cross-shaped piece of soft iron or steel. While the one shown in the illustration has four arms, either a greater or a lesser number may be used. The rotating part is carried on a shaft supported in suitable bearings, and rotates between the poles of

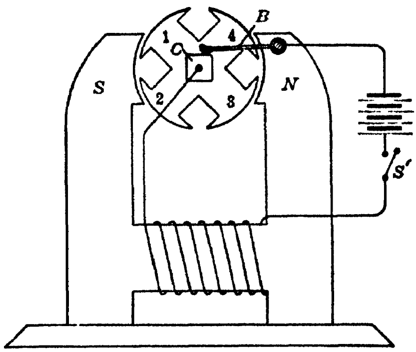

Fig. 7.

an electromagnet $N - S$. The magnet is wound with wire as shown. The current may be supplied from a battery or other suitable source. The path of the current is from the battery to a spring or brush B, through the cam C to the shaft, and by means of a sliding contact to the winding of the magnet through the switch S', and so back to the battery.

In the position shown the brush is just about to make contact with the cam, thus closing the circuit. As soon as this occurs the projection 1 will be attracted to the pole S and the projection 3 to the pole N. This will continue until 1 and 3 are directly in

line with N and S. At this instant the cam should have rotated just far enough so that the contact between itself and the brush is broken. The rotating part will then be carried on by its momentum until pole 4 is in about the same position as that occupied by pole 1 in the figure. At this instant the contact will again be made by the cam and the process will be repeated.

It will be noted that the torque or turning effort on the shaft of this motor is not constant but is, in fact, zero during a part of the revolution. This in itself would not be a serious matter, at least at reasonably high speeds, because the momentum of the rotating part would be sufficient to keep the motion practically uniform. The fact, however, that the torque is not constant means that the motor will not always be self-starting. If it stopped in such a position that the brush was in contact with the spring it would start itself when the switch was closed, but if the brush was not in contact no current would pass and there would be no tendency to turn. Perhaps an even greater defect of this motor from the practical standpoint is the fact that it sparks badly at the contact between the brush and the cam. A current flowing in a wire possesses a property very similar to a mass of matter in motion, namely, it resists very strongly any tendency to force it to stop suddenly. Matter manifests this property by the development of great force and heat; the electric current by the development of an electric arc and, of course, also by the production of heat. This phenomenon will be more fully treated later.

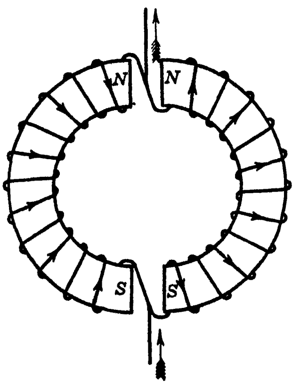

FIG. 8.

19. The Armature.—The ordinary electric motor is a more advanced type than the one just considered. While the machine will be described as a motor, the same machine *without any change whatever* will also operate as an *electric generator*. The reason for this will appear presently. In fact, this is a general rule. It will be found that any motor, whether for alternating current or for direct current, will also operate as a generator.

In Fig. 8 is shown a split ring wound with insulated wire. It will be seen that the wire is wound continuously around the

ring in the same direction. At two opposite points connections are made to the wires from a battery or other source of continuous current. As shown, the positive pole of the battery is connected to the lower lead. Here the current divides into two equal parts, half flowing through the left half of the ring and half through the right half. If we consider for the moment only one-half of the ring, say the left half, it is apparent that it will become

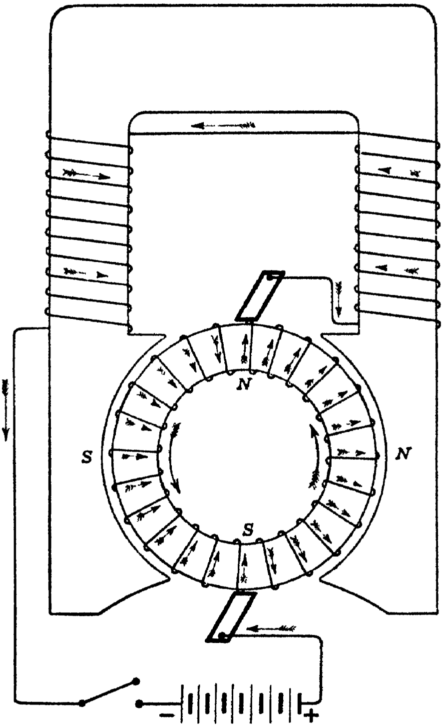

Fig. 9.

a magnet as soon as the current passes through the winding, and by making use of the rule that if we grasp the ring with the right hand, the fingers pointing in the direction of the current flow, the thumb will indicate the north pole, it will be seen that the upper end of the ring will be a north pole and the lower end a south pole.

If we consider the right half of the ring and apply the same rule to it, it will be apparent that the upper pole is likewise north and

the lower pole south. If now the ring should be joined at the split part so as to form a complete ring, we should have a north pole at the top and a south pole at the bottom, each of these being due to the combined action of the two poles present in the split ring.

In Fig. 9 is shown a ring of this character. The ring is so mounted on a shaft that it is free to rotate. Brushes marked + and − are arranged to make contact with the winding even though the ring is rotating. It is supposed that the wire is so arranged that the surface is smooth, so that the brush may make good contact at all times. If current is passed through the winding of the ring by means of the brushes, it will be seen that we shall have a N pole at the top of the ring and a S pole at the bottom. This will be true *even though the ring is in motion.* The poles will therefore *stand still* while the ring rotates. A ring arranged in this manner is called *an armature.*

20. The Field Magnet.—Surrounding the armature are shown the poles of a horseshoe magnet. This is commonly called the *field magnet* or simply the *field.* The current is carried a number of times around the field as shown, thus strongly magnetizing it. This is called a *series* connection and the motor is known as a *series motor.*

21. Operation as a Motor.—The action of the machine as a motor will now be readily understood. When current is passed through it, the upper part of the armature will become a N pole and the lower part a S pole. At the same time we have a N and a S pole in the field. It will be evident that the S pole of the armature will be attracted toward the N pole of the field, and the N pole of the armature toward the S pole of the field. At the same time the N pole of the field will repel the N pole of the armature and the S pole of the field will repel the S pole of the armature. All these four actions tend to *turn the armature in the same direction.* The top of the armature will move to the left as shown, or the armature will turn in a counter-clockwise direction.

When the armature first starts to turn, its poles will move a short distance with it. However, before the armature has turned through any great angle, the brush will cease to make contact with the wire it touched at first and the next turn will slide under the brush. As soon as this takes place the pole will move backward to the position it occupied originally. Thus

no matter how rapidly the armature may rotate the pole will stand nearly still, having merely a slight backward and forward motion. The machine will therefore continue to operate as a motor as long as current is supplied to it.

It will also be seen that this motor will have no dead points, that is to say, it will start from rest no matter what the position of the armature. Therefore it is greatly superior in this respect to the motor illustrated in Fig. 6. Moreover, the turning effort (which we call torque) will be constant during the entire rotation. This, although a minor point, is of importance in some applications.

22. The Commutator.—A few years ago many machines were constructed with the brushes trailing upon the winding exactly as

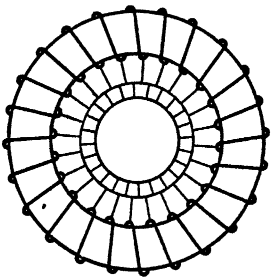

Fig. 10.

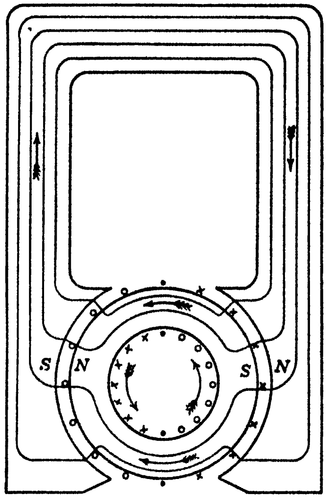

Fig. 11.

shown. Others were constructed with the brushes bearing upon the side of the winding. At the present time, this construction is rarely if ever used. It has been found cheaper and equally as efficient to provide a separate structure to make contact with the brushes. This structure is known as the *commutator.* Figure 10 shows an armature with a commutator. The latter consists of a number of bars of copper mounted upon a sleeve. All the bars are insulated from one another and from the sleeve. Each bar is connected to a turn of the winding. There may be as many bars as there are turns in the windings, at least in the case of large machines. In small machines there are frequently a great many turns for each commutator bar. The action of the machine is exactly the same as before.

23. Action of the Forces on the Conductors of a Motor.—The foregoing way of looking at the action of a direct-current motor, while simple, is open to some objection. For example, it is entirely possible to neutralize the "poles" of the armature by means of a compensating winding (see Art. 294), and the motor will still operate. Figure 11 will serve to give a more accurate idea of the actions which actually take place.

The currents in the armature are represented by small crosses and by small circles. The cross indicates a current from the observer; the circle a current toward him. The reader may readily remember this by considering the cross as the feathers on the end of an arrow. It will be seen that the currents flow in the same direction as in Fig. 9.

24. Force Acting upon a Current in a Magnetic Field.—To understand the action of these currents upon the magnetism,

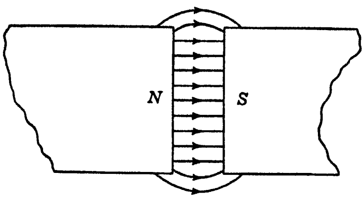

FIG. 12.

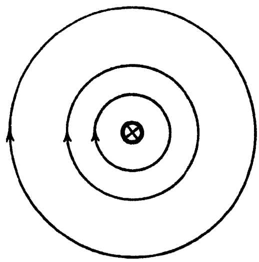

FIG. 13.

let us consider Figs. 12, 13 and 14. In Fig. 12 is shown the magnetic field between two opposite poles, the direction of the flux being from north to south as shown. Figure 13 shows the magnetic field which surrounds a conductor carrying a current of electricity. The lines of flux are circles having the direction shown and placed closer together near the wire. Figure 14 shows the field produced by the combination of these two elements. The lines of magnetism are close together just above the wire since the magnetic effect of the current in the wire and the magnetic field are both in the same direction. Just below the wire there is little or no magnetism since the effects are in opposite directions. Since the lines of magnetism act like stretched elastic bands, there will be a force acting on the wire and tending to force it downward out of the field. There will be an equal and opposite force tending to force the magnet

upward. Either the wire or magnet or both may move, depending upon which one is free.

The force which acts upon the wire and the magnet will be proportional to the strength of the magnetic field, to the current in the wire and to the length of the wire that is exposed to the magnetic field. Obviously, where a number of wires are acting, the total force will also be proportional to the number of wires.

Returning now to Fig. 11 we can readily see why the armature should rotate. All the wires in the gap on the left are carrying current toward the observer, and all will be pushed downward by the magnetic field. All of those on the right are carrying current from the observer, and all will be pushed upward. Both of these actions tend to turn the armature in a counter-clockwise direction as shown Since the position of the currents is not changed as the armature rotates, the action will be continuous.

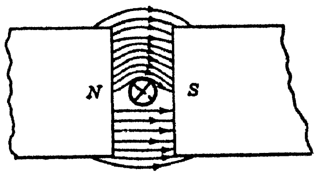

Fig. 14.

It will be seen that the conductors on the inside of the armature are not in the magnetic field. It is true that a few lines of magnetism may leak across this space, but the field will be very weak and practically negligible. Since these conductors are not in the field the force upon them will be zero. Therefore they have no part in the action except that they carry the current from one active conductor to the next.

25. Multipolar Machines.—All the machines we have considered have had two poles. Commercial machines, except in the smallest sizes, have in general four or more poles. This leads to a more symmetrical structure and is stronger and better mechanically. It can also be shown that the weight of iron or steel in the field is greatly reduced, thus leading to a cheaper and lighter machine. A multipolar machine is illustrated in Fig. 18.

26. Construction of the Drum-wound Armature.—The ring winding just described has one fatal defect which has caused it to become practically obsolete. In winding, it is necessary to carry each of the wires through the armature, one at a time, by hand. This is a slow and laborious process not adapted to modern methods of manufacture. The winding that is in almost universal use is called the drum winding. All the coils are

wound, and in most cases fully insulated, before being placed on the outside of the armature core. The appearance of typical coils may be seen from Fig. 15. There may be only one turn of wire in a coil although several are usual.

The surface of the armature is practically always slotted and

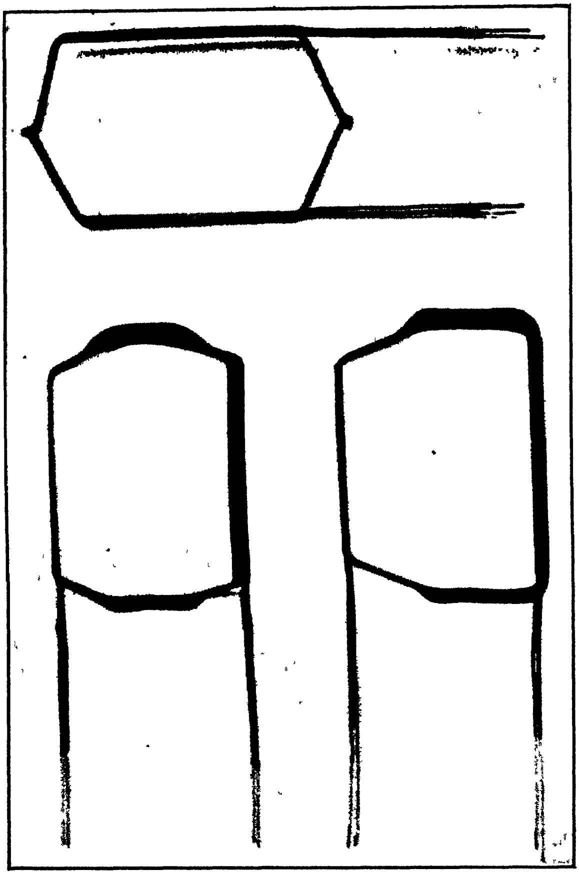

FIG. 15.

the coils are placed in the slots. There are a number of reasons for doing this. Winding is somewhat easier since the coils are held firmly in place while the connections are being made. The completed armature is far less likely to be injured by being placed on the floor or by being struck accidentally. Perhaps the most

obvious advantage of the slotted construction is the fact that the air gap may be made far shorter than would be possible if the coils were placed on the outside. It will be remembered that the permeability of air is unity, while that of iron at ordinary flux densities is about 1000. Therefore a far greater number of ampere turns is required to force flux at a given flux density across an air gap than is required for the same flux density through the same length of iron. Hence it is desirable that the gap be kept short so that the number of turns required upon the field may be small. However, one should not infer from the foregoing that it is desirable to make the gap as short as it can possibly be made and still give the necessary clearance. If this were done the number of turns required upon the field to force the flux across the gap would be small compared with the number of turns upon the armature, and in consequence the distortion of the flux would be too great. Consequently, the gap is made comparatively great, but by no means so long as would be necessary if the winding were placed upon the outside of the armature.

27. Connections of a Lap-wound or Parallel-wound Armature.—The number of turns in a coil is usually such that a coil fills only half the available space in a slot. Each slot, therefore, contains one side of each of two coils. The coils are arranged in a perfectly symmetrical manner so that one side of each coil occupies the upper half of a slot while the other side is in the lower half of another slot, distant approximately one pole pitch. The coils are so connected to the commutator that if the beginning of a coil is connected to bar No. 1, the end is connected to bar No. 2. The beginning of the next coil is then also connected to bar No. 2 and the other end of this second coil to bar No. 3. This constitutes a lap winding.

These connections are clearly shown in Fig. 16. This is supposed to represent a four-pole armature flattened out, or we may think of it as being a portion of a very large armature having a great number of poles, so great in fact that a small section of the armature is practically straight. Starting from one of the brushes, say A, the current divides into two parts, half going to the left and half to the right. If we trace the path of the part going to the left, it will be seen that after passing through the coil 1 the current arrives at the next commutator bar to the right of the one from which it started. This bar is insulated from all the other bars and the only possible path for the current is through

the coil 2. Passing through this coil in the same way, it arrives at the third commutator bar. As the current may continue in this way it is evident that it will finally arrive at the brush *B*, and be free to pass out.

In the same way the current passing to the right may be traced. It will be found that each time the current passes through a coil it moves one commutator bar to the left and finally arrives at the brush *B′*.

Alternate brushes are connected together as shown so that the total current going to the machine divides into as many parts as there are pairs of brushes and, as stated, each of these parts again divides into two parts at the brush. Finally all the currents

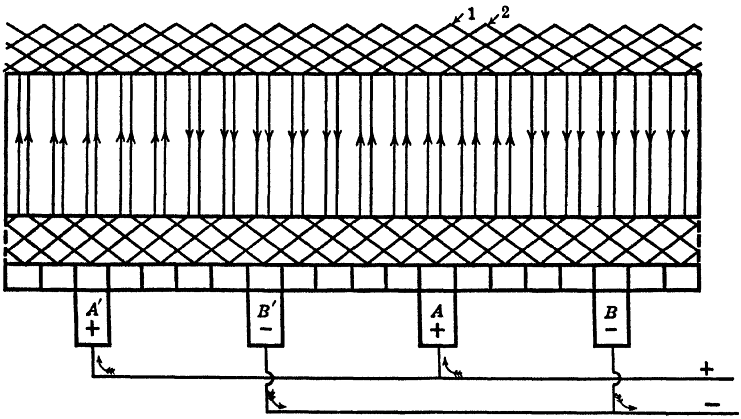

FIG. 16.

come out again at the proper brushes and are combined into a single current in the main leads.

If while tracing through the windings in this way we place arrows on the respective conductors to indicate the direction of the currents, the directions will be as shown in Fig. 16. Considering the armature as a whole, we have a number of broad bands of current, that is, all the individual currents in each band are in the same direction. The span of the coils and the positions of the brushes are such that the width of each of these bands is just the width of a pole span of the field, that is, there are as many bands as there are poles in the field. If the brushes are so placed that the point at which the band changes direction comes opposite the space between two poles, we shall have the

same condition as shown in Figs. 9 and 11, namely, a band or current lying under each pole. Each of these bands will be pushed in the same direction by the action of the field, and consequently the armature as a whole will tend to rotate or the machine will act as a motor.

28. The Wave Winding.—The lap winding or parallel winding described is the one in most common use, particularly in large machines. Many of the smaller machines have what is known as a wave or series winding. In this type of winding, the ends of a coil instead of being connected to adjacent commutator bars are connected to bars whose distance apart corresponds to nearly twice the distance between two poles. The connections are shown in Fig. 17. Starting from one of the brushes, say A, and

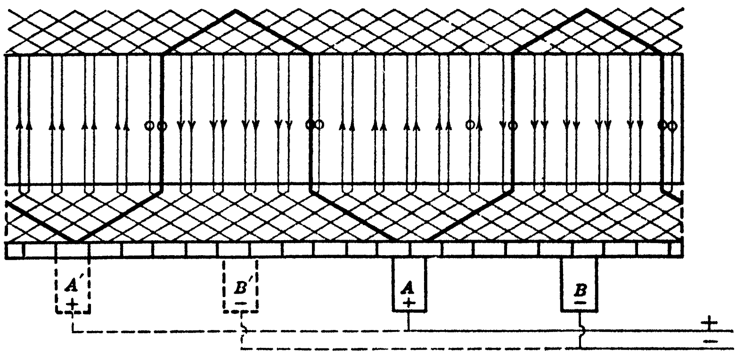

FIG. 17.

tracing the winding through to the brush B, it will be found that there will be current in all of the coils (with the exception of those short-circuited by the brushes), even though we ignore entirely the brushes A' and B'. Consequently, the machine will operate properly even though only two brushes are used. In small machines it is customary to use the two brushes only. In larger machines, where the current to be carried is large, one brush to each pole is usually employed. In this way sufficient contact area between the brushes and the commutator is obtained without making the commutator too long.

In some classes of motors, notably those used on railway cars, it is highly desirable that the brushes be readily accessible for inspection or renewal. With traction motors, therefore, it is

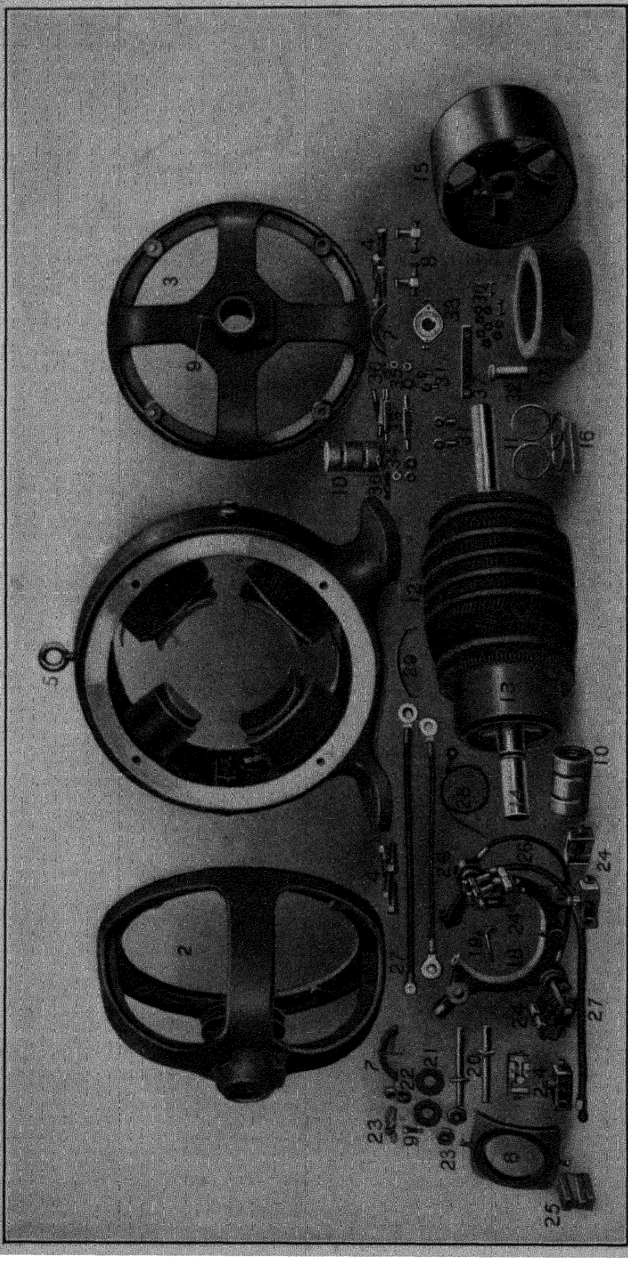

Fig. 18.

customary to use only two brushes. They are placed on the upper part of the commutator so that they may be easily reached through the car floor.

It will be seen that some of the conductors in Fig. 17 have no arrows placed on them. The reason is that the coils of which these conductors form a part are for the moment short-circuited by one of the brushes. The current is therefore in the act of reversing, and it would be difficult to say what the direction of the current is at the instant. While the lap winding of Fig. 16 shows twenty slots, twenty coils and twenty commutator bars, *any* number might have been used. This will be readily apparent if the reader will actually construct the diagram, using say twenty-one slots and the corresponding number of coils and bars.

29. Number of Coils in Wave-wound Armature.—It is impossible, however, to construct the *wave* winding of Fig. 17 with twenty coils, since the winding would close on itself the first time around. We are, therefore, forced to use an odd number and twenty-one was chosen. With six poles, the number might be either odd or even, but it would have to be a number made up by multiplying some integer by three and either adding or subtracting one. For a larger number of poles a corresponding rule would hold, namely, the number of coils must be obtained by multiplying an integer by half the number of poles and adding or subtracting one. We may express this by the equation,

$$N = \frac{n}{2} y \pm 1$$

where

N = number of coils or commutator bars,
n = number of poles,
y = pitch of winding.

30. Construction of Small Motor.—Fig. 18 shows the parts of a small motor or generator. The machine is of the four-pole type. The armature is slotted and the coils are beneath the surface of the iron. The armature is wave wound and four brushes are used. The bearings are of the ring-oiled type and are supported in cast-iron housings. The pole pieces are detachable so that a field coil can be readily removed if necessary.

CHAPTER III

GENERAL PRINCIPLES OF DYNAMOS AND MOTORS

31. Classification of Dynamos and Motors.—A dynamo-electric machine is an apparatus for converting mechanical energy into electrical energy or *vice versa.* The former is a dynamo or a generator, the latter is a motor. This definition would, strictly speaking, include static machines, but these are ordinarily considered separately.

It was once a common practice to classify electrical machines as generators and motors. However, this is not advisable when considering the subject in its broad aspects since any electrical machine may act as either a generator or a motor. Many machines are so operated. This applies even to such machines as the induction motor, and the static machine, although we do not frequently use the former as a generator or the latter as a motor.

Again, the classification into direct- and alternating-current machines was once satisfactory as one kind could always be distinguished from the other by the presence or absence of a commutator. To-day, however, we have many alternating-current motors provided with commutators and some direct-current generators without commutators. Hence this classification is also somewhat unsatisfactory. Perhaps the classification given below is as good a one as can be devised at the present time.

1. *Commutating Machines (Generally Continuous Current).*—These machines usually generate (or use, if employed as motors) a uniform current, that is, the strength of the current does not change materially, except at comparatively long intervals when the load is changed. Some alternating-current machines are however of the commutating type.

2. *Synchronous Machines, or Alternators.*—These machines generate or consume a current which is rapidly changing its direction, so that it flows alternately in one direction and in the other. The machines may have a single winding generating or consuming one current, in which case they are called

single-phase machines, or they may have two or more windings, in which case they are called polyphase synchronous machines.

3. *Rectifying Machines.*—These would be classed by many authors with commutating machines. They, however, generate primarily an alternating current either single phase or polyphase, and this is rectified by means of a commutator with few bars. The current, therefore, is not entirely steady, but has a rapid pulsation in value, although it does not reverse in direction. Such machines were formerly used in arc lighting, but are not in general use now.

4. *Induction Machines.*—These differ from the synchronous machines principally in the fact that the field, instead of being excited by means of a direct current, is excited by alternating currents. This gives the machine the characteristic that instead of operating at constant speed like a synchronous machine, its speed decreases with load as a motor and increases as a generator.

5. *Unipolar or Acyclic Machines.*—These are machines generating a continuous current, but so arranged that the conductors revolve at all times in a field of the same sign. This avoids the need for a commutator.

32. General Principles.—In all dynamos, whether used as generators or as motors, there are two principal elements which the author calls the Flux Sheet and the Current Sheet.[1] Since every element of current lying in a magnetic field is acted upon by the field in such a manner as to tend to force it across the field, it follows that if the current sheet is arranged to be perpendicular to the flux sheet, the current sheet as a whole will be forced in the one direction or the other across the flux sheet. Of course, the same force that is exerted on the current sheet will also be exerted on the flux sheet in the reverse direction. Thus either one or both may move. This movement is generally a circular motion about an axis or shaft. The direction of motion may be found by means of the rule of Art. 10.

If the machine is allowed to rotate in the direction in which the current sheet tends to move, the machine acts as a motor and consumes electrical power. If, on the other hand, the machine is forced to rotate in the opposite direction, it becomes a generator, consuming mechanical power and giving out electrical power.

[1] The term "sheet" is not entirely satisfactory since neither the flux nor the current is in exactly the form of a sheet. The meaning will, however, be evident.

In order that the torque and consequently the power of the machine may be as great as possible, it is necessary that the flux and the current sheets retain the same relative position. In general, there will be several flux sheets of alternately reversed polarity, and a corresponding number of current sheets, also of alternately reversed direction. Thus Fig. 19 illustrates the general distribution of the flux and the current sheets of a direct-current motor or generator. To obtain the maximum power from the machine it is necessary that the current sheets should change polarity at approximately half way between the flux sheets. For example, Fig. 19 shows that all the conductors between the lines A and B should carry current in the same direction, if the maximum torque is to be developed, since all lie in a magnetic field of the same sign. If the current in some of them were

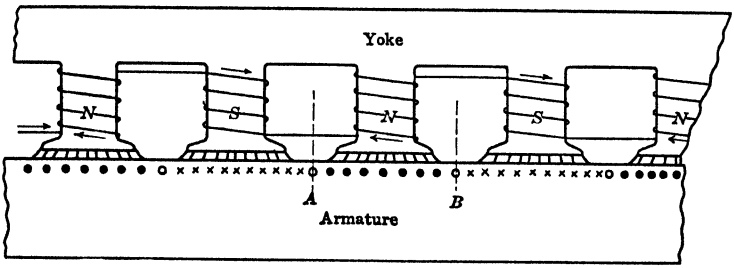

FIG. 19.

reversed, the pull of those conductors would be exerted in the reverse direction to that of the others, and the output of the machine as a motor would be decreased. It would likewise be decreased if the machine were operating as a generator, since it will be apparent that the e.m.f. induced in all of the conductors from A to B will be in the same direction. For action as a generator the current must flow in the same direction as the induced e.m.f. Consequently, the current should be in the same direction in all of the conductors.

The arrangement of the flux sheets and the current sheets shown in Fig. 19 is that of a direct-current generator or motor, The same arrangement in its essential features is common to a great variety of electrical machines. In fact, all machines both direct and alternating current, with the exception of the unipolar machine, employ essentially this arrangement. The principal

difference between the various types of machines comes from the different arrangements necessary to obtain this distribution.

Coming back to the case of the continuous-current machine, as shown in Fig. 19 it will be seen that the flux is produced by a number of poles. The flux passes through each of the cores and divides into two equal parts in the armature and also in the yoke of the field. Any even number of poles may be employed. However, machines are rarely built with less than four poles except in the smallest sizes.

To produce this flux, each pole is surrounded by a coil of wire. The different coils are usually connected all in series, although other groupings may be used if more convenient. It is also common to employ two coils per pole, in order to improve the action of the machine in certain respects. The current for the coils may be supplied in various ways as will be described later.

33. Commutators.—With a distribution of the flux in bands of alternately opposite direction, it will be obvious that we may produce a current sheet, stationary with respect to the flux sheet by passing *alternating* currents into the armature, care being taken to operate the machine at such a speed, that *the current reverses in each conductor once for each field pole passed.* The most advantageous position in which to reverse the current is when each conductor is in a position midway between the field poles. The best means for effecting this will be gone into later in connection with the synchronous machine.

In the direct-current machine, however, with the indicated distribution of the flux, it is necessary to use a commutator. Resting on the commutator are the brushes. These are usually of carbon or graphite, but in low-voltage machines, are sometimes composed of a mixture of ground copper and graphite. Usually, as many brushes or sets of brushes are used as there are field poles, although a smaller number are used for certain classes of windings.

Two of the simplest methods of connecting the conductors to the commutator bars have been shown in Figs. 16 and 17. Each coil may consist of a single turn or there may be several turns per coil.

If the two main conductors marked + and − are connected to a source of continuous current of corresponding polarity, current will flow into all of the brushes marked + and will flow out of all of those marked − . By tracing through the connec-

tions it will be evident that the direction of the currents in the individual conductors will be as shown in Figs. 16 and 17. The current will then be distributed in a series of bands, each band comprising ten conductors in the example shown. In practice there are always many more conductors in a band than this number. As the armature revolves, since the brushes are stationary, the bands of current will also be stationary. The current in the individual conductors will, however, not be constant, but will reverse in direction each time that the commutator bar connected to a conductor passes under a brush.

If an armature of this type be placed in a magnetic field as shown in Fig. 19, and if the brushes are in such a relation to the field structure that the points at which the current sheet changes sign are approximately midway between the poles, it will exert a torque and if not restrained will turn continuously in the one direction or the other, operating as a motor. The torque developed will be in proportion to the strength of the current in the armature and to the strength of the magnetic field.

34. Action as a Generator.—If instead of being supplied with current from some outside source and acting as a motor, the machine be rotated by power applied through the shaft, it will automatically become a generator capable of supplying power to an external circuit. Figure 19 shows that all of the conductors in the band between the points A and B are cutting the flux from the pole N in the same direction and consequently all of them will generate an e.m.f. in the same direction. The strength of the induced e.m.f. will be somewhat different in the different conductors, and in fact will be nearly zero in those near the points A and B but all the e.m.fs. in the band will be in the same direction. All of these conductors are connected in series with one another and are connected to the external circuit through the brushes. Figures 16 and 17 show that starting from any one of the brushes and tracing through the winding to the next brush, all of the e.m.fs. will be acting in the same direction. All of them will therefore add together giving a terminal e.m.f. equal to the sum of their individual values. No matter which path is selected, the number of conductors in series is the same. Consequently the e.m.f. generated will be the same in all of them, provided the field poles are of equal strength.

It will, moreover, be noticed that starting from any brush, there are two possible paths. No matter which of those are chosen,

an e.m.f. will be encountered in the same direction. All of the paths are in parallel, and since their e.m.fs. are the same, the action is similar to that of a corresponding number of primary or storage cells connected in parallel. The external voltage is that of *one path only*. The effect of the additional paths is to *reduce the resistance of the armature*. The current-carrying capacity is therefore in proportion to the number of these paths.

If when such a machine is in action, the terminals + and − are connected through a suitable resistor, a current will flow and the machine will act as a generator, changing mechanical power into electrical power. As soon as the current passes, there is, as in the case of the motor, a torque between the armature and the field. As before, this torque will be in proportion to the strength of the current and to the strength of the magnetic field. The engine or other prime mover is therefore required to exert a greater torque as the current increases, and in consequence requires an increased supply of steam, gas, water, etc.

35. Back E.M.F.—Referring again to the direct-current motor, it will be recalled that nothing was said about the e.m.f. It is apparent that in both the motor and generator, the conductors will cut the field flux, and consequently will generate an e.m.f. In the generator, the flow of current is in the same direction as the e.m.f., since it flows because of the generated e.m.f. In the motor, however, the current and the e.m.f. are in opposite directions, since the current must be reversed to give torque in the direction of motion.

The back e.m.f. in a motor therefore tends to cut down the current passing into the motor.

This phenomenon can be readily observed by connecting in series a small motor, a few cells of battery and an ammeter. If the motor be prevented from rotating, the current may be 25 amp. As soon as the motor is released and starts to rotate, the current will decrease, and if the motor is allowed to run without load, the current may drop to 5 amp. When this fact was first discovered, it was considered that it was very unfortunate since it apparently seriously limited the power of the motor. It is now known that the production of this back e.m.f. is absolutely essential to the working of the motor. The output of the machine is, in fact, proportional to the value of the back e.m.f. and without it no power would be developed. Neglecting certain other losses the efficiency of the motor is the quotient

of the back e.m.f. divided by the applied e.m.f. Therefore, in order that the efficiency of the machine may be high, it is essential that the back e.m.f. of the motor be nearly equal to the applied voltage. The current that flows is then only a small fraction of that which would flow if the motor were at rest and the same e.m.f. were applied. Hence the motor develops normally only a small percentage of its maximum torque.

36. Calculation of E.M.F.—The most important equation of the continuous-current generator or motor is that connecting the generated e.m.f. with the flux, the speed, the number of poles and the number of conductors on the armature. The calculation may be made very readily without the use of a formula. Assume a six-pole generator operating at a speed of 600 r.p.m. Let the flux per pole be 1,000,000 lines and assume the total number of conductors on the armature to be 840. The armature is supposed to have a two-path wave winding as shown in Fig. 17. The e.m.f. generated is simply the number of lines cut in a second, divided by 10^8. In one revolution one conductor will cut 6,000,000 lines and in 1 sec. it will cut 60,000,000 lines. In a two-path wave winding half the conductors are *in series* and the total cutting per second is therefore 420 times as great or 252×10^8. This is the generated e.m.f. in absolute units, or the machine is generating 252 volts.

If the armature had been lap wound, as shown in Fig. 16, the number of paths through it would have been six instead of two or we should have had 140 conductors in series. The generated voltage would have been one-third as great or 84 volts. It should be carefully noted that in this case the armature would have been capable of carrying three times as much current as it could carry wave wound, or the capacity of the machine would be the same in the two cases.

The foregoing facts can be readily expressed by means of a formula. Let Φ be the flux per pole, n the number of revolutions per second, N' the total number of conductors on the armature, P the number of poles and P' the number of paths through the winding. The number of conductors in series is $N' \div P'$, the cutting per revolution, $\frac{N'\Phi P}{P'}$, and the generated e.m.f. is given by the formula $E = \frac{\Phi N' n P}{10^8 P'}$. For simplicity we shall frequently substitute $N = \frac{N'P}{P'}$, in the above and write $E = \frac{\Phi n N}{10^8}$. Fre-

quently the number of paths is the same as the number of poles and in this case $P = P'$.

37. Methods of Field Excitation.—*The Magneto Machine and the Separately Excited Machine.*—The most obvious method of constructing a motor or generator is with a permanent magnet for the field. The chief objection to this is the high cost of the steel used in such magnets. This alone would make the cost of a machine of reasonable size prohibitive. In addition, it would be impossible to produce in this manner machines with as strong fields as are now considered advisable. The output for a given size would therefore be low and the cost per kilowatt correspondingly high. It would also be difficult to provide proper means for varying the voltage of such a machine, since the strength of the field could not readily be changed. These

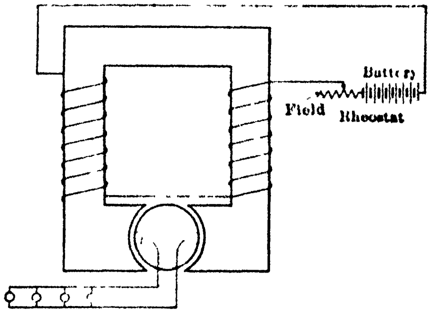

FIG. 20

objections have such weight that permanent magnets are never used, except in the case of small generators used for ringing telephone bells, or for the ignition of internal combustion engines. In these cases, the fact that the field is always present and is independent of the speed, is of more importance than the low ouput and high cost of such machines.

Figure 20 shows a separately excited machine. The source of current for the field may be a battery, a small generator supplied for this purpose or other source of continuous current. The brushes as shown are placed 90° from the position indicated in Fig. 9, which had reference to a ring-wound armature. Separate field excitation is almost universal in the case of synchronous machines, since on account of the alternating character of the current generated, it is necessary to provide some other source

of direct current than the machine itself. It is but rarely employed in continuous-current machines, and then generally in connection with some special method of regulation.

In most continuous-current machines, the machine generates the current to excite its fields, or if a motor, takes the field current from the same circuit as that for the armature. While it appears to be almost obvious that this may be done, the early builders of dynamos were very slow to appreciate this possibility and dynamo machines went through a long evolution before they were generally so built.

38. The Shunt-wound Machine.—The field coils of self-exciting dynamos may be connected in one of the three methods shown in Figs. 21, 22, and 23. The first method is known as the shunt connection. The current supplied to the field will be equal

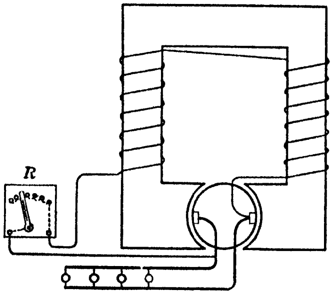

FIG. 21.

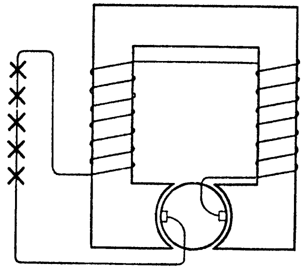

FIG. 22.

to the terminal e.m.f. of the machine divided by the resistance of the field circuit. If the terminal voltage of the machine is constant, the current through the shunt winding will be constant, and consequently the magnetic flux passing through the field and armature will also be constant. This type of winding is therefore particularly applicable to machines intended to deliver a substantially constant voltage.

When a shunt dynamo is at rest, there is of course no current in either the field or armature windings. When the armature is rotated there is generated in it a feeble e.m.f. due to the fact that some residual magnetism is left in the field. This small e.m.f. causes a current to flow through the field winding. This in turn increases the field, thus again increasing the e.m.f. This action continues until the field reaches a certain point of saturation. This action is called "building up."

39. The Series-wound Machine.—In series-wound machines (see Fig. 22), the whole current generated by the machine passes through a few turns of comparatively coarse wire. The field current is therefore proportional to the current which the machine is generating. The field flux will increase as the current output of the machine increases although not so rapidly as the latter on account of magnetic saturation. Hence the voltage of such a machine will increase as the load on the machine is increased.

40. The Compound Wound Machine.—The compound winding shown in Fig. 23 employs a combination of the shunt and the series windings. In general the series turns are so connected that they help the shunt turns. The machine will, therefore,

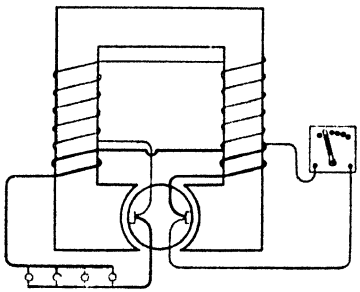

FIG. 23.

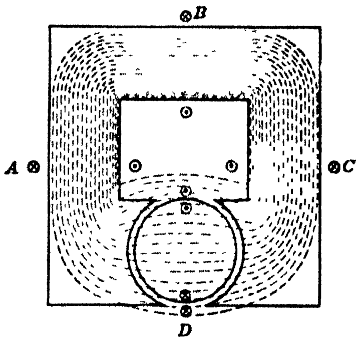

FIG. 24.

have intermediate characteristics, and will generally increase somewhat in voltage as the load is increased. The exact effect of these windings is treated in more detail later.

41. Magnetic Effect of the Armature.—Before considering more fully the voltage regulation of direct-current machines and the speed regulation of motors, it is necessary to investigate the magnetizing effect of the armature as well as that of the field winding. In general, it may be said that *all* of the current passing through a machine has a part in setting up the magnetization. In a continuous-current machine, the arrangement is purposely such that the magnetizing effect of the armature is a minimum. In other types of machines, as the synchronous machines, its effect may in many cases be very great, and in the induction motor, the effective magnetizing effect is due to the

difference between that of the stator (armature) and that of the rotor (field), so that the former is always the greater.

Figure 24 shows a section of a continuous-current machine. For simplicity the machine represented is bipolar. The path of the magnetic lines is approximately as shown. It will be seen that not all of the lines which pass through the field pass through the armature also. In order to force the lines across the air gap, it is necessary that a large portion of the magnetizing effect of the machine be concentrated at the air gap. On this account, and on account of the crowding of the lines of induction, some of the lines find it easier to take the path shown from pole to pole without going through the armature at all. In general, from 10 to 20 per cent. more lines will pass through the fields than through the armature.

Considering Fig. 24 again and neglecting the effect of magnetic leakage, the magnetizing effect of a certain number of ampere turns upon the magnetic circuit is the same *no matter where the turns are placed.* The most common location is at the points A and C the coil being divided into two equal parts, and half placed on each side. It is, however, entirely possible to use only one coil located at B, and many bipolar dynamos are so constructed. In multipolar machines, this location leads to great mechanical difficulties and is not used.

It would also be entirely possible to make use of a stationary coil slightly larger than the armature and located at the point D (so as to surround the armature). This is occasionally though rarely done.

42. Armature Reaction.—It is apparent that a turn located on the armature itself in the position D will have the same magnetizing effect while it is in the position shown as though it were stationary. This fact is at the bottom of the idea of armature reaction. In the actual machine, the turns are located as shown in Fig. 25. The dots represent currents coming toward the observer; the crosses, currents flowing from him. The angle which the line AB makes with the vertical, depends upon the position of the brushes. When the brushes are in such a position as to give the distribution shown, they are said to be in the neutral position, and the magnetizing effect of the armature upon the field is nearly zero. Thus considering any particular conductor as C there will be another conductor D symmetrically located

upon the armature, and carrying current in the opposite direction. Therefore the net effect of the two conductors will be nearly zero.

If, on the other hand, the brushes in Fig. 25 had been rocked 90° from the neutral position, the armature would have had its maximum magnetizing effect. Instead of being able to find for each conductor another conductor which would offset its action, we should be able to find for each conductor another so located as to help the magnetizing action of the first. Consequently, with the brushes in this position, the armature would have a powerful effect upon the magnetization of the machine.

Figure 26 shows the conditions in a machine in which the brushes are rocked a moderate distance from the neutral axis.

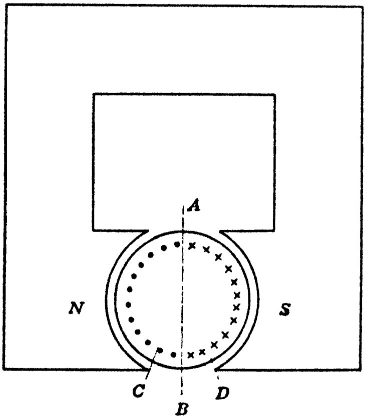

FIG. 25

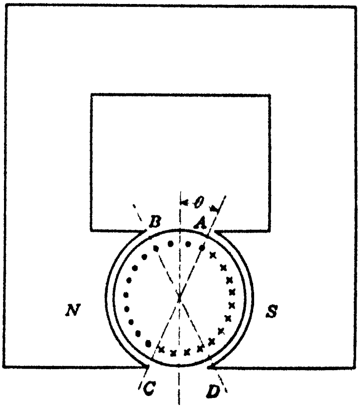

FIG. 26.

This corresponds to the general case in practice. The slight rocking of the brushes is to assist commutation, as will be explained later. It will be readily seen that taking the conductors from A to D and those from B to C the magnetizing effect of the one band will be exactly offset by that of the other, and the net effect will consequently be zero. The conductors in the bands A to B and C to D, however, are so situated that they assist one another, either to increase or to decrease the total magnetization of the machine. In a machine acting as a generator, it is necessary to rock the brushes forward of the neutral position in order to secure the best results in commutation. In this position the currents in the bands AB and CD are in such a direction as to tend to demagnetize the field, and the flux is thus weakened. In

a motor, it is necessary to rock the brushes backward. This would reverse the direction of the current in the bands, but in addition the current is reversed since the machine is acting as a motor. The net consequence is that the magnetizing effect of the bands is in the same direction as before, or they still tend to demagnetize the field.

PROBLEMS

12. In a dynamo the flux passes in succession through two air gaps each ⅛ in. long, section 100 sq. in.; an armature in which the mean path is 8 in., section 80 sq. in., permeability 500; two field cores each 7 in. long, section 60 sq. in., permeability 350; and a cast-iron yoke 9 in. long, section 120 sq. in., permeability 180. Assuming that the total flux is the same in all parts of the machine, how many ampere turns are required in order that there may be 50,000 lines per square inch in the air gap? If there are 2000 turns on the field, what is the current needed to obtain the above flux density?

13. A certain armature is 10 in. long. The flux density is 40,000 lines per square inch. What is the e m f. generated in one conductor moving at the rate of 4000 ft. per minute? If the pole shoes cover 70 per cent. of the armature surface what is the *average* voltage during a complete revolution?

14. A certain armature is 15 in. long and 25 in. in diameter. In order that sparking may be avoided it is necessary that there be generated in each conductor under the commutating pole 2 volts. What is the necessary flux density if the machine is rotating at the rate of 900 r p m. and if the interpole is 7½ in. long, measured parallel to the shaft?

15. A rod of copper is falling vertically at the rate of 100 ft. per minute. The horizontal component of the magnetic field of the earth is 0 029 lines per square inch. If the rod is 10 ft. long and is pointing due east and west as it falls, what is the voltage between the ends of the rod?

16. A certain six-pole direct-current generator has an armature 43 in. in diameter by 10 in. long. Each pole shoe is 10 in. by 12½ in. There are 534 conductors on the armature. There are two paths through the armature, so that half of the conductors are in series. If the density in the air gap is 56,000 lines per square inch, what must the speed be in order that the machine may generate 250 volts?

17. Each conductor on the foregoing machine carries a current of 200 amp. What is the force in dynes acting on a conductor under the pole face? In pounds? What on one not under the pole face? What is the force in pounds acting on the whole armature, taking account of the fact that not all of the conductors are under the pole faces?

18. What would be the power output of the foregoing dynamo if none of the voltage generated were lost? What is the horse power required to keep it in motion, using the answer to the above problem and assuming that the friction, etc., is zero? How many kilowatts are required? Does this correspond with the power output? Would this result be obtained in practice?

CHAPTER IV

SYSTEMS OF DISTRIBUTION

43. The Constant Current System.—Before taking up the subject of the regulation of dynamo machines, it is necessary to consider briefly the methods of distributing and utilizing the electric current. In the earliest electric generators, the machines were, in general, used to supply current to one device only. Thus, a certain generator might supply current for an arc light, and for no other purpose. The voltage would be regulated to the proper value, and no further regulation would be required.

As the use of electricity increased, however, it soon became apparent that it would be necessary to provide means whereby a great number of devices might be operated independently of one another from one generator. Thus at the present time it is not uncommon to find thousands of incandescent lamps, numerous arc lights, electric railway systems, electrified sections of main line railways, electrical heaters, thousands of horse power in electric motors, besides numerous other devices, all operated from the same power house, and taking current from the same generators.

Perhaps the simplest method of distribution is illustrated in Fig. 27 which shows the connections of the series or constant current system. The various devices are connected as shown so that the same current passes through all of them in series. Neglecting a possible slight leakage, the current is, of course, the same in all parts of the circuit. In order that it should be possible to operate any number of the devices separately or in any combination, it is necessary that the dynamo be so constructed that its *current* remains constant at all times. The *voltage* generated must vary in proportion to the load connected in the circuit at any time. Thus with two incandescent lamps, three arc lamps and a motor connected as shown in Fig. 27, and requiring the voltages shown to force the current through them, we should have to generate a voltage equal to the sum of the individual voltages

or 270 volts plus enough voltage to overcome the *RI* drop in the line. To cut out one of the devices the corresponding switch would be closed, thus providing a short circuit around the device, and causing the current to take this path instead of that through the lamp or motor. In order that the current may remain constant it is therefore necessary that the voltage should decrease as lamps are turned off.

There are various vital objections to this scheme. At present it is used only in certain special cases, for example, the operation of street lights. These are, as a rule operated upon alternating-current circuits, and even in this case, the regulation required is obtained from a special transformer and not from the generator.

The principal objection to this method of distribution will be apparent if we consider a typical case. The highest current that

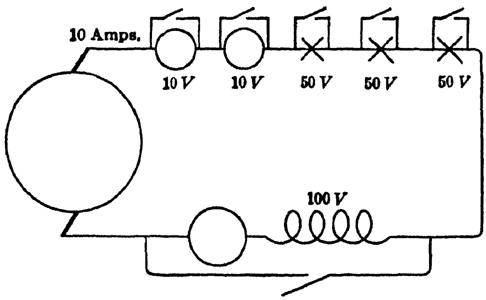

FIG. 27.

it is considered commercially practicable to use with arc or incandescent lamps is about 10 amp. Many arc lamps require about 500 watts, or 10 amp. at 50 volts. An incandescent lamp of moderate size would require 10 amp. at 5 volts or 50 watts. With a voltage at the dynamo of 6000 volts we should therefore be able to operate 120 arc lamps or 1200 incandescent lamps of the size mentioned. The power expended in the circuit would be $10 \times 6000 = 60{,}000$ watts or 60 kw. Since a horse power is equal to 0.746 kw., this would be equal to 80 hp.

It would not be practicable to increase the voltage above this figure since this is already higher than the maximum allowed by the municipal regulations of most cities. Neither is it practicable to increase the current beyond the figure given. Even at the voltage mentioned, the impossibility of attempting to pass the current into homes for general use will be apparent.

We should thus be limited to about 60 kw. or 80 hp. on each circuit and consequently to generators of a corresponding size. Such dynamos would be mere pigmies in comparison with the machines in modern power houses, where generators of 1000 kw. are very common and units of from 10,000 to 35,000 kw. are not unusual. Motors adapted to work on a constant current are also very unsatisfactory. Without further multiplying reasons, it will be apparent that such constant current distribution would be totally unsuitable for use in a modern system of electrical supply.

44. The Constant Potential System.—In modern installations, practically all of the power is distributed at constant potential.

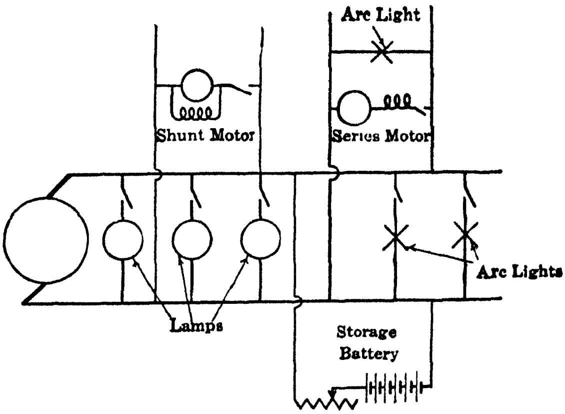

FIG. 28.

The general arrangement of such a system is illustrated in Fig. 28. All the receiving elements are "bridged" across the line as shown. It is evident that if the difference of potential across the lines remains the same at all times and all places, any one of the receiving units may be connected or disconnected without in any way affecting the remainder.

The conditions in such a system will be more clearly understood from the simpler diagram of Fig. 29. The generator is supposed to furnish current at a uniform pressure of 110 volts. If no lamps are turned on, the current will be zero, the voltage 110. Suppose that all of the lamps have the same resistance, 110 ohms. If one lamp such as A be turned on the current passing through it will be $I = E/R = 110/110 = 1$ amp. This same

current will flow through the generator, and if the lamp is adapted to this current, it will be properly lighted. If now another lamp be switched on, it also will take a current of 1 amp. and the generator will supply a current of 2 amp. It will be evident that either of the lamps may be turned on or off without any effect on the other.

The conditions when all four of the lamps are turned on is shown in the figure. The total current will be 4 amp. All of this current will flow in the section of the wire nearest to the generator. The next section will have only 3 amp. in it, and so on.

A little consideration will show that the assumed condition, namely, that the difference of potential between the wires is everywhere the same, can never be rigorously fulfilled. The conductors forming the mains must have some resistance, and this resistance will cause a drop of potential. Thus if the re-

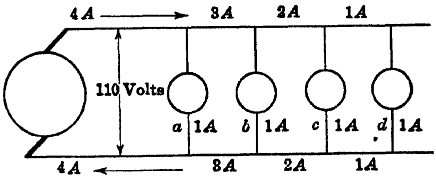

FIG. 29.

sistance in both of the wires connecting the first and the s lamps is 0.1 ohm, there will be a drop in voltage of $E =$. $3 \times 0.1 = 0.3$ volts. Then if the voltage across lamp A is 110, that across B will be 109.7 volts. In a similar way, it could be shown that the "drop" between B and C is 0.2 volt, and between C and D 0.1 volt. Thus the voltage applied to D is 110 minus all of these drops or 109.4 volts. The above is not quite exact, as we have neglected the fact that owing to the drop between the successive lamps, the currents in all the lamps except A would be slightly less than 1 amp. However, the computation is close enough for all practical purposes.

By using larger wire in the circuit, the drop between the lamps could have been made less. At the same time, the loss of power in the circuit would have been reduced. No matter how large the wire, there would, however, be some drop and some loss of power. Both of these can, however, be brought within commercial limits without a prohibitive expense for conductors.

The exact size of the wire to be used in any given case is frequently determined by the relative interest and depreciation on the cost of the copper, and the value of the power wasted. Sometimes, the deciding feature is the allowable drop that may be present without interfering seriously with the regulation of the lights, and frequently the size is determined by the fact that the wire must be large enough to carry the current without undue heating, which, if present, would lead to a serious fire risk.

45. Regulation of Generators.—As shown in the preceding pages, it is necessary that a generator be constructed to hold its voltage or its current constant. The former requirement is the more common and will demand more study.

At an early period in the development of electrical engineering, particularly while the constant-current system was in vogue, frequent attempts were made to govern the current or the voltage of the generator by changing the speed of the prime mover. Thus a solenoid might be employed connected in series with the circuit in a constant-current system or in shunt with it in a constant-potential system, and so connected with the governing mechanism of the prime mover as to increase the speed when the current or the voltage dropped below the specified value. Several facts have combined to cause this practice to fall into disrepute. In the first place, it will appear from what follows that it is a simple matter to construct a generator which will hold its voltage substantially constant without the addition of more or less complicated governing mechanisms. Such a system, moreover, would be somewhat unsafe because a trifling disarrangement of the electrical connections of the solenoid might cause the governing mechanism to increase the speed without limit and thus let the prime mover run away. On account of these and other reasons the attempt to govern the current or voltage of the generator by changing the speed has been practically abandoned. Engines, water wheels, etc., intended for use in driving electric generators are therefore usually provided with governors adapted to hold the speed at a reasonably constant value.

46. Regulation for Constant Potential.—*The Separately Excited Generator.*—The connections of a machine of this type are shown in Fig. 30. The action of the machine as regards regulation is best shown by means of a curve connecting the terminal voltage and the current output of the machine. If the flux were absolutely constant in value and were not distorted and if the

speed of the machine did not vary, the internal or actual generated voltage would be constant. The *terminal voltage* would be less than this on account of the drop in voltage due to the passage of the current through the armature. The relation between these quantities is shown in the following equation:

$$E = E_m - RI,$$

in which E is the terminal voltage of the machine, E_m is the voltage generated in the armature, I is the current in the armature, and R is the resistance of the armature and brushes. As will appear presently, the same equation applies to continuous current motors as well as to generators. The $-$ sign before RI is then changed to $+$ as the current is in the opposite direction, and we

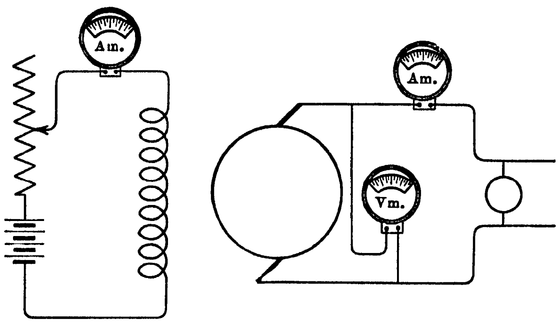

FIG. 30.

call E_m the back e.m.f. of the motor. E_m is always less than the terminal e.m.f. in a motor and greater in a generator.

In addition to the RI drop in the armature, we must consider the effect of armature reaction. As has been shown, this acts to reduce the flux. If then, the speed remains constant, the generated voltage will decrease slighty as the current is increased. The drop due to armature reaction can not be computed exactly, since it depends upon the shift of the brushes from the neutral position as well as upon the dimensions of the machine. We have then, the following two factors tending to cause the terminal voltage to fall as the load is increased:

A. Drop due to armature resistance = RI.

B. Decrease in flux due to armature reaction.

Figure 31 shows the curve connecting volts and amperes in the case of an 11-kw.. 110-volt, 100-amp. direct-current generator.

The field was separately excited and the field current and the speed were constant. It will be seen that the drop in voltage at full load is 13 volts. The resistance of the armature of this machine is 0.04 ohms. Since the full-load current is 100 amp., the drop in the armature is $100 \times 0.04 = 4.0$ volts. The remainder of the drop, 9 volts, therefore is due to armature reaction.

As previously mentioned, the separately excited machine is not used to any extent in direct-current practice. To use it would require an exciter, thus adding to the cost and complication.

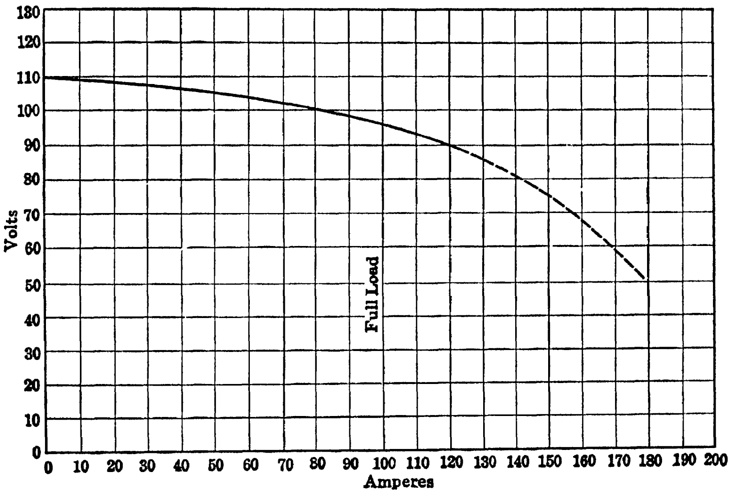

FIG 31.

47. The Shunt-wound Generator.—The connections of a shunt wound machine are shown in Fig. 32. In a machine connected in this way, the two causes for drop in voltage, discussed in connection with the separately excited machine, are still operative. However, another factor is introduced which causes the voltage to drop still more. In the separately excited machine the field current is constant, since it is not influenced in any way by the current taken from the machine. In the shunt machine, however, the field current is equal to the terminal voltage divided by the resistance of the shunt plus that of the connected rheostat. This resistance is constant, but since the terminal voltage drops somewhat, due to the two causes already given, the current taken by the shunt will also decrease as the load is increased. The

terminal voltage will therefore drop off more in the shunt-wound machine than in the separately excited generator.

For light loads, the drop in voltage of the shunt-wound machine is about double that of the separately excited generator of the same capacity. At heavier loads, the action becomes cumulative and the voltage falls very rapidly. Near the point C in Fig. 33, any increase of the armature current causes a drop in terminal voltage, and this in turn causes a decrease in the field current. This again reacts to make the voltage drop to a still lower value. At this load, the voltage of the machine has become unstable, and any attempted increase of current will cause the machine to lose its excitation completely. The current will then drop nearly to zero. There will still be some current due to the fact that there

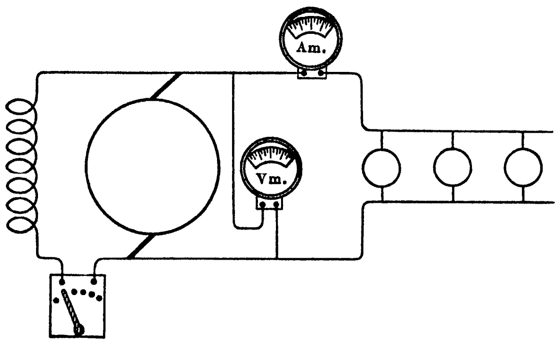

FIG. 32.

is some residual magnetism in the field magnets. A reduction of the external resistance to zero will cause the current to drop to the value shown at S and the external voltage to become zero.

It is clear from the above that no harm would result if a shunt machine were short-circuited and then put in motion. The field would fail to build up, and all the current that would flow would be that due to the small voltage generated by the residual magnetism. It should not, however, be inferred that a machine *in operation* could be short-circuited without damage. A small machine might not be injured, but in a large generator, the field magnetism would not die down instantly when the short circuit was established. For an appreciable time the machine would be operating on short circuit with considerable magnetism in the fields, and during this period it would generate an excessive

current. This condition would last only for a few seconds, but during this time considerable damage might be done.

Figure 33 shows the characteristic curve of the same generator that was used in getting the curve of Fig. 31, but the machine was shunt wound instead of being separately excited. It appears that the voltage drops much more for a given current than is the case in Fig. 31. The rapid drop in voltage when the current reaches approximately 130 amp. should be noted.

For good operation of incandescent lamps, the variations in voltage should not exceed 1 or 2 per cent. The drop in voltage from no load to full load will be far more than this with any shunt

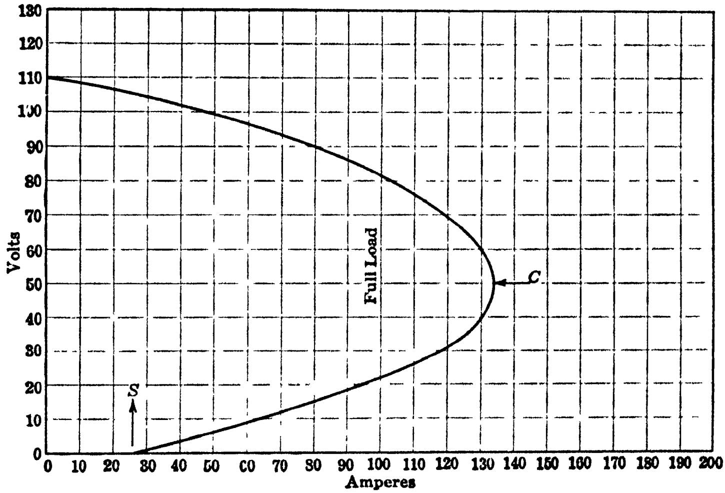

FIG. 33.

generator of reasonable size. To use such a machine in constant-potential service, it is necessary to regulate the voltage as the load changes by varying the resistance of the field rheostat. Usually this would be done by hand, but it might be accomplished by some automatic regulator. The compound-wound generator, however, is generally preferable for constant-potential work, as it can be adjusted to maintain its voltage constant or even to cause the voltage to rise as the load increases.

48. The Series-wound Generator.—In discussing the action of the series machine, it is necessary to consider the magnetization or saturation curve of a generator. Consider a shunt, series or compound-wound machine, connected as a separately

excited dynamo as shown in Fig. 30. The field current is varied from zero to its maximum value. No load is connected to the machine and consequently the ammeter in the armature circuit reads zero. If we plot the field currents and the armature volts, the result is a curve like the upper curve of Fig. 34. With zero current in the field circuit, there will be a small voltage due to the residual magnetism in the field. As the field current is increased, the voltage will also rise. The voltage will, however, not be in proportion to the field current. It will be strictly proportional to the flux passing through the armature, and may, in fact, be used as a measure of this flux. The flux will be nearly

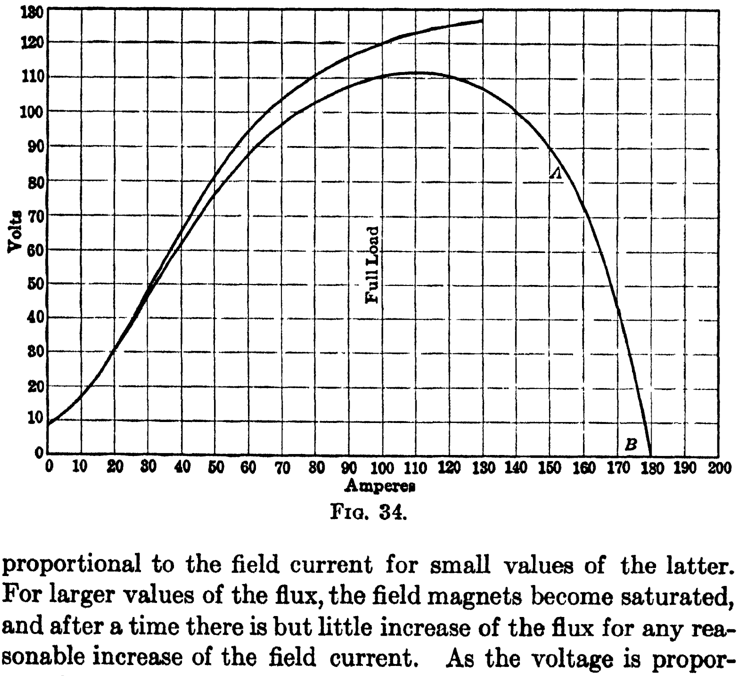

FIG. 34.

proportional to the field current for small values of the latter. For larger values of the flux, the field magnets become saturated, and after a time there is but little increase of the flux for any reasonable increase of the field current. As the voltage is proportional to the flux, it will increase rapidly at first, then more slowly, and finally will increase but little. It is clear, however, that it will never decrease for an increase of the field current.

In taking the volt-ampere curve of a series machine, the connections are made as in Fig. 22, an ammeter being connected in the circuit and a voltmeter across the terminals of the machine. This characteristic will be similar to the magnetization curve just discussed. The terminal voltage will, however, be lower than

that shown on the magnetization curve for any given current on account of the same two factors previously treated, namely, the armature *RI* drop and the effect of armature reaction. In addition there is an *RI* drop in the series field. The curve obtained is shown in the lower curve of Fig. 34. These curves were derived from the same small generator used in taking the curves of Figs. 31 and 33, only the series winding being used. It will be seen that for large currents the terminal voltage drops as the current is increased, since the increased effects of the armature reaction and the armature and series field *RI* drop are greater than the slight increase of generated voltage due to the larger field current. In the machine discussed the resistance of the armature and brushes is 0.04 ohm. That of the series field is 0.02 ohm, or 0.06 ohm in all. At 100 amp. the difference of voltage between the two curves is 9 volts. The drop due to field and armature resistance is $100 \times 0.06 = 6$ volts. The difference, or 3 volts, is due to armature reaction.

It will be apparent at once that a characteristic of this sort is useless in a machine designed for constant potential service. However, the portion of the curve *AB* approximates a constant current. This will be particularly the case in machines with large armature reaction. Such generators were formerly used to a large extent for constant current service, but the inherent regulation of the machines was hardly good enough. This regulation was usually supplemented by automatic regulators, operating to shift the brushes, shunt some of the current from the field coils, or perform some similar action to keep the current constant.

49. The Compound-wound Generator.—The connections of the compound-wound machine are shown in Fig. 23. In the case of generators, the series coil is generally wound to assist the action of the shunt coil. The characteristic of the machine will be a compromise between that of the shunt machine and that of the series machine. By properly proportioning the number of shunt and series turns, any characteristic between that of Fig. 33 and that of Fig. 34 may be obtained. Thus in Fig. 35 are shown two characteristics of the machine previously referred to. The upper one was taken with twenty-two series turns on the machine, the lower with sixteen. In the case of the lower curve the generator is said to be flat compounded, since the voltage varies but little from zero to full load. In the upper, where

more turns are used, the generator is overcompounded, in this case about 10 per cent.

A characteristic of the type given by a compound generator is ideal for constant potential distribution, and since it can be obtained with practically no extra expense or complication, compound-wound machines are in practically universal use for this class of service. The generators are usually overcompounded somewhat to allow for the drop in speed of the prime mover, and to allow for the loss in voltage between the generator and the load. By using a proper amount of compounding, the machine may be so adjusted that the voltage at the load remains prac-

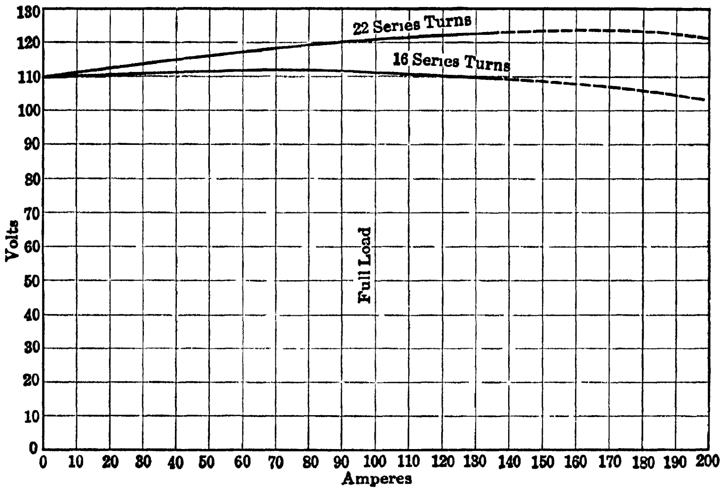

FIG. 35.

tically constant with increase in load. The voltage at the generator will of course rise somewhat.

The exact action of the series coil may be made more apparent by a numerical example. The machine from which the foregoing curves were taken has 4180 turns in the shunt field. Operating at rated speed and without load, it was found that the shunt field current necessary to give a terminal voltage of 110 was 2.40 amp. Under full load, it was necessary to increase the field current to 2.80 amp. to maintain the voltage. The ampere turns at no load were

$$4180 \times 2.40 = 10{,}032$$

and at full load

$$4180 \times 2.80 = 11,704$$

The difference was 1672 ampere turns. Since the full-load current of the machine was 100 amp., it is evident that sixteen series turns, carrying the entire current of the machine, will give $16 \times 100 = 1600$ ampere turns, or nearly the increase in the shunt field ampere turns required in order to maintain the voltage. It is apparent from the curve that this number is correct.

Occasionally a generator is differentially compounded, that is, the series turns oppose the shunt turns. This winding is used when it is necessary to limit the output of a generator to a certain current, as when a dynamo is driven from a windmill operating without a governor, and is used to charge a storage battery; or in the similar case of a lighting generator used on an automobile to keep a battery charged for operating the lights and ignition. In both of these cases, if a shunt-wound generator is positively driven so that its speed is proportional to that of the prime mover, the current output will be excessive at high speeds. This may be prevented by the use of the differentially compounded machine, since the series winding opposes the shunt, and if the current should rise to a high enough value, would completely neutralize it. This current is then the limiting current of the machine, and it may be placed at any desired value by a correct design of the shunt and series fields.

50. Method of Testing Regulation.—The regulation of a direct-current machine may be shown by curves, as has been explained. It is convenient, however, to be able to express the regulation by a simple number. Thus if we say that a certain shunt-wound generator regulates within 5 per cent., we at once have a basis of comparison with other machines.

To obtain the regulation of a shunt or separately excited machine, we put full load upon the dynamo and adjust the field until the voltage is the rated voltage of the machine. The speed, of course, is also adjusted to the rated value. The main switch is then opened so as to remove all of the load from the machine. The speed will doubtless increase, and it will be necessary to adjust it to normal again or else make a correction for the changed speed. The voltage is again read. The rise in voltage divided by the full-load voltage is the percentage regulation. Thus if the voltage with full load were 250 and this rose to 275 when the

load was removed, the speed remaining constant, the regulation would be

$$\frac{275 - 250}{250} = 0.10 \text{ or } 10 \text{ per cent.}$$

Regulation of compound-wound machines is not of great importance and is rarely required. It would be obtained by taking the characteristic curve as shown in Fig. 35 and obtaining the maximum deviation of the curve from a straight line connecting the full-load voltage and the no-load voltage. This deviation divided by the full-load voltage is the regulation.

The percentage of overcompounding is defined as follows:

$$\text{Per cent. overcompounding} = \frac{\text{full-load e.m.f.} - \text{no-load e.m.f.}}{\text{no-load e.m.f.}}$$

Thus if a machine gave 220 volts at no load and 250 volts at full load at the same speed, we should have

$$\text{Per cent. overcompounding} = \frac{250-220}{220} = 0.136 \text{ or } 13.6 \text{ per cent.}$$

In the case of the series machine the term regulation as used above has no particular value and is not employed.

51. Parallel Operation of Generators.—In early days of electrical engineering it was the custom to operate each generator upon a circuit of its own. Each power house had a number of separate circuits all ending upon a common switchboard. When the load was light all of these circuits were supplied from a single generator. When the load increased to such a value that a single machine was not capable of carrying it, a second machine was started up and some of the circuits switched to it. A third generator was added when necessary, and so on.

With this method of operation it was impossible to adjust the load on the different machines to the exact value desired. Moreover the current was interrupted, at least momentarily, when the load was transferred from one machine to another, causing an objectionable flicker of the lights. With motors on the circuit, there was considerable chance of injury, if the interruption was for more than a fraction of a second.

This method of operation has been entirely abandoned. It is now the custom to have all the generators feed into a common system of mains, known as bus-bars. All of the feeders to the different circuits are connected to the same bus-bars. Any

number of machines from one to the total number installed may be operated at one time, and any one of them may be started up or shut down without the slightest disturbance of the system.

This applies to alternating- as well as to direct-current circuits. In fact, the tendency is to extend the idea, particularly in the case of alternating-current systems, and operate a number of power houses in parallel. These may be separated by considerable distances from one another. This method tends to insure continuity of service, since the crippling of a single power house does not interfere with the supply of power from the other stations. It also tends to economy since water power plants may be operated in parallel with steam stations, the former supplying the power when the flow of water is sufficient, the latter being called upon to supply all or a part of the output during the dry season. It may also happen when operating a number of water power plants in parallel that the water for the different plants comes from different sources, and consequently the dry seasons may not occur at the same time. Thus a steam reserve may be avoided.

52. Shunt Generators in Parallel.—The connections of two shunt machines with the necessary instruments are shown in Fig. 36. Each generator has a shunt field rheostat, a double pole, single throw switch and an ammeter. A voltmeter is also provided and a switch to connect it to either of the machines. Assume that the machine *A* is in operation and that the load is such that it is necessary to start generator *B* also. The steam engine, water wheel or other prime mover connected to *B* is set in motion, and run up to full speed. The attendant then reads the voltage of the machine *A* or, what is nearly the same thing, takes a reading of the voltage directly across the bus-bars. He then connects his voltmeter to the machine *B* and, by means of the field rheostat, adjusts the voltage of *B* until it is the same as that of the bus-bars. The switch *S′* is then closed, thus connecting *B* to the load. If the adjustment of the voltage is exact, no current at all will flow when the switch is closed, since the voltages of the line and that of machine *B* will be exactly balanced. To make *B* take its share of the load, it is merely necessary to cut out some of the resistance in its field circuit. This raises the voltage of *B* and causes it to force current into the line. Since the e.m.f. across the line does not change materially, the total current flowing to the load does not change. It follows, therefore, that

the current of A must decrease by the amount of current B is generating. The procedure as described results in a slight increase in voltage across the line. It is therefore better to cut resistance into the field of A at the same time that it is being cut out of the field of B. In this manner it is possible to transfer the load from one machine to another with no disturbance to the voltage or interruption of service.

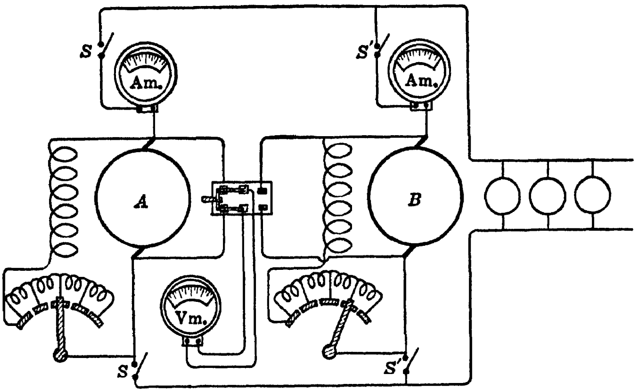

Fig. 36

53. Compound-wound Generators in Parallel.—Although the foregoing arrangement of apparatus gives good results, shunt-wound machines are rarely used. At a very small additional expense, compound-wound machines can be bought. These will have the very great advantage of holding their voltage very nearly constant with changes of the load, or of increasing their voltage if desired with increase of load. Since they present practically no added complication or greater cost, they are almost universally preferred.

However, with compound machines it becomes necessary, at least if the machines are at all overcompounded, to introduce an added connection, called an equalizer, between the machines. The connection in simplified form is shown in Fig. 37. Suppose that both machines are overcompounded and that the equalizer is not present. Let the load on the two be the same, and suppose that for some reason the steam engine driving one of the machines speeds up a trifle. The voltage of this machine will be increased, and its current will also increase. This increase of current will cause the voltage to rise still more, since all the current passes

through the series field. This action will continue until one machine has lost all its load and has in fact reversed and is operating as a motor, while the other machine will be carrying all the load and driving its mate as well. The combination would therefore be unstable and would not operate long in this way.

All this is remedied by the simple device of the equalizer. This is made of heavy copper so that its resistance is negligible compared with that of the series fields. The result is that when the current comes to the equalizer it divides into equal parts, if the two machines are identical, even though the currents in the two armatures are not the same. Since the two shunt fields are the same while the two series fields are kept the same by the equalizer, it is evident that the voltages of the two machines

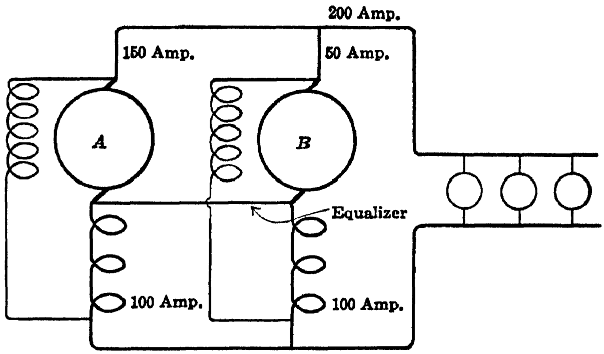

FIG. 37.

must be nearly the same, and they will therefore always divide the load in the proper proportion. If the machines are of unequal sizes, the larger machine will have the lower resistance in its series field. This field will therefore take the larger current as it should.

A more elaborate connection diagram is given in Fig. 38. Starting from the bus-bars, the current first passes through a double-pole circuit breaker, designed to protect the machine from overload. The main switch has three poles so that the equalizer connection is closed at the same time that the main circuit is closed. The operation of connecting a new generator to the line is the same as in the case of the shunt machine, except that the voltage should preferably be slightly below that of the bus-bars when the switch is closed. As soon as the equalizer connection

is closed, some current will flow through the series field of the incoming machine, and will raise its pressure a few volts. After a little experience, the attendant can readily make allowance for this. The output is then increased by cutting resistance out of the shunt field circuit of the incoming machine. To shut down a machine, the current is reduced to zero or at least to a small value, the main switch is quickly opened and the circuit breaker is tripped.

Any number of such machines may be operated in parallel. Two power houses may also be connected in parallel. If they are close together, it may be necessary to install an equalizer between them. However, they are usually at some distance, and

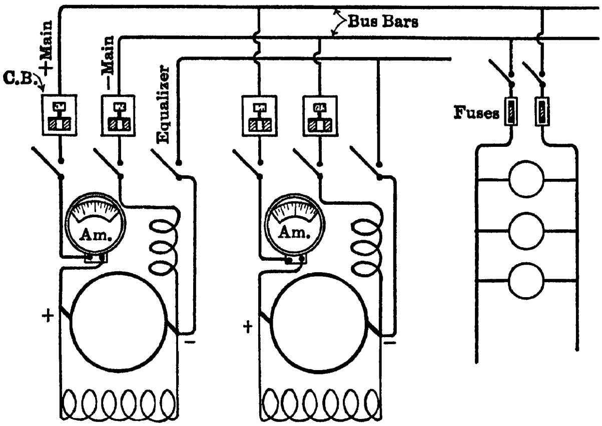

Fig. 38.

the drop in potential is generally great enough to prevent the voltage from rising as the load is increased. The equalizer therefore becomes unnecessary.

54. Effect of Voltage upon the Amount of Copper Required.— The higher the voltage at which we operate a circuit, the less the weight of copper required; in fact, for a given loss, the amount of copper required varies inversely as the square of the voltage. This can be readily shown as follows:

$$\text{Power lost} = P = I^2R = \frac{E^2}{R}$$

or for a given loss the resistance varies as the square of the voltage. Since the weight of the wire varies inversely as the resistance,

the weight varies inversely as the square of the voltage. Thus if we should compute the weight of copper required to transmit a certain amount of power a given distance at a pressure of 220 and again at 2200 volts, we should find that for the same loss the former case would require 100 times as much copper as the latter. The importance of working at the highest possible voltage will be apparent. Long-distance transmission lines are now operated at pressures as high as 150,000 volts.

55. The Three-wire System.—Most incandescent lamps are constructed for a potential of approximately 110 volts. Lamps are built for double this voltage, but for the same candle power the filament is twice as long and of half the cross section. The lamps are therefore far more fragile and burn out quicker in service. They are also more expensive to make. From the standpoint of the amount of copper required to transmit the power to them, they are far superior to the 110-volt lamps, but

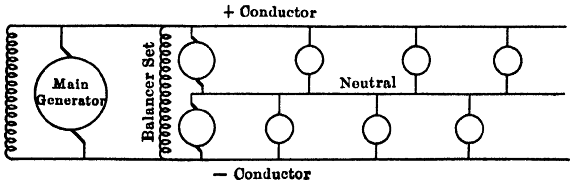

FIG. 39.

they are so far inferior in other respects that they are but little used.

By using the three-wire system illustrated in Fig. 39 it is possible to operate at a voltage of 220 and yet apply a voltage of only 110 to the lamps. If the lamps on the two sides of the system take exactly the same current, no current will return by way of the neutral wire. By care in the grouping of the lamps on the two sides this condition can be approximated in practice, and the drop on the neutral wire becomes negligible.

If the neutral wire were not present, this system would require only 25 per cent. as much copper as the 110-volt system. The neutral is usually made of half the cross section of the outer wires. The total amount of copper is then 25 per cent. + 6.25 per cent. = 31.25 per cent. as much as in the two-wire system. The saving of copper is so great that the three-wire system is universally used whenever a large amount of direct-current power must be distributed over a considerable area.

Figure 39 shows that the main generator operates at 220 volts. The two small dynamo machines are mounted on a common bedplate with their shafts coupled together. They are shown shunt wound, but are frequently compound wound. If the system is balanced, both machines operate as motors without load. If one side of the system is loaded more than the other, its voltage tends to drop. As soon as this takes place, the voltage of the line on that side becomes less than the back e.m.f. of the corresponding machine and the latter operates as a generator. Meanwhile the voltage on the other side has risen and the other machine develops more power as a motor and drives the machine which is operating as a generator. In this way, the load on the two sides is equalized and the voltages are kept near their proper values.

PROBLEMS

19. Four incandescent lamps are connected to 110-volt mains in the same manner as those of Fig. 29. Their resistances are respectively 100, 110, 120 and 130 ohms. What is the current in each lamp? The total current? What is the resistance of all the lamps in parallel, *i e*, what single resistance would take the same current from the line as the four resistances in parallel?

20. If the above lamps are 1 mile from the generating station and are fed through a pair of No. 10 B and S wires, what is the voltage at the generating station if the voltage at the lamps is 110? The resistance of No. 10 wire is 1 ohm per 1000 ft.

21. The potential at the terminals of a shunt-wound d.-c. generator is 250 volts, and the current is 1000 amp. If the resistance of the armature is 0.0035 ohm, what is the generated voltage?

22. A compound-wound dynamo generates 230 volts when operating at no load and at normal speed. At full load with the same shunt field current and an output of 300 amp. it generates 250 volts at the terminals. What is the internal voltage if the resistance of the armature is 0.012 ohm? What is the percentage increase in the flux passing through the armature due to the series field?

23. A certain d.-c. generator gave its rated e.m.f. (110 volts) at rated speed with a shunt field current of 2.5 amp. With full load of 100 amp. applied it was necessary to increase the shunt field current to 2.90 amp. to hold the voltage constant, the speed being the same. If there are 1600 turns of wire in each shunt field coil, how many series turns per coil will it be necessary to add in order that the machine may be flat compounded? The drop due to the series coil may be neglected.

CHAPTER V

CHARACTERISTICS OF MOTORS

56. Characteristics of Motors.—In the case of continuous-current generators, the speed is constant or approximately so, and the variables are the terminal voltage and the current. In plotting characteristic curves, the voltage and current are therefore usually the quantities chosen. In the case of motors, since practically all are operated on the constant potential system, the terminal voltage may be considered as constant. The principal variables are then the speed, current, torque, kilowatts input and output in kilowatts or in horse power. The ratio of the output to the input or the efficiency is also frequently required. In performance curves of motors, currents are usually plotted as abscissæ, and the other values as ordinates.

Of the above values, the speed and the torque are perhaps the simplest to study in gaining an elementary notion of the operation of a motor. They are also of value in comparing the electric motor with other sources of power.

57. Operation of same Machine either as a Generator or as a Motor.—As previously explained, the action of a dynamo is essentially the same whether it is operating as a generator or as a motor. Assume that a shunt machine is driven by some outside power at the proper speed and that by means of the field rheostat its voltage is adjusted so as to be the same as, and in the opposite direction to, that of a pair of constant potential mains. A voltmeter should be used which will reverse its deflection if the voltage is reversed. By connecting first to the mains and then to the machine, the operator can assure himself that the two voltages are equal and opposite. (See Fig. 36.) If the switch connecting the machine to the line be closed, no current will flow. The e.m.f. of the line will be just balanced against that of the machine and the resultant voltage acting around the circuit will therefore be zero. The machine is acting neither as a generator nor as a motor.

If, now, the speed of the prime mover be slightly increased, the

e.m.f. of the machine will become greater than that of the line and consequently there will be an unbalanced voltage. A current equal to the difference of the two voltages divided by the resistance of the armature of the machine will flow. This current will be in the same direction as the e.m.f. of the dynamo, and the machine will therefore act as a generator and deliver power to the line. As explained in Art. 32, since the current and the e.m.f. of the machine are in the same direction, the torque due to the current will be in such a direction as to oppose the motion. The prime mover must therefore develop more power in order to maintain the rotation.

If, on the other hand, instead of increasing the speed, we decrease it, the reverse action will take place. The voltage of the machine will become less than that of the line. The difference of the two voltages will act as before to force current through the armature of the machine, but since the line e.m.f. will now be the stronger, the flow of current will be in the reverse direction. The torque will also be reversed or the machine will act as a motor.

If now the load, *i.e.*, the torque on the motor shaft, be increased, the machine will slow down. This immediately results in a still further reduction of the back e.m.f. of the motor and a greater difference between the line voltage and that of the machine. More current will now flow through the armature of the machine and the latter will develop more torque. The slowing down of the machine will continue until the increased torque, due to the larger current is sufficient to overcome the resistance of the load. The motor will then continue to operate at this speed.

The slowing down of the motor in order to allow the necessary current to pass need not be large. Thus in the case of the machine used as a generator in obtaining the curves of Figs. 31, 33, 34, and 35, since the resistance of the armature is 0.04 ohms, a voltage of only $100 \times 0.04 = 4$ volts is required to force full-load current through the armature. Since the terminal potential is 110 volts, this requires a reduction in back e.m.f. and a corresponding reduction in speed of only $4 \div 110 = 3.64$ per cent.

Instead of changing the speed as explained above to cause the machine to act either as a generator or as a motor, the same result can be secured in another way. If, when the machine is operating at such a speed that the voltage generated by

the machine is the same as that of the line (and in consequence the current is zero) the field is strengthened by cutting out some of the resistance in the field circuit, the voltage of the machine will exceed that of the line. The machine will then force current in the direction of its own e.m.f. or it will become a generator. A further increase in the field strength will furnish still more current. In this manner the load on each of a number of machines operating on the same circuit is adjusted to make each take its proper share of the load.

If, on the other hand, the field is weakened, the e.m.f. of the line will be greater than that of the machine, current will flow against the back e.m.f. or the machine will operate as a motor. A further weakening of the field will cause the machine to take more current and consequently develop more power and torque as a motor. If the resisting torque of the load remains the same, the motor will develop more torque than is required by the load, and the motor will speed up. This in turn will increase the back e.m.f. until it is more nearly equal to the line e.m.f. The current will then decrease until just sufficient torque is developed to keep the load in motion. It is important to note that a *weakening* of the field results in an *increase of speed.* This is contrary to what one might expect at first thought.

58. The Fundamental Equation of the Direct-current Motor.— The foregoing principles can be better brought out by a consideration of the fundamental equation of the motor. This was developed in Art. 46; in connection with the study of the machine as a generator. As explained, the action as a generator or as a motor, is essentially the same. The difference is merely a question of whether the voltage of the machine is higher or lower than that of the line to which it is connected. The sign of the current is changed since it is in the opposite direction in a motor to that in a generator and we may then write

$$E = E_m + RI$$

As we have previously shown the back e.m.f. developed by the motor is (see Art. 36)

$$E_m = \Phi n N \div 10^8$$

Omitting the constant 10^8 for the sake of simplicity we may rewrite the equation in the following form:

$$E = \Phi_1 n N + RI$$

and solving for n we obtain the expression

$$n = \frac{E - RI}{\Phi_1 N}$$

In which n = the revolutions per second, E is the line voltage, R is the resistance of the armature, I is the current in the armature, Φ_1 is the magnetic flux per pole (measured in units of 10^8 lines), and N is the number of conductors connected in series between brushes, times the number of poles.

In considering the formula, it must be remembered that the quantity RI in motors of reasonable size, is always very small compared to the value of E. In a small motor at full load, it may be as much as 10 per cent. In a motor of say 1000 hp. it would probably not be greater than 1 per cent. This means that the speed n will not vary more than this percentage when the load and consequently I, is varied from zero to full-load value, *providing Φ remains constant.* This is nearly the case in a shunt-wound motor. To be more exact, in the shunt machine the brushes usually have a slight forward lead and in consequence of the armature reaction Φ will usually decrease somewhat as the armature current increases. (See Art. 42.)

59. Speed Torque Curve of Shunt Motor.—The speed torque curve of an actual shunt motor is plotted in Fig. 40. The torque is proportional to the number of conductors on the armature, the current and the flux. The number of conductors N is constant. The current I is a variable. The flux Φ is nearly constant but decreases somewhat as the current increases. On this account the curve of speed and torque is not a straight line.

The point of full-load torque is shown on the curve, and it will be seen that the torque at this point is only a small fraction of the maximum torque that the motor can develop. The maximum overload torque is also indicated. It is important to note that the motor is not worked at anywhere near its maximum torque. If an attempt were made to do so, it would heat up very rapidly and burn out quickly. Moreover if allowed to rotate and develop anywhere near its full torque, there would be prohibitive sparking at the commutator, and the efficiency would be very low.

These remarks apply particularly to large motors. With a motor of a fraction of a horse power it is frequently possible to

apply the full potential of the line to start the motor, so that we are operating for an instant at the point S. This method of operation would not do for large motors, and in this case we must use a starting rheostat. (See Chap. VI.) It is clear that the motor has the ability to develop enormous starting torque and there is therefore never any difficulty with a shunt- or series-wound direct-current motor in developing all the torque that is necessary to start any load within the capacity of the motor. This is by no means the case with alternating-current motors as will appear later.

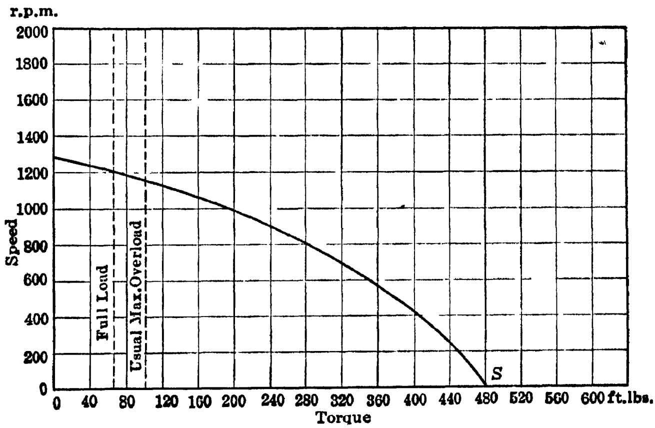

FIG 40

60. The Series-wound Motor.—The analysis of the action of the series-wound motor is made somewhat more difficult by the fact that the flux Φ is not constant. The whole current taken by the motor passes through the series field and consequently an increase in current results in an increase in the field strength. We cannot embody the facts in a mathematical equation because the field strength bears no simple relation to the strength of the current through the field winding. This is on account of the magnetic saturation. It might appear that we could derive an empirical equation. This could be done in the case of any particular motor, but the expression would not be applicable to any other machine since the quality of the iron, degree of saturation, etc., would be different. It is enough for the present to know that Φ increases with an increase in the value of the cur-

rent, but that the rate of increase becomes very small with large values of the current.

Considering again the equation of the motor, an increase of the torque of the load connected to a series motor results, as in the shunt motor, in a decrease of speed. The back e.m.f. at once becomes less and since the difference between the back e.m.f. and the line voltage is increased, the current increases. This, in turn, results in an increase in the flux. This latter action was absent in the case of the shunt machine. The increase of flux results in a higher back e.m.f. than would otherwise be present, and consequently part of the effect of the decrease in speed is neutralized. It follows that to obtain a given increase in current, the series motor must slow down far more than the shunt machine.

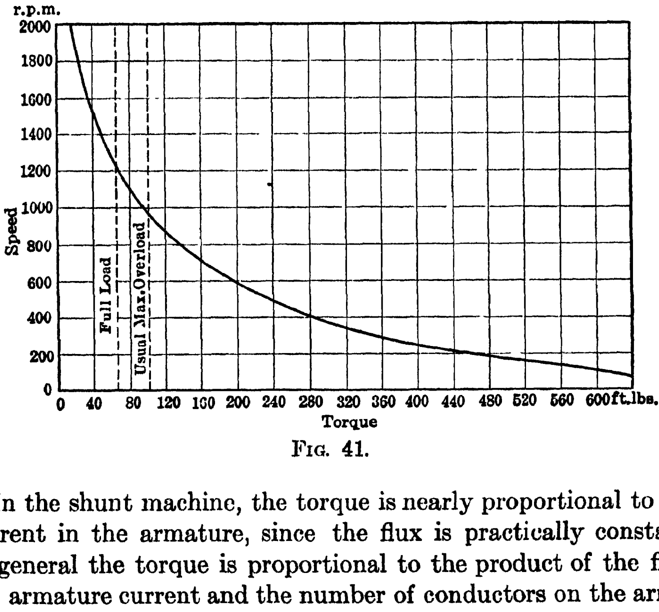

FIG. 41.

In the shunt machine, the torque is nearly proportional to the current in the armature, since the flux is practically constant. In general the torque is proportional to the product of the flux, the armature current and the number of conductors on the armature. In the series motor the flux increases as the current increases. If the increase of flux were proportional to the increase of current, the torque would be proportional to the square of the current. This is nearly the case for small values of the current. For larger currents, however, the increase of the flux is far less than the increase of the current, and the torque therefore increases faster than the current but not in proportion to the square of the current. The net result of these actions is a speed torque

curve of the shape indicated in Fig. 41. This curve shows that the line will never cross the axis of zero torque, no matter what the speed is. An increase in speed results in a decrease of current and a consequent weakening of the field strength. Therefore no matter how fast the machine may be driven, its back e.m.f. will never exceed that of the line, and consequently it can not reverse and become a generator. Also if the torque required to maintain the load in motion is very small or is entirely removed, the motor will speed up to an excessive speed. It might readily reach such a speed that the conductors would be thrown out of the slots by centrifugal force and the motor seriously damaged. Series motors are therefore used only on loads which can not be reduced to a small value or they are applied to classes of service where an attendant is always present to control the speed.

61. The Compound-wound Motor.—The series coil on a compound-wound motor may be wound to assist the shunt or to

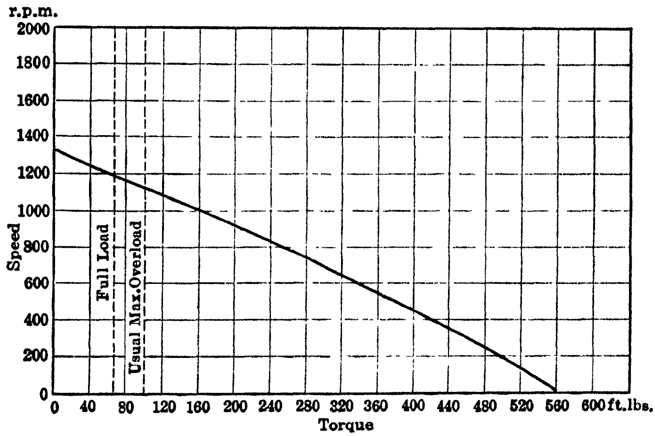

FIG. 42

oppose it. The latter connection is rarely used, while the former is in common use. The two connections are known, respectively, as cumulative compounding and differential compounding.

A cumulative compound motor is intermediate in its characteristics between the series- and the shunt-wound motor. There is always present a certain amount of flux in the field, due to the shunt winding and this is independent of the current in the armature. In addition, as the armature current increases, the flux is increased by the action of the series coil. The character-

istics of the motor will, therefore, be nearly the same as those of the shunt motor if the number of series turns is small, or on the other hand, they will approach those of the series machine if the number of series turns is large. It is standard practice with stock motors to supply approximately 15 per cent. as many series ampere turns as shunt ampere turns. With this proportion a motor will have a speed torque characteristic approximating that shown in Fig. 42. The motor has a limiting speed like the shunt machine, and if driven above this speed it will act as a generator. It, however, slows down more under load than the shunt machine. This is advantageous in certain applications as will be shown presently.

62. The Differential Compound Motor.—Compound-wound motors have been constructed in which the windings were so

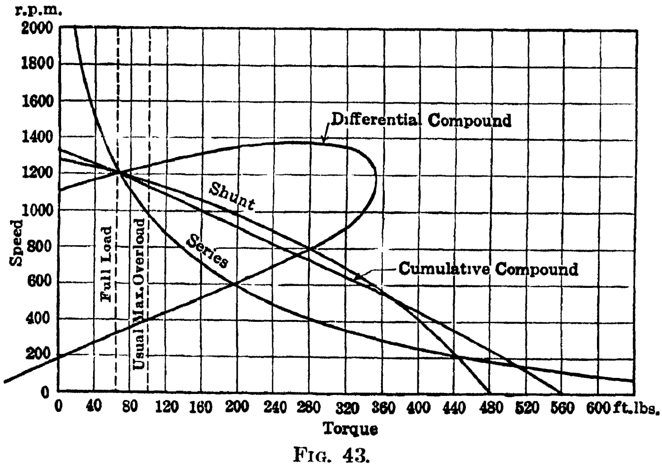

FIG. 43.

connected that the series field opposed the shunt field. An increase of load and consequently of current in such a motor results in a weakening of the field. This weakening of the field flux may be sufficient to cause the motor to increase somewhat in speed as the load increases.

The speed regulation of the ordinary shunt motor is sufficiently good so that there is little demand for a motor with still better speed regulation or even an increase in speed with an increase in load. Moreover there are several rather serious objections to such a motor. Perhaps the most obvious is that if an attempt is made to start with a large load and consequently a large current,

the action of the series coil will be so powerful that the flux will be very much reduced and an excessive current will be required. If a still larger current is passed the torque will be reversed and the motor will tend to rotate in the opposite direction. Such motors are rarely used. The speed torque curve of this motor is shown in Fig. 43.

In Fig. 43 are plotted on a single sheet the same curves as are plotted in Figs. 40, 41, and 42 to aid comparison of the characteristics of the different machines. The curves are those of a motor rated at 15 hp. at 1200 r.p.m. The same armature and field structure are used in all three cases and the only difference is the type of winding on the field.

63. The Choice of Motors for any Particular Service.—*The Shunt Motor.*—For work which requires a practically constant speed, irrespective of the load, the shunt motor is well adapted. Examples of such work are the driving of line shafts to which a number of machines are belted, the operation of individual machine tools, such as lathes, drill presses, planers, milling machines, woodworking tools, etc. In all of these the starting duty is moderate, and heavy overloads are not common.

64. The Series Motor.—As the torque of the series motor is approximately proportional to the square of the current, it follows that for starting heavy loads requiring more than full-load current, it is superior to the shunt motor, since the same torque will be developed with less current. For loads less than normal, the series motor is correspondingly inferior in starting to the shunt motor. Thus, suppose a motor were required to exert merely full-load torque during starting. Either type of motor would require full-load current, and both would be equally effective. However, if it were necessary to exert four times full-load torque, the shunt motor would require four times full-load current. The series motor, on the other hand, if the fields were unsaturated, would require only twice full-load current. This current would produce a field of double strength, and this in combination with the double armature current would give four times the normal torque. As a matter of fact, the field would be saturated to a considerable extent in a commercial motor, and the motor would require more nearly three times full-load current, instead of only twice this quantity. Even with this qualification, the superiority of the series motor will be apparent.

If, on the contrary, the motor were required to develop only

one-half of normal torque, the shunt motor would require one-half of normal current. The series motor would need $\sqrt{0.5} = 0.707$ of full-load current.

The series motor also differs from the shunt motor in that it is to a certain degree a constant-power machine; that is, as the torque increases on a series motor, it slows down very much. This means that less power is required to keep the load in motion than would be the case if nearly full speed were to be maintained. Hence a given series motor will be able to handle torques that would be too great for a shunt-wound motor. This feature, also results in a lessened demand upon the source of power. This may be a point of considerable importance.

As an example, consider the application of electric motors to the propulsion of cars. Series motors are always used in this service. As compared with shunt motors, they consume less power in starting and accelerating the car. This is very important, as practically the whole duty of the motor in the case of city cars is to give this acceleration to the car, which is then allowed to coast for some distance, and finally is stopped by the application of the brakes.

In case a hill is to be climbed or heavy snow is met, the series motor will slow down, and continue to propel the car with less current consumption than the shunt motor, but at a reduced speed. Thus it continues the service, whereas the shunt motor would be so overloaded that it would be in danger of burning out.

It has been mentioned that the shunt motor, if driven above normal speed will act as a generator and return power to the line. It would seem that this property might be exploited to return power while the car was descending hills. This can be done, but the number of hills in the usual urban or interurban line is too small to make the possible saving from this source important. However, in the case of a long uniform grade up a mountain side the possible return of power while the cars were descending might be important enough to justify the use of shunt motors.

Hoisting service as applied to cranes is very similar in many respects to car service. Series motors are exclusively employed for this class of work also. The fact that a light load is automatically hoisted rapidly, and a heavy one more slowly is of great practical importance.

65. The Compound-wound Motor.—A typical example of the class of service to which the compound-wound motor is adapted

is the operation of a punch press. The average power requirement of a punch press is rather low, being in many cases not much more than that wasted in friction. Just at the moment of punching, however, the power rises momentarily to a high value. A load of this character is best handled by installing a flywheel of considerable size on the motor shaft. While the punch is passing through the metal sheet, the flywheel slows down and delivers the energy required to punch the hole. If a shunt motor were used, it would take a large current from the line on account of the slowing down, and would be accelerated at a correspondingly high rate to the original speed. This would result in drawing a large momentary current from the line and would necessitate a correspondingly large motor to commutate the current. The compound-wound motor on the other hand does not take so large a current for a given drop in speed. A smaller motor can therefore be used, and the power demand is more uniform. A series motor is not suitable since, if allowed to run idle for some time it would attain a dangerous speed.

Compound motors are often preferred to shunt machines in cases where close speed regulation is not necessary and the load is of such a nature that a large starting torque is required. Compressors for refrigerating plants often fall in this category.

66. Direction of Rotation of Motors and Generators.—The direction of rotation of a shunt motor may be reversed by reversing the connections of either its armature or its field. The direction of rotation is *not* reversed by reversing the applied voltage. The field is reversed, which alone would cause a reversal of rotation, but since the current in the armature is also reversed, the result is that the direction of rotation remains the same. The same remarks apply to the series motor. In fact, as we shall see, a series motor with certain alterations to prevent excessive losses and heating, may be operated on alternating current.

The procedure in the case of a compound-wound motor is slightly different. To reverse the direction of rotation, we should reverse the connections of the armature only, or else those of both the shunt and series fields; that is, we should *not* treat the armature and series field as a unit, since if we did so the action of the series field on reverse would be the opposite of that desired.

The actions of the corresponding machines as generators can be readily deduced from their actions as motors. Thus, a shunt machine may generate with a certain polarity of the brushes.

This polarity may be reversed without change in the connections or reversal of direction. If we take a given machine which has been generating with a certain polarity, and pass a current through the shunt field in the reverse direction from an outside source of power, we shall probably reverse the residual magnetism of the field. When the machine is started, the polarity will be reversed, current will flow through the shunt in the reverse direction and the machine will "build up" with reversed polarity.

A shunt machine will not generate at all if rotated in the wrong direction. The residual magnetism will cause a small voltage to be generated in the reverse direction. This will force current around the fields in the wrong direction, reducing the residual magnetism present, and the voltage will quickly drop to zero. Similar considerations apply to the series and the compound-wound generators.

It has been previously shown that the shunt machine will operate either as a generator or as a motor with the same direction of rotation. The same is true of the compound machine. When the machine changes from a generator to a motor, the current in both the armature and the series field reverses. If, therefore, the machine operates as a cumulatively compounded generator, it will be a differentially compounded motor. Since the latter is rarely used, if we wish to use a compound generator as a motor, it is generally necessary to reverse the connections of its series field.

The case is different with the series machine. As seen from Fig. 41, with a given connection and direction of rotation, it will never act as a generator. The direction of the back e.m.f. is opposed to the current as a motor. If an attempt is made to pass current in the same direction as the e.m.f., the current of the series field will be reversed and consequently the magnetism of the field will be rapidly reduced to zero. The same considerations will apply if we start with the opposite direction of current flow. It is then apparent that to act as a generator, the series machine must rotate in the opposite direction to its rotation as a motor, or else the connections of its armature or field must be reversed.

PROBLEMS

24. A certain d.-c. shunt-wound motor operating at 230 volts has an armature resistance of 0.03 ohm. If driven by outside power at a speed of 900

r.p m. the machine takes no armature current from the line although connected to it. What will be its speed when taking 150 amp armature current as a motor? What at 75 amp.? The effect of armature reaction may be neglected in the above.

25. What will be the speed of the same machine when acting as a generator and delivering the above currents, the field current being assumed to remain constant?

26. A 230 volt d.-c. shunt-wound motor when running light, *i.e.*, with no load except the losses in the armature takes an armature current of 10 amp and rotates at a speed of 600 r p m What will be its speed when the output is 50 hp ? The loss in the field may be neglected and the resistance of the armature taken as 0.03 ohm.

27. If the armature of the foregoing machine were blocked so that it could not rotate, what would be the armature current when full voltage was applied? If the normal current of the machine is 200 amp how many times full-load torque will the above machine develop under the above circumstances? (The effect of armature reaction would greatly reduce this but may be neglected in solving the problem)

28. At standstill the above machine would develop zero power. It can be readily shown that the maximum power is developed (neglecting friction and hysteresis) when the machine is rotating and taking half of the locked current. With this current what is the input? What is the armature copper loss? Neglecting all other losses, what is the output? What is the efficiency? What is the torque in terms of the full-load torque?

29. A certain motor has an armature resistance of 0 05 ohm. The applied e m.f. is 230 volts The full-load speed with 50 amp in the armature is 1200 r.p m. At what speed would the motor have to run to take zero current from the line? If the applied e m f is reduced 10 per cent. what will be the speed at which the armature current is zero, it being assumed that the field is unsaturated so that the change of flux is proportional to the change of voltage? What will be the full-load speed?

30. What will be the corresponding speeds if the field is so strongly saturated that there is practically no change of flux with a 10 per cent. change in applied e m f.?

31. What will be the corresponding speeds if the machine is separately excited?

32. A certain shunt-wound motor is wound to operate at a speed of 600 r.p.m. with zero load at a line voltage of 115. At what speed will it operate if the line voltage is increased to 230 and enough resistance is inserted in the shunt field circuit so that the field current is the same as before? The machine will be able to carry the same current in the armature as before If the original rating was 25 hp., what will be the new rating?

CHAPTER VI

ACCESSORY APPARATUS

67. Starting Rheostats, Series Motors.—It is evident from a consideration of Fig. 43 that any of the ordinary types of motors, if of reasonable size, will develop a very great starting torque when connected directly to a constant potential line. The motor, of course, takes a correspondingly large current. The enormous torque would be liable to result in great damage to the connected machinery, and to the belts or gears used in transmitting the power. The large current would also cause flashing at the commutator and possible damage to the motor by heating. For these reasons, it is necessary to use resistors to limit the starting current with all except the smallest motors.

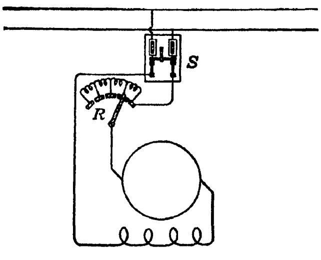

FIG. 44.

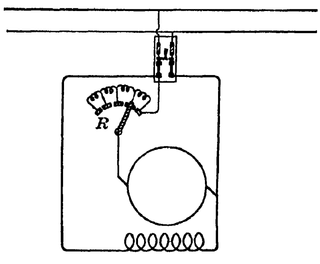

FIG. 45.

Figure 44 shows the connections of a starting rheostat for a series-wound motor. This consists merely of a resistor connected in series with the motor and means for cutting out the resistance as the motor speeds up. In some cases a mechanical interlock may be provided so that it is impossible to close the main switch *S* unless all of the resistance is in circuit.

68. Starting Rheostats for Shunt Motors.—The case of the shunt or compound motor is somewhat more complicated. The current which passes through the shunt field winding is not affected by the speed of rotation. Moreover this current is always small compared to the total current of the motor. In

order that the motor should start promptly and with the minimum current in the armature, it is essential that the field should be *as strong as possible*. Hence it is necessary that the connections be made in such a manner that the shunt field is of full strength at all times. A connection which will accomplish this is shown in Fig. 45. When the switch S is closed, the shunt field takes its full current. The resistance of the rheostat, R, is such that approximately full-load current flows through the armature. The motor having full shunt current and full-load current in the armature develops full-load torque, and if the load is not above normal, will start at once. The rheostat handle is then moved steadily across the contact unit all of the resistance has been cut out and the motor is operating at full speed.

The foregoing connection is perhaps the simplest that can be made, but it is open to a serious objection. The proper method of stopping the above motor would be to open the main switch, wait until the motor ceased to rotate and then move the rheostat handle to the "start" position. If instead of doing this the operator stops the motor by first moving the rheostat handle to the "start" position and then opening the main switch, the result will be that a heavy arc will be drawn at the contacts of the main switch. This is due to the large voltage induced in the field on account of the sudden dying down of the magnetism. Not only are the contacts burned, but there is grave danger of puncturing the insulation of the field because of the high induced voltage.

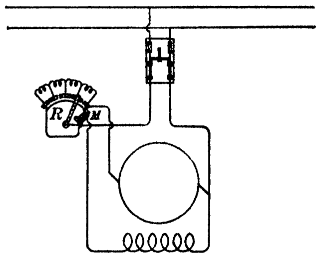

FIG. 46

Both of these dangers are avoided by using the connection shown in Fig. 46. If the motor is stopped by moving the starting handle to the "start" position, the field circuit is not broken, and the armature supplies a gradually decreasing current to the field, the machine acting for a time as a generator to the extent of supplying the field current. When the motor is started by moving the starting arm on to the first contact, a fraction of a second is required for the field to attain its full strength. An appreciable time, therefore, is needed for the motor to develop its full torque after the contact is made. This is a slight

advantage as it avoids giving a blow-like impulse to the connected mechanism.

69. The No-voltage Release.—Figure 46 shows a small electromagnet *M*, connected in series with the shunt field. This serves to hold the rheostat arm in the running position against the pull of a spring. If for any reason, the supply of power to the motor is interrupted, the motor slows down and ultimately stops, the field current falls to zero and the magnet *M* releases the arm, which then returns to the starting position. This protects the motor from injury when the line again becomes energized.

In Fig. 47 is shown a perspective view of two-motor starters of the type discussed. The construction will be clear from the illustration.

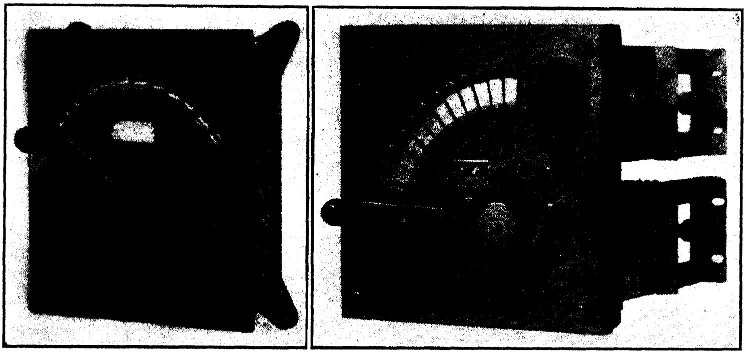

Fig. 47.

70. Protective Apparatus.—It is essential that a motor or other piece of electrical apparatus be protected from overload. The simplest way of doing this is to include in the line supplying the motor a piece of readily fusible metal. This is so proportioned that if a current capable of injuring the apparatus passes for a considerable time, the metal strip or wire (called a fuse) will melt and break the circuit.

It is, of course, essential that the fuse be properly protected so that when it melts the heated metal will not be thrown where it may ignite combustible material. Formerly, porcelain blocks were used to enclose the fuse. These were not entirely safe, and it was too easy for a workman to replace a blown fuse with a larger size of wire, or even with a piece of iron wire, a nail or anything else that happened to be handy. At present the enclosed fuse

is almost universal. The fuse itself is enclosed in a tube, and the space between the fuse and the tube is packed with incombustible material. The arc due to the rupture of the fuse is thus effectually smothered. It is not easy to refill these fuses and the workman (except car motormen) does not usually have replacing fuses available. As a consequence the matter is reported to the responsible person and necessary measures are taken to prevent a recurrence of the overload.

71. Circuit Breakers.—When an enclosed fuse burns out, the expense of replacement is appreciable. Moreover, there is

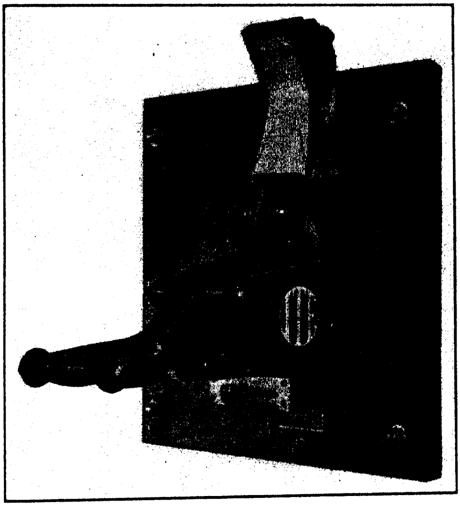

Fig. 48.

always a delay in replacing it and getting the machinery in motion again. The money value of the fuse and the lost output while the machinery is shut down may be considerable. To meet conditions of this character, the circuit breaker is well adapted.

Figure 48 shows a modern single-pole circuit breaker. The instrument consists essentially of a switch with such additions that it opens automatically when a certain current is exceeded. Either an electromagnet or a solenoid is connected so that the entire line current passes through it. When the current for which it is set is exceeded, an iron plunger or armature is lifted against the force of gravity by the attraction of the magnet. As the plunger rises it strikes and releases a latch, and a spring then

quickly opens the breaker. As the armature rises it approaches nearer to the magnet; hence it is more strongly attracted and rises rapidly, striking a strong blow. The first contact when the breaker is closed and the last as it opens is made upon renewable carbon blocks. Since the arc due to the opening is taken upon these blocks, the main contacts are protected.

PROBLEMS

33. In the case of a certain series-wound motor the full-load current is 50 amp. The voltage is 230. The resistance of the armature is 0.08 ohm and the series field 0.05 ohm. What must be the resistance of the starting box in order that the starting current shall be 125 per cent. of the full-load current?

34. What would be the desired resistance if the above machine were shunt wound?

CHAPTER VII

RATING OF MACHINES

72. Influence of Speed.—The output of a given electric generator or motor is nearly proportional to its speed. Thus, to take a simple case, assume a separately excited, continuous-current generator, rated, say, at 100 kw., 125 volts, and operated at a speed of 100 r.p.m. Such machines are frequently used direct-connected to Corliss engines, except that they would rarely be separately excited, and would usually be compound wound. If the same generator were operated at a speed of 200 r.p.m. by being direct connected to a high-speed engine, it would generate twice the voltage, or 250 volts. Since the winding has not been changed it would be capable of carrying about the same current and would, therefore, rate at twice the output, or 200 kw. It would not be safe to carry this principle too far in practice since various troubles in connection with sparking, balance, strains due to centrifugal force, etc., would be encountered. It should also be noted that the shunt winding would not be adapted to the higher voltage, although it could be used by providing sufficient field resistance.

It follows from the foregoing that the cheapest and lighest machines can be produced by operating at a high speed, and conversely, machines for direct connection to slow-speed engines or water wheels are correspondingly heavy and costly.

The speed of a generator is usually fixed by the requirements of its prime mover, since most generators are direct-connected. The speed of motors is sometimes limited in the same way, as, for example, when they are direct-connected to centrifugal pumps. Where either generators or motors are belted, it is advisable to use the highest possible speed, other things being equal, on account of the saving in first cost of the machines. Nevertheless, excessive speed should be avoided on account of rapid wear on the bearings and commutator, too much noise, vibration, etc. The action of the belt at high speeds offers, however, a still more serious objection to the employment of too high speeds. Centrifu-

gal force tends to lift the belt from the pulley, and thus to offset the effect of the belt tension. On this account, it is necessary to limit the peripheral speed of belts to a value of about 5000 ft. per minute. This means that if excessive speeds were attempted, the diameter of the pulley would have to be small, and the length would have to be correspondingly great. This would soon lead to impracticable sizes of pulleys.

73. Heating.—The speed at which the generator or motor is to operate having been determined in accordance with the foregoing principles, the other factors which determine the rating in kilowatts or horse power of the generator or motor remain to be considered. An electric machine is inherently capable of carrying enormous overloads. Any one of several factors may, however, be the limiting cause to force a lower rating of the apparatus.

The limiting factor which most frequently determines the rating is heating. There are, in the machine, whether it be a generator or motor, several sources of loss. There is mechanical friction of the journals and commutator and air friction. In addition, there is a loss due to the alternate magnetization and demagnetization of the iron of the armature. These losses are present whether the machine is carrying load or not, and their value is not seriously changed by the magnitude of the load. In addition, when the machine is called upon to carry a load, there is an I^2R loss in the windings of the armature and the series field. This loss varies in proportion to the square of the current the machine is carrying. There is also an I^2R loss in the shunt field. This is independent of the load in a shunt motor, but may either increase or decrease slightly in the case of a compound-wound generator, depending upon whether or not the machine is over- or under-compounded.

These losses cause the temperature of the machine to rise above that of the surrounding atmosphere. The rise continues until the difference of temperature is such that the heat is radiated as fast as it is generated. After this temperature is reached, there is no further rise or fall in temperature unless the load is changed. A small machine may reach this steady temperature in an hour or less, while a very large machine, may require 24 hr. or more.

Experience has shown that for the usual insulating materials the temperature should not rise higher than to about 95°C. (203°F.). The room temperature in hot engine rooms may be as

high as 40°C. (104°F.). This leaves an allowable rise *above the room temperature* of 55°C. (99°F.), and this is the figure usually adopted. The temperature is frequently measured by thermometers applied to the outside of the completed machine. Since the temperature at some points in the interior must be higher, the (American Institute of Electrical Engineers) rules specify that 15° be added to the thermometer temperature. This leaves an observed rise of 40°C. A temperature of 125°C. is allowable if the machine is insulated with mica, asbestos or other material capable of resisting high temperature.

If a machine is to be used in intermittent duty, the rating as far as the heating is concerned may be made much higher. For example, if a motor were to be applied to a hoist it would be impossible to have the full load on the motor all of the time, and the heating for the same maximum load would be less. Hence, it would be allowable to rate the motor at a higher horse power than if it were to be used in some service in which it would be subjected to continuous load.

Motors for the electric propulsion of cars are rated upon a somewhat different basis. In fact, the service of a traction motor is of such a character that a horse-power rating means little or nothing. The motors are habitually worked far above their rating for a short time, while the car is accelerating; are then disconnected from the line, and allowed to revolve idly while the car is coasting and later are brought to rest when the car is stopped by the brakes. Moreover, the motor is cooled to a large extent by the rapid current of air past it. The latest railway motors are even fitted with special means for internal ventilation. In view of these facts, many makers assign a horse-power rating to railway motors only under protest, if at all. If any rating is given, it is usually based upon a rise of 75°C. in 1 hr. on a block test in the shop. Of course, this gives little information as to what the motor will accomplish in actual service under the car.

There are three standard types of motor construction; open, semi-enclosed, and enclosed. A given motor will have a lower rating for continuous duty if built semi-enclosed and a still lower one if entirely enclosed. For this reason, enclosed motors are little used except for intermittent service and where protection against moisture, dust, chemicals, etc., is a factor.

74. Efficiency.—To a certain extent, the point of best efficiency may be taken into consideration in fixing the rating of a motor

or generator. Usually this is not the determining factor, since the efficiency of electrical machinery is so high that no material gain in average efficiency could be expected by basing the rating upon efficiency. This is not by any means the case with other types of mechanism. Thus the rating of steam engines and boilers is almost universally given as the output at the point of best efficiency. An engine or boiler can usually develop twice its rating without injury or greatly increased rate of depreciation, but the pounds of coal burned per horse-power hour will be increased.

75. Sparking.—For machines used in intermittent service (and to a lesser extent for those used for steady service), the commutation is sometimes the determining feature in assigning the rating. To understand this fully we shall have to consider what happens during the period while the current in a coil is being reversed.

Referring to Figs. 16 and 17 it will be recalled that the current in the armature of a continuous-current machine is distributed in a number of bands. Thus under the north poles, all the currents may be flowing away from the observer while under the south poles, they are flowing toward him. It is evident that the current in any one conductor must flow in one direction as long as the conductor is under a given pole and that it must suddenly reverse its direction and flow in the opposite direction as long as the conductor is under the following pole. The reversal takes place while the coil of which the conductor is part, is short-circuited by reason of the brush connecting together the two commutator bars to which the coil is connected.

The curve of current in an armature conductor of a direct-current machine is represented in Fig. 49. During the period AB the current remains constant. At the time B the brush begins to short-circuit the coil. At the time C one of the commutator bars to which the conductor is connected passes from under the brush. The current must then be the same as that in the other conductors with which it is in series or it must assume the value CG equal to its former value AF. This process must be repeated each time the conductor moves a distance equal to a pole pitch.

There is no question about the straight portions of the curve, namely, the portions AB and CD. The portion of the curve BC may, however, differ widely from that shown on account of

various causes. The form shown represents practically the best possible condition of commutation.

Just before the coil undergoes commutation a current I is circulating in it. This current sets up a small stray magnetic field surrounding itself and thus requires a certain amount of energy for its establishment. This energy must be dissipated and an equal field built up in the opposite direction while the coil is undergoing commutation. The inherent difficulty of doing this leads to most of the trouble experienced in obtaining satisfactory commutation.

The energy stored in a coil of the armature is equal to $\frac{1}{2} LI^2$ where L is the coefficient of inductance (see Art. 121) and I is the current in the coil. The expression is exactly similar to that giving the energy stored in a moving body, namely $\frac{1}{2}$

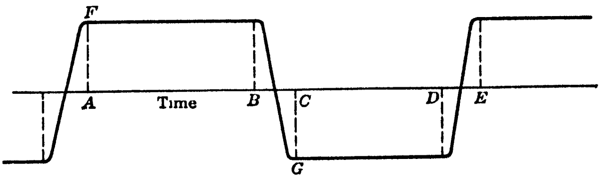

FIG 49

MV^2 where M is the mass of the body and V is its velocity. To stop the current and start it in the opposite direction is similar to stopping the motion of a moving body and setting it in motion at the same speed in the opposite direction. In the case of a moving body, if the motion is stopped by an obstacle in its path heat will be generated. This may not be apparent in the case of a small body moving at moderate speed, but is very evident in the blow of a steam hammer or the impact of a projectile. The attempt to stop the electric current leads to a similar evolution of heat, usually manifesting itself in the form of a spark.

76. Resistance Commutation.—Figure 50 shows a diagrammatic representation of the armature and coils of a continuous current machine. The coils are represented as of very short span in order to make the diagram clearer. It will be understood that the two conductors of a coil carrying the current across the face of the armature would be separated by approximately the distance corresponding to one pole pitch. Imagine that the armature is at rest and that current is entering at the brush shown. It will divide into two equal parts passing respectively to the right

and left. These currents will pass through the windings and finally emerge at the next brushes to the right and left of the one shown. Of the various coils, those marked *a* and *e* will carry the full current of the winding, *i.e.*, half the current passing into the brush. Each of the next two coils *b* and *d* will carry somewhat less current since some of the current will pass into the winding by way of the commutator bars 1 and 4 and this current will not pass through the conductors *b* and *d*. The conductor *c* when situated as shown will carry no current.

It is evident that if we were to cause the armature to revolve very slowly, and were to plot the current in one conductor, that the shape of the current curve would resemble that of Fig. 49. Thus we should have a gradual rise and fall of the current and

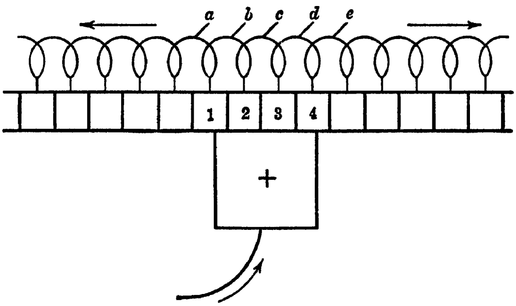

FIG. 50.

excellent commutation. However as soon as the motion of the armature becomes at all rapid, another factor enters. As previously mentioned, a current passing through an inductive circuit acts like a body in motion and resists being stopped. The consequence is that we do not have zero current in the coil *c* when the armature is in motion. If the motion is from left to right, the current in *a* will be in the same direction as that in the coils *a* and *b*. If the motion is rapid, it may even happen that the current will still be flowing in the original direction in the coil *d*. This will lead to sparking since when the coil *d* finally passes to the position occupied by *E* (and the current *must* pass in the new direction), there will be an action equivalent to trying to set a body in rapid motion in a very short interval of time. To do this the force applied must be very great. Hence a very great e.m.f. must be applied to the coil to effect the reversal and to start the current in the new direction. This e.m.f. may be

enough to cause the current to "spill over," as it were, at the point which separates the two commutator bars, thus giving rise to sparking.

The use of carbon or graphite brushes greatly assists the commutating action because such brushes have a far higher contact resistance with the commutator than metallic brushes. Thus, when the coil is short-circuited by the brush the current dies down to zero much sooner and commutation is facilitated. In fact the action is equivalent to operating the machine at a slower speed, thereby giving the current more time to reverse.

77. Effect of Rocking the Brushes.—In the foregoing it was tacitly assumed that no e.m.f. was induced in the coil except that due to the flux set up by its own current. It will be evident, however, that it is possible to provide a magnetic field such that an e.m.f. will be induced in the conductors undergoing commutation. If this e.m.f. is just strong enough and is in the right direction, it will reverse the current already present in the coil and build it up to the same strength in the opposite direction. This will result in perfect commutation, since, as the coil leaves the brush, it will be carrying the same current that will flow through it until it reaches the next brush.

This may be accomplished in an imperfect way in a generator by rocking the brushes forward or in a motor by rocking them backward. Forward rocking in a generator is required since it is necessary that the e.m.f. generated in the coil be in the reverse direction to that which it has been generating. It is therefore evident that the coil must be in a position where it is to some extent subject to the influence of the next pole. In a motor, the reverse will apply since the current is flowing in opposition to the e.m.f. generated in the armature, and the brushes must be rocked backward.

It will be apparent that this is a very simple way of meeting the difficulty, but it is subject to very serious limitations. First, it is entirely inapplicable to motors which have to be reversed, since the lead of the brushes would be wrong when the motor was operating in the reverse direction; second, the effect of armature distortion must be considered. In the shunt machine as shown in Art. 42, this results in weakening the leading pole of a generator or the trailing pole of a motor. It is, moreover, toward these poles that the brushes must be shifted in order to assist the commutation. Thus, the lines of magnetic induction

which should assist commutation are weakened as the load on the machine is increased. This is exactly the reverse of the action desired, since these should increase in proportion as the current increases. Hence the best that can be expected from this form of commutation is some help to the resistance commutation previously described. With the series machine, the case is not so bad since the strength of the field increases as the current increases. Very fair commutation can, therefore, be obtained in this way in series machines. Unfortunately series generators are rarely used, and practically all series motors are employed in service where it is essential that the motor shall operate in both directions. This prohibits the shifting of the brushes and the use of this property to assist commutation.

78. Commutating Poles.—The best method of commutation requires the use of auxiliary poles. These are commonly known as commutating poles or interpoles. A diagrammatic repre-

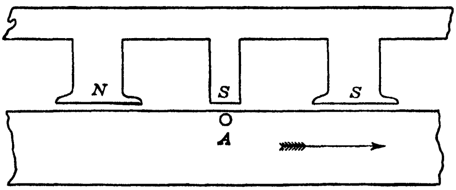

FIG. 51.

sentation of a dynamo machine using such poles is shown in Fig. 51. In addition to the main poles, small auxiliary poles are provided as shown. The winding of the main poles may be shunt, series or compound. The commutating poles are always series wound. Their width need be only sufficient to cover the conductor during the whole period of commutation. As they are in series with the armature, the strength of these poles will always be in proportion to the strength of the current in the armature, and consequently just the proper strength to give correct commutation.

A motor provided with commutating poles has its brushes set at the geometrical neutral point. It will give correct commutation operating in either direction since to reverse the motion the current in the armature, and consequently that in the series winding on the commutating poles, is reversed. Moreover, the commutation will be correct for any speed because as the speed

increases the difficulty of reversing the current increases but the voltage generated in the coil undergoing commutation by the flux from the interpole also increases in the same proportion. This is a point of great importance in adjustable speed motors. A motor with properly designed commutating poles will usually carry many times full-load current without sparking.

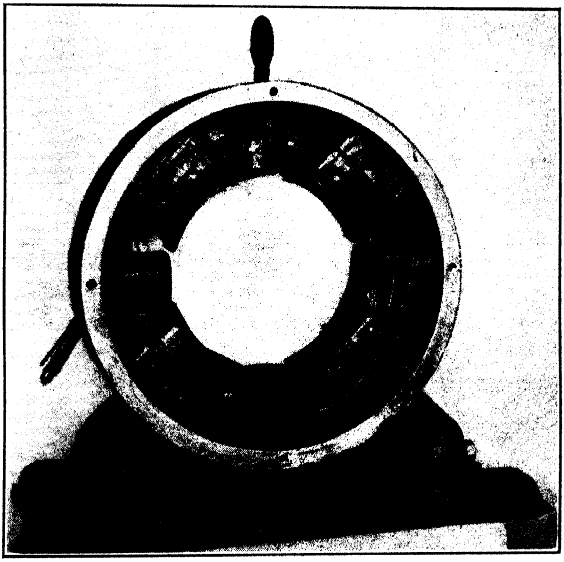

FIG. 52.

The limit is generally found in the ability of the brush to carry the current without glowing. Figure 52 shows the field of a modern motor equipped with interpoles. Many generators are also now provided with commutating poles. In fact it would be impossible otherwise to build generators with satisfactory commutation for large outputs and high speed particularly in turbine-driven continuous-current machines.

CHAPTER VIII

EFFICIENCIES AND LOSSES

79. Efficiency.—The efficiency of any machine is defined as the output divided by the input. The output and input are usually measured in watts or in horse power, *i.e.*, in terms of power, but may be measured equally well in work using as units, kilowatt hours or horse-power hours. Both experience and reason teach us that this ratio can not be greater than unity, and in fact since all machines contain imperfections, must be less than unity. Of course, it is understood that the machine at the end of the test is in the same condition as at the start; that is, we exclude such experiments as taking a partially charged storage cell, adding a small amount of energy to it, and taking out far more energy than was put in. Obviously the cell would not be in the same condition as at the start, and the process would not be capable of continuous repetition.

80. Methods of Determining Efficiency.—*The Brake Test.*—In determining the efficiency of any mechanism, there are two possible methods of procedure. The most obvious is to measure directly the work or power put in and the corresponding amount of work or power taken out. In a gas engine, for example, the quantity of gas consumed during an hour may be measured, and from a knowledge of the calorific properties of the gas used, the work or energy put into the engine can readily be computed. At the same time, the power output can be measured by means of a prony brake or similar appliance. The product of the power and the time will give the work performed by the engine. The output so obtained divided by the input reduced to the same units will give the efficiency.

This procedure is often carried out and is fairly accurate. However, it gives little information regarding the *way* in which the losses occur and is therefore of little assistance to the designer of the engine, although it may give all the information the user desires.

81. The Stray Power Method.—Another method of procedure is based upon the principle of the conservation of energy, namely, that all of the energy which goes into the engine must reappear in some form or other. A part of this appears in the form of

useful work. Another part is wasted in friction of the parts of the engine, air friction, etc. Another large part is wasted in heating the exhaust gas, and another in raising the temperature of the cooling water used to keep the cylinder within the working temperature. If the amount of work wasted during the given interval in these ways could be accurately measured and this quantity be subtracted from the work put into the engine in the same period, it would be certain that all of the remainder appeared as useful work. We should then be able to compute the efficiency and, in addition, should have information regarding the magnitude of the individual sources of loss. The designer would than be in a position to lessen these losses, if excessive, in his next design.

In the case of electrical machinery the stray power method is the one almost universally employed. This results from reasons of convenience, small quantity of power required, and on account of the greater accuracy obtainable.

Considering the last reason first, it may appear strange that an indirect method can be more accurate than a direct one. The condition arises from the high efficiency of electric machinery. Thus a large motor might easily have an efficiency of 95 per cent. If the output and the input are measured separately, with an error of 1 per cent. in each, making the output too high and the input too low, an efficiency of approximately 97 per cent. would be obtained for the final result. If, on the other hand, we had measured the loss and made a corresponding error of 1 per cent. we should have obtained a loss of 4.95 per cent. instead of 5 per cent. The result would be that the computed efficiency would be 95.05 per cent. in which the error is negligible. At the same time by using this method we should obtain the value of the individual losses, which information might be of great value.

82. Losses in Direct-current Machines.—The principal losses in a direct-current machine have already been mentioned. We shall here treat them more in detail and also study the methods of measuring them. As a simple example consider the case of a shunt motor already installed in a factory and whose efficiency is to be determined. The connections of the motor to the line are as shown in Fig. 53. This is the ordinary connection of a shunt motor except that the instruments have been added. *R* is the starting rheostat, used to limit the current in the armature of the motor during starting. An ammeter and a voltmeter should

be provided and connected as shown by the full lines. The ammeter should be capable of measuring a current equal to about 10 per cent. of the full-load current of the motor. The value of the latter can usually be found stamped on the name plate. The ammeter should be short-circuited during the process of starting the motor since the starting current with a commercial starting box will be approximately equal to the full-load current of the motor and this would be liable to injure a meter capable of measuring only one-tenth of this current.

As soon as the motor is running at full speed the short circuit may be removed from around the ammeter, and the reading will be within the limits of its scale unless there is excessive loss from some cause or other. The product of the readings of the

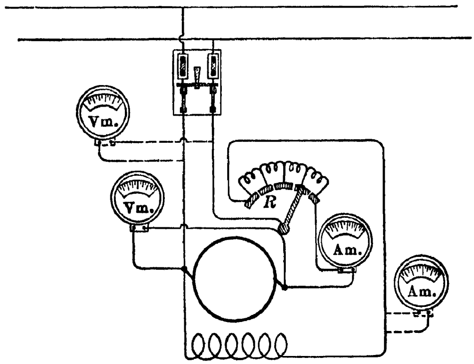

FIG. 53.

voltmeter and that of the ammeter will be the power in watts expended in the *armature* while the motor is running without load.

83. Stray Power Loss.—The power shown by the instruments is expended solely in keeping the motor in rotation. A part of it is wasted in *bearing friction* and *air friction*. The greater part of the remainder is expended in *hysteresis and eddy current losses*. If we consider a certain small portion of the iron of the armature it is evident that when this portion is under a north pole the lines of magnetic force will pass through it in a certain direction. As soon as it has passed under a south pole the lines will pass in the opposite direction. To cause this repeated reversal of the magnetism of the iron, requires a certain amount of power. The exact amount depends upon the rapidity of the reversals of the flux, the flux density in the iron and the quality of the iron.

Besides this, there is a loss in the iron due to the formation of eddy currents; that is, small stray currents in the iron of the armature. This latter loss can be made very small by making the laminations thin enough.

In a shunt motor, all of these losses are practically constant, irrespective of the load on the motor, and together constitute the stray power loss of the machine. It is true that some of these losses increase slightly with the load while others decrease but to say that the total is constant is near enough to the truth for any ordinary test. It is evident that the bearing loss will be greater when the load is large, particularly if the machine is belted. With a direct-connected machine, there may be little difference. The loss due to air friction will on the other hand be less, since the motor slows down somewhat as the load increases, and a loss of this nature varies nearly as the cube of the speed. The hysteresis loss will be nearly constant since the magnetic field is nearly constant, but will decrease somewhat as the load increases on account of the slight reduction of speed. On the other hand, the distortion of the magnetic flux under load tends to increase the loss. The same conclusion applies to the eddy current loss.

84. Shunt Field Loss.—There is also a constant loss in the shunt field circuit. The value of this is readily obtained by shifting the ammeter from the armature circuit to the shunt field circuit as shown by the dotted connections. The loss will be the product of the volts at the terminals of the shunt field times the current in the field. The voltmeter connections should also be shifted to the positions shown by the dotted lines unless the motor is running at full speed when the measurement is taken, *i.e.*, unless the resistor R is completely cut out. If this is the case, the reading will be the same in either position.

It may seem feasible to obtain both the shunt field loss and the stray power loss with one reading by connecting the ammeter in the main circuit so as to include the current in both the armature and the field at the same time. This is entirely allowable unless the separate losses are required.

85. Armature Copper Loss.—To determine the armature copper loss we must know the resistance of the armature. This is readily obtained by *preventing the armature from rotating* and taking a reading of the voltmeter and the ammeter when connected as shown in the full lines. The simplest method of pre-

venting rotation is to disconnect the shunt field. The brushes must be on or near the neutral point as otherwise the armature might magnetize the field by armature reaction enough to set the motor in motion. The simple blocking of the armature will be equally effective if it can be more conveniently done. Nearly all the starting resistance should be in circuit while this reading is being taken, and care should be taken to secure the readings within about half a minute, as the starting resistor is not sufficiently heavy to stand being in circuit continuously. It will be found that the voltage across the armature terminals is very low, rarely more than 5 per cent. of the normal running voltage of the machine when full-load current is passed. To secure accurate readings it is therefore advisable to use a low reading voltmeter, unless the original voltmeter has a separate low reading scale. It is also advisable to take several readings with say $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ and full-load current in the armature. This is done because the resistance will be found to vary somewhat as the current is changed. The variation is in the contact resistance of the carbon brushes on the commutator. A resistance of this character has a tendency to vary inversely as the amount of current passing. For the greatest accuracy the resistance should be taken while the armature is being rotated slowly by hand. Of course there must be no current in the field while this is being done, as this would introduce an e.m.f. due to the cutting of the lines of force. Once the armature resistance for various currents is known, the loss for any current is readily obtained by multiplying the square of the current by the resistance corresponding to that current.

The stray power loss of the armature taken in the manner just explained includes an I^2R loss in the armature conductors, but the loss due to this current is so small that it may be neglected without appreciable error. The stray power loss and the armature I^2R loss are of nearly the same magnitude when the machine is operating under full load. While measuring the stray power, the armature current will rarely be more than 5 per cent. of the full-load current. Since the copper loss is proportional to the square of the current, it will be reduced to 0.25 per cent. of its full-load value. Hence, it is small enough to be neglected in most cases. If greater accuracy is desired it is very easy to compute its value and subtract it from the power found in the stray power measurement.

If the test is an important one the temperatures of the windings at the time the resistances are measured should be taken. The resistances of the windings at 75°C. should be computed and used in obtaining the efficiency.

86. Calculation of Efficiency of a Shunt Motor.—All of the losses of the machine have now been considered. The input may be taken as given by the name plate rating at full load. Subtracting from this value the losses as determined will give the output in watts. This is readily changed to horse power by dividing by 746 and the value so obtained should agree closely with the horse-power rating of the machine.

It will be noticed that it has not been found necessary to load the machine at all in order to obtain its efficiency, so that one may well inquire how it is known that the machine will carry the load assumed at all. All the power that goes into the machine must reappear in some form or other. If all of this power is not accounted for in the shape of losses, the remainder will be available as useful power at the shaft of the motor. It may be quite true that the machine *might not be a satisfactory machine* at the load assumed, that is it might spark badly or overheat. To determine these features a separate investigation is required. This is carried out most readily by actually running the machine under its rated load for a sufficient time to allow the temperature to rise to its highest value, and at the same time, observing the character of the commutation.

Although the computations may be made without the use of a formula it may be desirable for certain purposes to express the efficiency by means of a formula as follows:

$$\eta = \frac{EI - P - EI_s - I_a^2R_a}{EI}$$

in which η = Efficiency.
P = Stray power.
E = Terminal voltage.
I_s = Shunt field current.
$I_a = I - I_s$ = Armature current.
R_a = Armature resistance.
I = Total current, *i.e.*, line current.

An actual example may serve to make this statement clearer. A certain motor is rated at 25 hp., 900 r.p.m., full-load current 97 amp., volts 220. The machine is shunt wound. Connected

as shown in Fig. 53 and running at full speed with all of the starting resistance cut out, the machine required 3.6 amp. in the armature at a pressure of 220 volts. With the ammeter transferred to the shunt field circuit, 2.75 amp. at a pressure of 220 volts was indicated. With a higher reading ammeter, and a lower reading voltmeter substituted for those of Fig. 53, and with the starting lever on the first notch, a current of 100 amp. passed through the armature and the voltmeter indicated 11.7 volts. The armature was blocked so it could not rotate. The computations are as follows:

Resistance of armature and brushes, 11 7 ÷ 100 =	0.117 ohms.
Stray power loss, 3.6 × 220 =	792 watts.
Shunt field loss, 2.75 × 220 =	605 watts
Armature copper loss $(97–2.75)^2 \times 0.117$ =	1,040 watts.
Total losses,	2,437 watts
Input, 97 × 220 =	21,340 watts.
Output = input − losses =	18,903 watts
Output in horse power = 18,903 ÷ 746 =	25 4 hp.
Efficiency = output ÷ input =	88 5 per cent

The same method may be applied to any other load. Thus, if it is desired to know the efficiency at ¼, ½, ¾, 4/4, and 1¼ load, we start by estimating the current required at these various loads. It is known from experience that at ¼ load, the efficiency of a well-designed motor will be lower than at full load. Consequently somewhat more than one-fourth of full-load current will be required, say in this case 26 amp. It is not essential to come very close to the exact current. If on completing the computation it is found that the assumed current gives a horse power too far from the value sought, we can readily select one more nearly correct and perform the computation again. Selecting 50 amp. for ½ load, 73 amp. for ¾ load, and 121 amp. for 1¼ load, the values for the following table can be computed:

Loads	¼	½	¾	4/4	1¼
Amperes (estimated)	26	50	73	97	121
Stray power	792	792	792	792	792
Shunt loss	605	605	605	605	605
Armature copper loss.	63	261	577	1,040	1,636
Total loss	1,460	1,658	1,974	2,437	3,033
Input	5,720	11,000	16,060	21,340	26,650
Output in watts.	4,260	9,342	14,086	18,903	23,617
Output in horse power	5 70	12.5	18.9	25 4	31 7
Efficiency, in per cent	74 5	84 8	87.6	88.5	88.5

87. Performance Curves.—The foregoing figures are embodied in the curves of Fig. 54. These curves can be used in various ways. Thus, suppose the motor from which these data were taken is belted to a line shaft, and it is found that the normal current taken by the motor is 85 amp. An inspection of the curve will show at once that the motor is developing an output of 22 hp. Another reading taken when no machines are connected to the line may show that a current of 60 amp., corresponding to 15.2 hp. is required. These results will at once suggest the probability that more efficient operation of the factory can be obtained by a rearrangement of the shafting or machines, since the

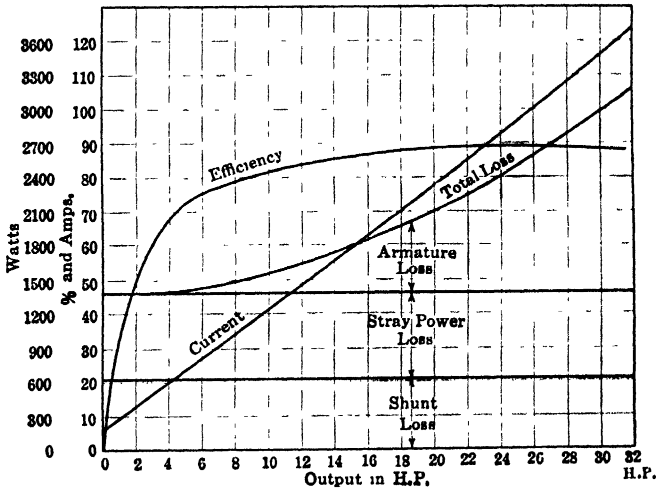

FIG. 54.

power required to operate the shafting alone is 69 per cent. of the power needed for the operation of the shafting and machines. So valuable are tests of this character that in some places graphic recording meters are installed in connection with important motors, in order that a continuous record of the conditions of operation may be obtained.

88. Efficiency of a Generator.—In a very similar manner, the efficiency of a generator can be obtained. The stray-power loss can be obtained as for a shunt motor, namely by running the machine from some other source of electric power as *a shunt motor* under no load. The stray-power loss so obtained may be considered as constant if the machine is shunt wound or flat-

compounded. If overcompounded, the stray power should be corrected in proportion to the voltage generated. The results so obtained will not be strictly correct, but will be close enough for ordinary purposes. The shunt loss may be obtained in the same manner as before. If the terminal voltage of the machine varies, the shunt loss must be corrected in proportion to the square of the voltage. The armature and brush resistance may be obtained in the same way as for the shunt motor. The resistance of the series field may be determined separately, or the voltmeter may be applied outside of the series field terminals, in which case the resistance of the armature, brushes and series field may be obtained with the one measurement. The output should be taken from the name plate and the losses added to it in order to obtain the input. The ratio of these two will be the efficiency at full load.

The curves of Fig. 54 show that the maximum efficiency occurs at slightly over full load. This would be an economical motor if it were to operate at practically full load all the time. In the case of a motor in which the load is variable and frequently drops to a low value, it would be preferable to have the maximum efficiency occur at about three-fourths of full load. This would give a better average efficiency for the range of work performed. The curve will also show the inadvisability of using a large motor to carry only a small load. Not only will such a motor be more costly, but the average efficiency will be lower than that of a motor of such proper capacity.

89. Change of Efficiency with Speed.—In general, motors of high or moderate speed will be cheaper in first cost and more efficient than those of low speed. Thus, if the foregoing motor had been operated at 450 r.p.m. instead of 900, its rating would have fallen to about $12\frac{1}{2}$ hp. The first cost would have been nearly the same, about the only saving being in the starting box. The copper losses in the armature and in the shunt field would be the same. The stray-power loss would be less, perhaps one-half as much, at the lower speed. At full load, the total losses would become $1040 + 605 + 396 = 2041$ watts. The input would be half as great as before or 10,650 watts. Subtracting the losses, leaves an output of 8609 watts (11.54 hp. and the efficiency is $8609 \div 10{,}650 = 0.8083$ or 80.83 per cent. Thus the efficiency is decidedly lower at the lower speed. The heating will be nearly the same as before. It is true that the losses are somewhat less,

but on the other hand the armature is not revolving so fast, and the opportunities for radiation are therefore not so good. In practice the motor would undoubtedly be rated at 12½ hp. or a little larger load than that assumed.

A similar analysis applied to generators would show that they also are lower in first cost and higher in efficiency at high speeds than at low ones. This accounts in a measure for the success of the high-speed turbo-alternator.

PROBLEMS

35. A certain series-wound motor operating at 75 volts and 30 amp has a speed of 914 r.p m. The machine exerts a pull of 10 lb 6 oz at the end of a brake beam 17⅞ in. long. What is the horse-power output of the motor? What is the output in kilowatts? What is the efficiency of the motor?

36. In determining the efficiency of a shunt-wound motor the following data were obtained. With the motor stationary, the current through the armature was adjusted by means of a rheostat to 100 amp The drop across the brushes was 6 2 volts With no load the machine had a speed of 1200 r p m. and took a current of 5 1 amp through the armature and a shunt field current of 3 5 amp, the line voltage being 230 For inputs to the motor of 25, 50, 75, 100 and 125 amp compute the stray power loss, the armature copper loss, the field copper loss, the outputs in kilowatts, the horse-power outputs, the torques in foot-pounds and the efficiencies. Arrange the results in the form of a table like that on page 95.

37. The foregoing machine has four poles and the armature is lap wound The armature is reconnected with a wave winding, using the same armature coils. This has the effect of reducing the paths through the armature from four to two The field is not altered What is the resistance of the rewound armature? At what speed will the armature take no current from the line? What will be the stray power loss? (This loss may be taken as being approximately proportional to the no-load speed.) Compute the foregoing quantities for the rewound motor for currents of 12 5, 25, 37 5, 50 and 62 5 amp. (These currents give approximately the same current in each conductor of the armature as before)

38. If the motor as originally wound was rated at 25 hp., what is the new rating, the rating in each case being based upon heating?

39. A certain 10 hp 115-volt shunt motor with a speed of 1200 r p.m. has a field loss of 200 watts and a stray power loss of 450 watts. The resistance of the armature is 0 02 ohm. Find the value of the armature current for which the efficiency is a maximum and compute the efficiency at this current This problem is based upon the fact that the efficiency is a maximum when the fixed and the variable losses are equal.

CHAPTER IX

DIRECT-CURRENT MEASURING INSTRUMENTS

90. Voltmeter and Ammeters.—In continuous-current work the measurements most frequently made are those of current and e.m.f. An instrument for the measurement of the former is called an ammeter; for the latter, a voltmeter.

Occasionally very crude methods serve a useful purpose in indicating roughly the value of a current or an e.m.f. Thus, it is related that one of the early power houses had a comparatively small copper wire in series with each of the generators. When the wire became red hot due to the passage of the current, the attendants knew that it was time to start up a new machine. At the same period, it was a very common practice to use an incandescent lamp as a crude voltmeter, the attendant estimating the voltage from the color of the filament. All are also familiar with the fact that it is possible to detect the presence of a moderate voltage by allowing the current to pass through the body. Naturally this is not a method that one would recommend for general adoption.

91. The D'Arsonval Type of Instrument.—The most common type of direct current measuring instrument is based upon the action of a magnetic field upon a current. Such an instrument, known as the D'Arsonval type, is illustrated in Fig. 55. A permanent magnet of the horse-shoe type is provided with pole pieces of soft iron, and to make the magnetic circuit more nearly closed, a cylinder of soft iron is supported inside the pole pieces. A short air gap is left between the cylinder and the pole pieces and a coil of fine wire is so mounted that it can turn, restrained by means of one or more fine hair springs. The current is led to the coil by suitable flexible conductors. The hair spring may be used as one of these conductors or two springs may be employed, the current being passed in through one of them and out through the other.

As previously noted, when a current is passed through a conductor lying in a magnetic field, the conductor tends to move across the lines of induction. In the instrument described, it is evident that one side of the coil will be urged in one direction

and the other in the opposite direction, and the coil will tend to turn upon its axis. Since the spring restrains the motion, the angle through which the coil turns will be dependent upon the strength of the current. By providing a suitable pointer and scale, it is possible to measure the current.

In order that the instrument may be sensitive, it is necessary that the coil be very light, and, therefore, wound with fine wire. Moreover, it would be impracticable to provide flexible leads for any but a very small current. Therefore the instrument can not be used as it stands for any but small currents.

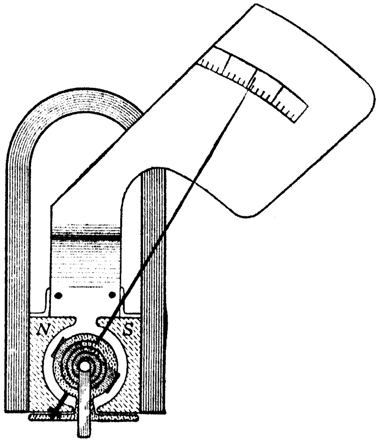

FIG. 55.

This condition is readily met by the use of shunts. Suppose that we had an ammeter with an extreme range of 0.1 amp. and a resistance of 1.0 ohm. If we connect in parallel with it as shown in Fig. 56, a shunt of slightly more than 0.001 ohm (0.001001 ohm to be exact), the current will divide, 999 parts passing through the shunt and one part through the instrument. It is obvious that the instrument is

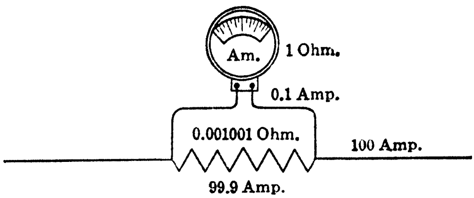

FIG. 56.

now capable of measuring a current as large as 100 amp. In a similar way we may construct as many shunts as we wish and an ammeter with its outfit of shunts may have any range within reason. In many cases the shunt may be enclosed in the same case as the ammeter, forming a self-contained instrument of large range.

92. The Voltmeter.—The same instrument may also be used as a voltmeter. To do this we actually measure the *current* through a known resistance and calculate the corresponding voltage. By choosing a suitable value for the resistance, the calculation becomes very simple, or by using a suitable scale the reading may be made direct. Thus, if the instrument considered be connected in series with 999 ohms (see Fig. 57), the total resistance of the circuit will be 1000 ohms. If we should apply 100 volts to this circuit, a current of 0.1 amp. would flow. This we have assumed will give the full scale deflection of the instrument. If we mark the point to which this current deflects the pointer 100 it is evident that the reading will be direct.

Similarly, if 99 ohms be connected in series with the instrument, its maximum reading will be 10 volts, while 4999 ohms would give it a range of 500 volts.

FIG. 57.

It will be apparent from the foregoing that one instrument, provided with suitable shunts and resistors, may be used to measure a very wide range of currents and voltages.

93. The Plunger Type of Instrument.—In the foregoing instrument a coil of wire is caused to move, due to the action between it and a permanent magnet. Instead of using this construction, the coil may be fixed in position and the permanent magnet allowed to move. However, it is necessary to take into account the action of the magnetic field of the earth upon the permanent magnet. For this reason it is customary in this type of instrument to use a piece of soft iron instead of the permanent magnet.

In one construction of the plunger type, the coil is in the form of a solenoid and the soft iron plunger forming the core is pulled down into the solenoid by the action of the current. The movement is resisted by springs or by the action of gravity. Such an instrument is not readily portable, and a large amount of power is required on account of the weight of the moving parts. For these reasons, this form of instrument is not much used at the present time.

An ingenious modification of this form of instrument is shown

in Fig. 58. The coil is inclined at an angle of approximately 45°C. with the horizontal. The "plunger" consists of a very small piece of soft iron, and it is also arranged at an angle with the axis of the coil. When current is passed through the solenoid, the soft iron plunger tends to place itself parallel to the axis of the coil. In doing this it rotates upon its axis and carries the pointer with it. This turning is resisted by a spring. This form of instrument is cheap to construct and gives very satisfactory results for ordinary commercial measurements. It requires more power to operate than the D'Arsonval type; is not so sensitive and suffers the disadvantage that the scale is somewhat crowded at both ends, that is, the deflections are not proportional to the currents

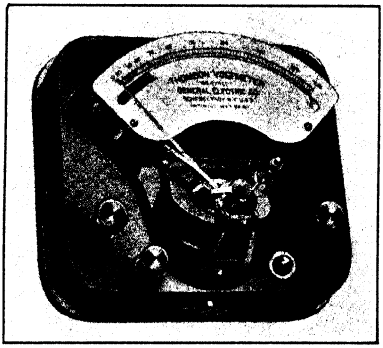

FIG. 58.

passing through the coil. It may be used for alternating as well as for direct currents.

94. Measurement of Power.—The power in a continuous-current circuit is usually measured by taking the product of the volts and the amperes in the circuit. It is also possible to use a wattmeter of the form described in Chap. XIII, but the former method is usually the simpler.

95. Measurement of Work.—*The Watthour Meter.*—In charging for direct-current energy it is necessary that we have an instrument that will add up or integrate the total amount of energy used during a given time. This is usually accomplished by means of a small continuous-current motor, connected by gearing to a counting mechanism which records the revolutions of the meter. The motor differs from the common type of power motor in that usually no iron is used in either the field or the

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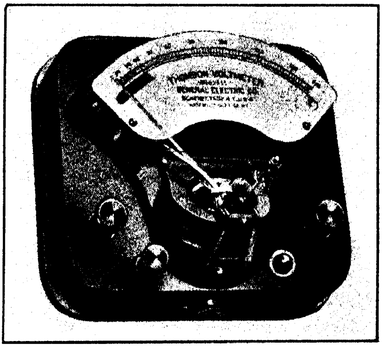

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armature. This is so that any effect due to the saturation of the iron may be avoided.

The armature is connected as a shunt across the line and the field is connected in series with the line. A high resistance is connected in series with the armature. This resistance must be great enough so that the drop across it will be large in comparison with the back e.m.f. of the motor; that is, the current passing

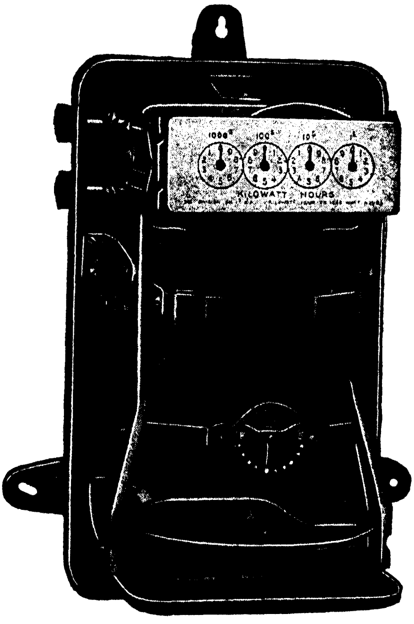

FIG. 59.

through the armature must be dependent solely upon the e.m.f. of the circuit and must not be influenced by the speed of the motor. Under these circumstances the torque of the motor will be proportional to the e.m.f. of the line and also to the current flowing, or to the power in the circuit.

The rotation of the armature is resisted by a disc, usually of aluminium, mounted on the shaft and rotating within the field of a permanent magnet. This produces a drag or counter-torque proportional to the speed. The net result is that the speed of the motor is proportional to the power in the circuit and consequently the reading of the counting mechanism in a given period

is proportional to the product of the power and the time; or to the energy. As will appear later, this same form of meter may be used on alternating-current circuits, but is not usually so used since cheaper and possibly more satisfactory forms are available for such circuits. Figure 59 shows the construction of a continuous-current watthour meter.

PROBLEMS

40. A certain direct-current D'Arsonval type instrument is intended for use both as an ammeter and as a voltmeter. The scale is divided into 100 parts, the resistance of the instrument is 5 ohms and the current required to produce a full-scale deflection is 10 milliamp. (*i e.* 0.010 amp.) What resistance must be used in series with this instrument in order that it may be used as a voltmeter, giving a full-scale deflection with 1 volt? With 100 volts? With 500 volts?

41. What would be the resistance of the shunt used with the same instrument if it is to give a full-scale deflection with 1 amp ? With 10 amp.? With 1000 amp.?

42. An incandescent lamp has a resistance of 500 ohms and the difference of potential across its terminals is 100 volts. A voltmeter having a resistance of 5000 ohms is connected directly across the terminals of the lamp and an ammeter whose resistance is 0 05 ohm is connected so as to measure the current going to both the lamp and voltmeter. What is the percentage of error in the determination of the current in this manner? Of the voltage? Of the power? What would be the corresponding errors if one terminal of the voltmeter were changed so that the current to supply the voltmeter did not go through the ammeter? Which connection is preferable? Which would be preferable in case the power loss in a very low resistance were to be measured?

CHAPTER X

ADJUSTABLE SPEED MOTORS

96. Adjustable Speed Motors.—An adjustable speed motor is one whose no-load speed may be adjusted to any value within a certain range and which will maintain approximately that speed for any load within the capacity of the motor. Motors of this type are frequently required for driving lathes, drill presses, planers and other machine tools. When an ordinary shunt motor is applied to drive a lathe, it is generally used to drive the counter-shaft and the speed changes can be obtained in the ordinary way by means of cone pulleys and back gears. This method of operation, however, leaves large gaps between the various speeds, and it is frequently desirable to provide a means for obtaining any speed of rotation of the work and not simply a few speeds differing considerably from one another. This need is supplied by the adjustable speed motor.

With a motor applied as described to a standard lathe, the range of speed control would not need to be great. Probably a range of one to one and one-half would be sufficient. However, in the case of this or other machine tools it sometimes seems desirable to reduce the range of mechanical speed adjustment and increase the electrical speed adjustment. Therefore, speed ranges of as high as one to four are sometimes called for.

97. Shunt Field Control.—The fundamental equation of a direct-current motor is (see Article 58):

$$n = \frac{E - RI}{\Phi_1 N}$$

It will be evident from an inspection of this formula, that for a given value of the current I, the speed may be changed by a change in any one of the other factors. The external voltage, E; the resistance in the armature circuit, R; the number of conductors on the armature, N; or the flux per pole, Φ_1; may be changed and will produce a corresponding change in the speed. These various methods will be examined separately.

The last method, varying the flux of the motor, is the one most frequently employed. The simplest way of doing this is to use an adjustable rheostat in the field circuit. The connections are shown in Fig. 60, the starting box being omitted for the sake of simplicity. Varying the resistance, *R*, changes the current flowing through the shunt field circuit, and consequently the strength of the magnetic field. This method can be applied to any standard shunt motor if the range of speed adjustment desired is small, say 10 or 15 per cent. The cost is low and the results are entirely satisfactory.

When larger ranges of speed are desired, it generally becomes necessary to adopt special motors. This necessity arises particularly from the difficulty of avoiding sparking, and the tendency of such a motor to become "unstable" in speed. Both of these difficulties are the result of armature reaction. It has been shown that the armature current distorts the magnetic field. In a motor the field is shifted in the direction opposite to the rotation, and at the same time is weakened. The result of the shifting is that the neutral point is no longer midway between the poles but is shifted backward. Consequently the coil undergoing commutation instead of being in a zero field or in one in a direction to assist commutation, may be in a field such as to oppose commutation. This tendency is greatly augmented when the main field is weakened to secure high speed, since, on account of the field being weak, it is easier for the armature current to distort it. The result is that the commutation is poor at high speeds.

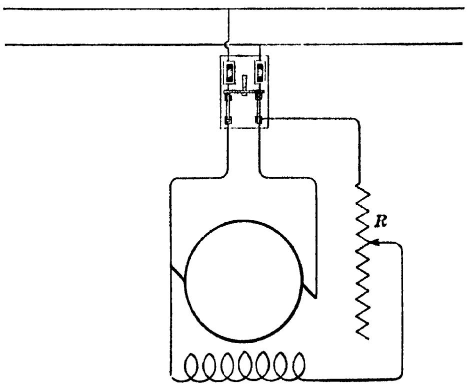

FIG. 60.

To keep the coils undergoing commutation in a better field, the brushes often receive a backward lead. This, as already pointed out, is not a complete cure and at the same time it introduces the difficulty that the armature current, in addition to distorting the flux, also weakens it. Thus, an increase of load may cause a decrease of flux. This causes a decrease of the back e.m.f. and a corresponding increase of current. In turn, this again weakens the field and leads to a still greater current. The action may readily become cumulative, and the current will continue to increase until the fuses blow or the circuit breaker acts and opens the circuit. Before this takes place, the motor may have attained such a high speed that the wires are thrown from the armature or the machine is otherwise injured.

98. Use of Commutating Poles.—Among the many methods used to overcome the two foregoing defects, the use of commutating poles is perhaps the simplest. These, by providing a separate field for commutation, allow effective commutation at all speeds, and with any current in the armature within reasonable limits. At the same time, they permit or even require the setting of the brushes at the geometrical neutral and thus the possibility that the field will be seriously weakened by the armature current is removed.

99. Methods of Changing the Magnetic Circuit.—Besides the method of weakening the field by using a rheostat in the shunt circuit, the flux can also be weakened by changing the reluctance of the magnetic circuit. One method of doing this is to form the field cores with a sliding plunger as part of the pole. The construction of the Stow adjustable speed motor is shown in Fig. 61. Mechanical means must be provided to withdraw the cores. In the motor illustrated, this is done by means of a number of bevel gears connected together by shafts and all operated by means of a hand wheel.

In another motor which depends upon somewhat the same principle, the armature is slightly tapered and the bore of the field is also turned to the same taper. Means are provided to slide the armature lengthwise. This results in an increase or decrease of the air gap, and a corresponding change in the flux. In both of these methods the m.m.f. of the main field is not altered. The flux is consequently not shifted to the same degree as would be the case with a plain shunt machine. They are therefore not so liable to spark or be unstable in speed with a weakened field.

The objection to these forms is that they involve somewhat complicated mechanical structures.

100. Speed Variation by Means of Resistance in the Armature Circuit.—The connections for this method of control are shown in Fig. 62. Referring to the formula, speed variation in this case is secured by varying the resistance R. In the discussion of the formula R has been considered as the resistance of the armature and brushes. Ordinarily this is the only resistance in the armature circuit. In this method of control additional resistance is purposely introduced. This causes the motor to run more slowly in order that there may be sufficient difference between the applied and the back e.m.f. to force the current through the in-

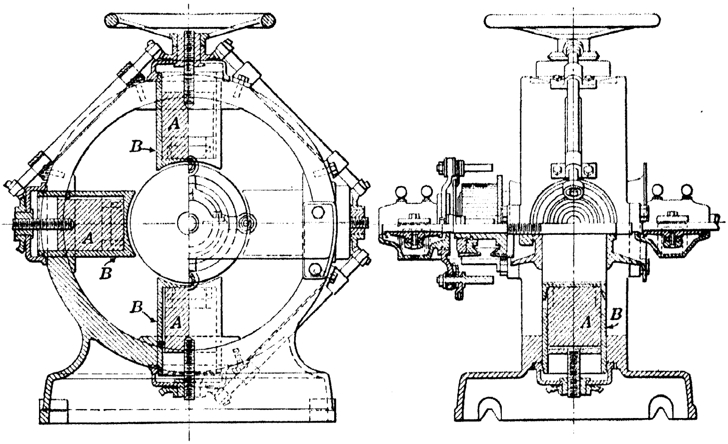

FIG. 61.

creased resistance. The result is that the speed falls off more rapidly with increase of the torque. The shape of the speed torque curve depends upon the amount of resistance inserted in the armature circuit. Thus, in Fig. 63 the highest curve represents the conditions when the resistance is a minimum, *i.e.*, when all the external resistance is cut out. The lower curves represent the effect of successively greater resistances. It will be noted that the speed is the same in all cases at zero torque. Since a small torque is required to overcome the resistance of the bearings, the air friction and the iron losses, the motor running with no load will not quite attain these speeds, but will run at some such speed as that represented by the intercepts

with the dotted line *AB*. The distance *OA* is the torque required to overcome the resistances mentioned.

This method of control is seriously defective in two respects. In the first place, it does not comply with the definition of an adjustable speed motor. Thus, if the motor is operating with such a resistance in circuit that its speed torque curve is represented by No. 3, and is acting against a torque *OF*, such that its speed is *FD*; if the torque be increased to *OG*, its speed will fall to *GE*. This may be decidedly objectionable in certain applications. If the motor were driving a lathe operating on a piece of such shape that the tool was not at all times in the cut, the speed would in-

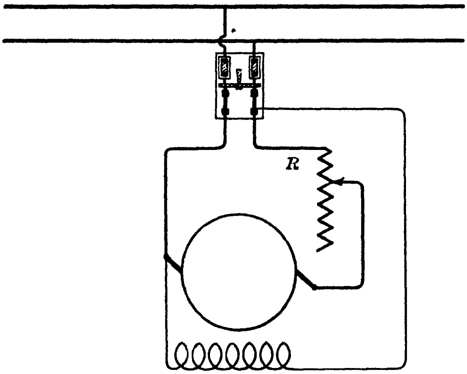

FIG. 62.

crease to a great extent when the tool ran out of the cut. This would lead to a serious shock when the tool again began to cut the metal.

A second objection is that the efficiency is low if large reductions in speed are secured. If the speed is reduced to half of normal, the current required for a given torque will be the same as though the motor were operating at full speed. The useful work done, however, will be only half as great, since the speed is halved. The net efficiency can not therefore be more than 50 per cent. in the example cited. A reduction of speed to one-fourth of normal would result in an efficiency of less than 25 per cent. and so on.

The horse-power output of the motor is also reduced in proportion to the speed. The maximum torque that can be developed remains practically the same no matter what the speed,

since this is determined by the current-carrying capacity of the armature. Since the speed is reduced, the capacity in horse power is reduced in the same proportion.

The two foregoing methods are often applied in combination to fan motors. The rheostat is usually so arranged that moving a lever from left to right first cuts out the resistance in the armature circuit. After all this is out, a further movement of the lever cuts in the field resistance thus further increasing the speed. Since the power to operate a fan varies about as the cube of the speed, the power required at low speed is very small and the low efficiency is not so objectionable.

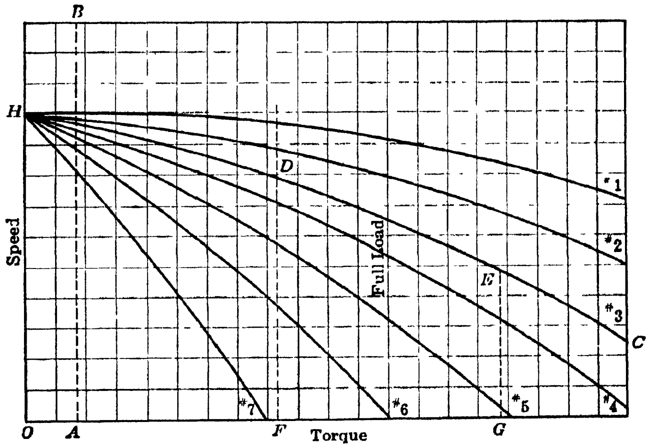

FIG. 63.

101. Motors with Two Commutators.—Variation of the number of conductors in series is most easily made by providing a double winding on the armature. Each winding is connected to its own commutator, and the two windings are entirely insulated from one another. Thus, suppose one winding is provided with 100 conductors in series and the other with 160. If the no-load speed with the former were 500 r.p.m., the speed with the other would be inversely proportional to the number of conductors or $500 \times 100 \div 160 = 313$ r.p.m. Moreover, the two windings may be operated in series, giving the equivalent of 260 conductors or in opposition giving the equivalent of 60 conductors. The respective no-load speeds would be 193 and 833 r.p.m. Intermediate speeds can be secured by the use of resistance in the shunt

field. The efficiency will be high for all speeds, and the normal horse-power rating of the motor will be constant no matter what the speed. This method is little used at the present time. This condition results not so much from any inherent defect of the method, as from the fact that the same result can be secured with somewhat simpler apparatus by the use of the shunt motor with commutating poles.

102. The Multi-voltage System.—We have still to discuss the possibility of obtaining speed adjustment by changing the applied voltage, E. This is unfortunately not readily done except in special cases. One of these occurs when the generator is used exclusively to furnish current for a single motor. This case

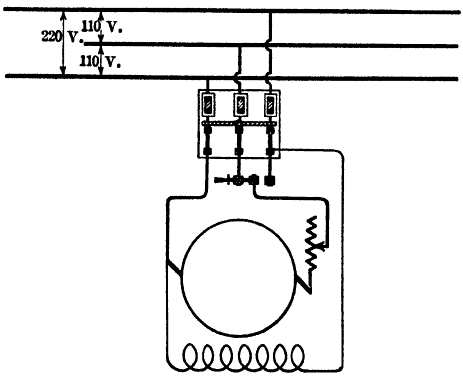

FIG. 64.

will be considered presently. There is also an opportunity to employ this method when three-wire distribution is used to supply the factory where the motors are located. In this case there will be a certain voltage between one of the wires and either of the others, and twice this voltage between the other two. The connections are as represented in Fig. 64. The shunt field is supplied at a constant voltage. The armature is so connected to a single-pole, double-throw switch that it may be connected to the neutral wire and one of the outside wires, or to the two outside wires. In the example shown, the voltage applied to the armature may be either 110 or 220 volts. The speed would be twice as great with the latter as with the former. This method of speed variation is entirely satisfactory where three-wire service is available. It may be combined with the method of

control using field resistance or with that using resistance in the armature circuit.

The connections of the shunt field should not be changed at the same time as the armature connections. This if done would result in weakening the field current in the same proportion as the armature voltage; the flux would also be changed but not in the same proportion. If it were not for magnetic saturation the field would be weakened to the same extent as the armature voltage and the speed would not be changed. In practice, the speed would be less with the lower voltage but not half, as might be expected.

103. The Ward-Leonard System.—In Fig. 65 is shown a method of control which may be used to great advantage when a

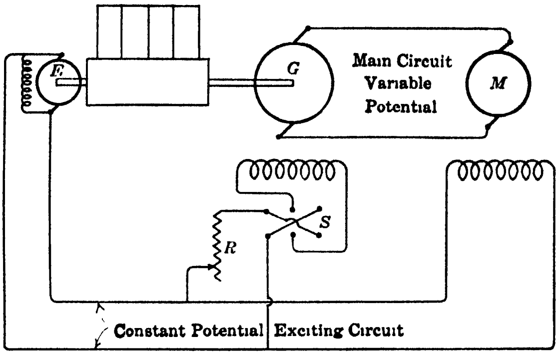

FIG. 65.

single motor (or a group of motors all used for the same purpose) takes its power from a generator used to supply power to this motor only. The main generator marked G in the diagram, may be driven by a gas or steam engine, a continuous-current motor, an alternating-current motor, either single phase or polyphase or other source of power. The driving motor whatever its type, is usually arranged to operate at practically constant speed.

The generator G and the exciter E are driven from the engine or motor by direct connection, belting or in any other suitable manner. The exciter operates at constant potential, and is therefore usually compound wound. If a continuous-current driving motor is used, this exciter may be dispensed with, and the exciting current may be taken directly from the supply mains (not from the main generator). In circuit with the

field of the generator are a regulating rheostat R and a reversing switch S. The former serves to vary the field current from zero to full strength, and the latter serves to reverse this current. To start the motor M, the field circuit of the generator is closed and the field current gradually increased from zero to a strength sufficient to give the speed desired. To reverse the motor, the field strength of the generator is reduced, reversed and strengthened in the reverse direction. Thus the entire control of the motor is accomplished by varying the small field current of the generator, and it is never necessary to break the large current flowing between the generator and the motor. In order to handle the full-load armature current without sparking at weak field strengths, the generator is usually provided with commutating poles.

Applications.—It is obvious that the foregoing system is far more costly than direct connection between the prime mover and the load. Its use is therefore justifiable only in special applications where the increased flexibility is important enough to offset the added cost. Some of these applications will be discussed in the following pages.

104. Rolling Mills.—In a rolling mill there is always a sudden demand for great power when the billet enters the rolls. It is also frequently necessary that the rolls be promptly reversed in direction. The current supply is commonly three-phase alternating. A three-phase alternating-current motor is not well adapted to quick reversals of direction since the starting torque is small. The continuous-current motor on the other hand can be very quickly and easily reversed. The writer has in mind the case of a 1200-hp. rolling mill motor which is brought from full speed ahead to full reverse in 4 sec. The Ward-Leonard system may be used in this work. A three-phase motor, taking its power from the supply system may drive a direct-current generator and a heavy flywheel mounted on the same shaft. The direct-current motor which drives the rolls receives its power from the generator. The sudden rush of power required when the ingot enters the rolls is supplied largely by means of the flywheel. This slows down while it is delivering this power and is subsequently accelerated by the alternating-current motor. Thus the power output of this latter is made more nearly constant and a smaller motor may be employed. When being stopped, the continuous-current motor returns power to the generator and

flywheel, since the generator field is weakened at this time and the motor voltage is consequently higher than that of the generator. This results in a large power saving.

105. Propulsion of Ships.—In the last few years, the application of the internal combustion engine and the steam turbine to the propulsion of ships has received great attention. Both of these prime movers operate most economically at speeds considerably above the most efficient speeds of the screw propeller. This is particularly the case with the turbine. Moreover, the turbine is absolutely irreversible, and it is necessary to provide entirely separate turbines for reversing the ship. The internal combustion engine also suffers in comparison with the steam engine in regard to its ability to reverse promptly and with certainty, and in its ability to operate at low rates of revolution while maneuvering. By using an electrical method of transmitting the power from the engine to the propeller, the speed of each may be chosen without reference to that of the other and reversal becomes very simple. The engine would be of the governed type operating at full speed. The control elements may be located in the pilot house, thus making the transmission of signals with their delay and possibility of mistake unnecessary.

It should, however, be pointed out that in the case of the turbine ship, an alternating-current generator and induction motors would probably be used, at least in the larger sizes, since it is difficult to construct very large direct-current generators to operate at turbine speeds. The use of continuous-current propulsion is therefore limited to small vessels.

106. Operation of Gas-electric Cars.—Cars operated by gasoline or oil engines are finding considerable favor at the present time. They are of use as local cars in connection with trunk lines and as independent systems when the frequency of operation is such that it would not pay to install electric traction with its high cost for the overhead conductor or the third rail. The internal combustion engine operates best at nearly a constant speed. It has no starting torque and consequently provision must be made so that it may be in operation before the car is started. It is then necessary to provide some means of connecting the engine to the wheels and varying the speed ratio between them. Mechanical gearing similar to that used on automobiles has been employed in some cases. An electric connection is, how-

8

ever, undoubtedly more flexible, and it is the opinion of many builders of this class of apparatus that it is also more reliable.

For this purpose, the series motor is preferable to the shunt. Usually two or four motors are used. This is done so that more of the wheels may be used in driving the car, thus enabling the car to produce a greater tractive effort. The different series motors may be connected in parallel. Reversing the direction of the voltage of the generator would not reverse the direction of rotation of the series motors, since the field and the armatures would be reversed at the same time. The reversing switch in the generator field is therefore omitted and instead a switch

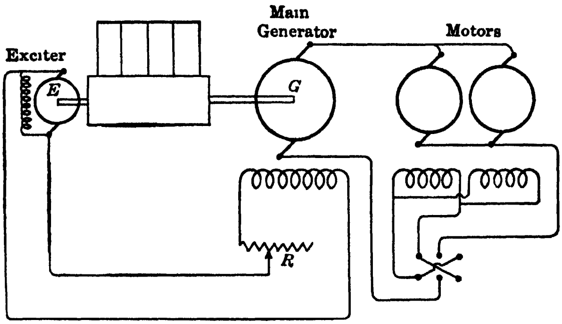

FIG. 66.

is provided to reverse the series fields of the motors. Unfortunately, in this case we are obliged to have a switch in circuit with the main current, but the controller is so arranged that this switch can not be opened until the field circuit of the generator is broken. There is therefore no current flowing in this circuit when the fields are reversed and consequently no burning of the contacts can occur on account of breaking the circuit. One way of connecting such a set is indicated in Fig. 66. Provision is also usually made so that the motors may be connected either in parallel or in series. This is done so that the motors may be operated in series for starting or at low speeds. Only half as much current is required for a given torque with two motors in series as with the same motors in parallel.

PROBLEMS

43. A motor connected as shown in Fig. 62 takes a current of 100 amp. through the armature at a pressure of 250 volts and operates with no external

resistance in the armature circuit at a speed of 600 r.p.m. giving an output of 27 hp. If the resistance of its armature is 0.05 ohm, how much resistance must be connected in series with it in order that the speed may be reduced to 300 r.p.m., the current remaining the same? What is the output of the motor at the reduced speed?

44. In the case of the same motor with the armature resistance as determined in circuit, what will be the speed if the load is reduced so that the armature current falls to 50 amp.? What if it is increased to 150 amp.? To 200 amp.?

45. A motor is connected as shown in Fig. 64. The voltages are as indicated. The resistance of the armature is 0.12 ohm and the armature current is 50 amp. If the speed is 700 r.p.m. when the armature is connected to the 110-volt circuit, what will be the speed when it is connected to the 220-volt mains? What would be the approximate speed if both the armature and the field were connected to the 110-volt circuit? Can this be stated definitely with the data at hand? Is this a usual connection?

46. If the stray power loss of the motor in the above problem is 500 watts at 110 volts or 1000 watts at 220 volts and the resistance of the shunt field is 150 ohms, what is the efficiency at 110 volts and 100 amperes armature current? At 220 volts and the same current?

CHAPTER XI

ALTERNATING CURRENTS

107. General Principles.—In studying the principles of alternating currents, it is highly desirable to keep clearly in mind the analogy between the laws governing the action of electric currents, and those applying to tangible matter. These laws are almost the same in the two cases. Where there are differences, it will usually be found, contrary to the general impression, that the laws governing ordinary matter are more complex than those of electricity. In fact, paradoxical as it may sound, the simplicity of the laws of electricity is what causes

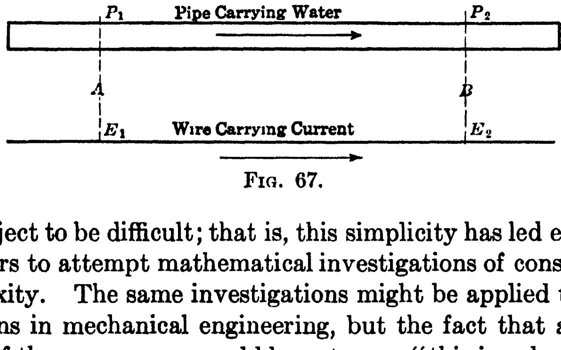

FIG. 67.

the subject to be difficult; that is, this simplicity has led electrical engineers to attempt mathematical investigations of considerable complexity. The same investigations might be applied to many questions in mechanical engineering, but the fact that at many stages of the process one would have to say, "this is only approximately so," would rob the result of much, if not all, of its value.

As a simple illustration, consider the electric circuit and the corresponding water circuit of Fig. 67. Let the electrical pressures at the points A and B be respectively E_1 and E_2, and let $E_1 - E_2 = E$. We may then write the expression

$$I = \frac{E}{R}$$

where R is the resistance of the circuit and I is the current. Similarly, in the hydraulic circuit, $P_1 - P_2 = P$ and we have $C = \frac{P}{R}$ where as before, R is the hydraulic resistance of the part

of the circuit between A and B, and C is the "current" of water. By "current" is meant the "rate of flow," *i.e.*, the gallons per second in the case of water, or the coulombs per second in the case of electricity. In electricity, this unit is named the ampere. In hydraulics it has no name, and must be designated in the somewhat awkward manner used above.

In the case of the electric circuit, the equation is *exact;* that is, R is a true constant, and does not vary at all with the current. In the hydraulic analogy, however, the equation is nothing more than an approximation, and a correction would have to be used if an attempt were made to apply it to a wide range of pressures. In other words the resistance R is not a constant, but is a variable and a function of C. Many other examples might be given, illustrating the fact that in general, the electrical phenomena are the simpler. Occasionally, it is true, the situation is reversed, and the mechanical problem is the simpler. These cases will be pointed out in their proper place.

108. Definition of an Alternating Current.—An alternating current is one in which the direction of flow is rapidly reversed. We usually add to this the proviso that the current shall pass in the two directions following the same law of change, that is, so that it would be represented by the same curve on the two sides of the zero axis. For example, in the secondary of an induction coil, the same *amount* of electricity passes in each direction, but in one direction, that corresponding to the break, the current passes in the form of a violent rush of short duration; while at the make, the same amount of electricity passes but the current is weaker and lasts enough longer to make the quantity the same. We would, therefore, hardly call this an alternating current, as generally understood.

109. Wave Shape.—One method of representing an alternating current is by means of a curve as shown in Fig. 68. The abscissæ are the times, the ordinates, the currents at the corresponding times.

The wave shown in Fig. 68 is known as a sine wave. This shape is the one aimed at in all alternating machinery, although, on account of inaccuracies in workmanship, the necessity of providing teeth on the armature surface, distortions introduced in the magnetic flux by the current generated and other factors, the best that can be obtained is an approximation. In Figs. 69 and 70 several waves as given by commercial machines are

they are not used at the present time, except in old installations. Other frequencies occasionally encountered are 40, 33, 30 and 15 cycles. The last has been proposed and used to a slight extent in the electrification of railroads.

Of the two common frequencies, 60 and 25 cycles, the former is largely used for lighting and most of the smaller power applications. The latter is used where the motors on the system are mostly of large size, and particularly if a large part of the power is to be transformed to direct current by means of rotary converters.

111. Construction of Sine Curve.—As before stated, the sine wave is considered the standard alternating-current wave. The reasons for this choice will appear gradually as we progress. It will suffice to point out at the present time that sinusoidal motion

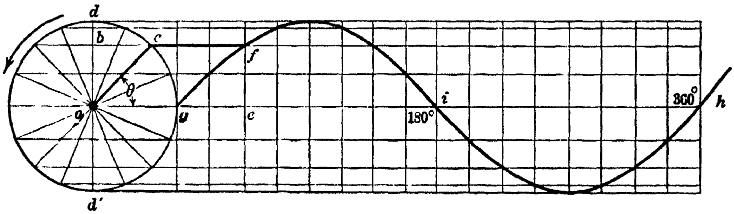

FIG. 73.

(also called simple harmonic motion) is the simplest of all to-and-fro motions. The motion of the piston of a steam engine if the connecting rod is extremely long, is simple harmonic. So also is the motion of a pendulum, if the decrease in the amplitude due to the friction of the air and of the suspension be neglected. The motion of a weight suspended on a spring, the discharge of electricity from a condenser and many other natural phenomena, also follow the same law.

The method of constructing a sinusoidal curve is shown in Fig. 73. The point c is supposed to be rotating around the circle in a counter-clockwise direction as shown with a uniform angular velocity of ω. We may think of the angle θ as increasing without limit, *i.e.*, not limited to 360°, and we then have the relation

$$ob = oc \sin \theta = oc \sin \omega t$$

and we say that the point b executes simple harmonic motion along the axis dd'. In constructing the curve, we may consider the horizontal distances as representing either angles or time,

since with uniform circular motion, the one is proportional to the other. For any angle $\theta = goc$, we lay off the distance ge, in which the ratio of ge to gh is the same as the ratio of the angle θ to 360°. At the point e determined in this way, we erect a perpendicular, ef, equal to ob. In this way we can construct as many points as we desire. The smooth curve connecting these points will be a sine curve.

112. Methods of Treating Alternating-current Waves.—In this book, three methods of representing alternating-current waves will be considered. Perhaps the simplest and most direct is to use rectangular co-ordinates, giving a curve like that shown in Fig. 73. The great advantage of this method is that it presents to the eye a picture of what is happening in the circuit. It will, in general, be employed in our first consideration of a piece of apparatus. The intention is that the student should actually *see* in his own mind just what must occur, before starting to take up the subject from the mathematical standpoint.

113. Analytical Method.—The second method of representing the current or e.m.f. in an electrical circuit, is by means of a mathematical expression. Thus, the sine wave of Fig. 73 may be expressed mathematically as

$$i = I \sin \omega t$$

in which i is the instantaneous value of the current, I is the maximum value which the current attains, ω is the angular velocity, and t is the time which has elapsed. It may be further explained that ω is the actual angular velocity of the alternator supplying the current if it is provided with two poles; it is twice the actual angular velocity, if the machine has four poles, etc.

This method of treating the subject has great advantages for many purposes. It is, however, very difficult for many students to grasp the true significance of the equations. In the author's opinion it is better suited to the investigation of more advanced problems than to the explanation of the more simple relations. It should also be noted that the expressions for waves of the non-sinusoidal shape, become very complicated, and the effort is rarely made to work with them.

114. Vector Method.—The third method which we shall employ is the vector method. Strictly speaking, an alternating current can not be represented by means of a vector, as a vector has magnitude, direction and sense. An alternating current has no direc-

tion, considered over a period of time. Even considered at an instant, it has only one of two directions, and that only at a particular point of the circuit.

Referring to Fig. 73, we have shown how the sine curve may be laid off by means of the circle with a point rotating uniformly around it. A sine wave, if present alone in a circuit, and if we are not interested in its instantaneous value, may be completely defined by one property alone, namely, its maximum value. Thus in Fig. 73 the radius or vector *oc* may be considered as respresenting the sine wave *gfih*. We may also think of the vector *oc* as rotating in a counter-clockwise direction, and, we may consider the instantaneous value of the current or e.m.f. represented by the vector *oc* as being the projection of this vector upon the vertical line *od*. The value of this projection will obviously be

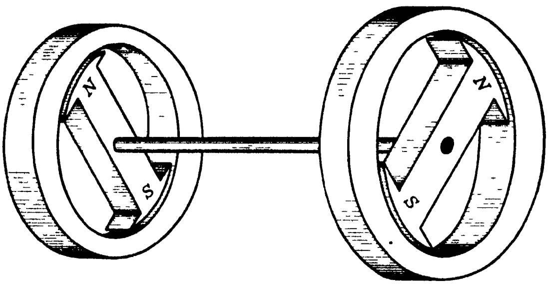

FIG. 74.

the same as the value of the ordinate of the sine wave *ef*, in fact the sine curve was constructed by plotting these projections as ordinates with the times or the angles as abscissæ.

115. Phase Difference.—Usually, we have in a circuit at least two waves to consider. These may be two waves of current or e.m.f., or a wave of current and one of e.m.f. Thus in Fig. 74 we have two alternators connected in series. The two machines are supposed to be mounted on the same shaft, and are in such an angular relation that the e.m.fs. of the two do not reach their maximum values at the same time. The two curves would be drawn as shown, in Fig. 75. They could be considered as being constructed by taking the projections of the two points *B* and *C* on the perpendicular line *ad* in the manner previously described.

116. Addition of Two Waves.—To get the total or the combined e.m.f. wave of the two machines from their individual waves

we should proceed *at any given instant* exactly as though we were dealing with a direct-current circuit. Thus, at the time corresponding to the point a we should take the instantaneous e.m.f., ab, of the machine X and add it to ac, the e.m.f. of the machine Y, giving as the combined e.m.f. the value ad. The same would be done with other points, and the smooth curve as shown by the dotted line drawn through these points will be the curve of the combined machines. This method could be applied no matter what the shapes of the two waves. It would not even be necessary that they be of the same shape, or frequency.

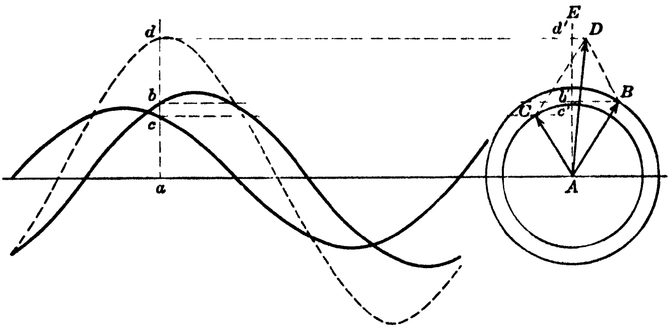

Fig 75

117. Vector Addition.—If, however, *we are dealing with sine waves* as in the case shown, we may arrive at the same solution in a simpler manner by using vectors. This is done by drawing the lines CD and BD parallel respectively to the lines AB and AC. The resultant AD will represent the resultant wave in both magnitude and phase. This will be readily apparent if we consider that at any instant the projection of the vector AD on the line AE is equal to the sum of the projections of the two lines AB and AC. Thus Ad' is the sum of Ab' and $b'd'$. Ab' is the projection of AB and $b'd'$ is equal to Ac', the projection of AC. Hence, at any time, the projection of the vector AD will be equal to the sum of the projections of the two vectors AC and AB and will represent their combined value. This gives us a very simple method of dealing with problems in alternating currents. It must, however, be kept clearly in mind that *solutions obtained in this manner apply only to sine waves of current and e.m.f.* Confusion is frequently caused by losing sight of this fact. Certain conditions frequently arise in circuits which tend to produce greatly

distorted waves. In this case, vector solutions have little or no value.

The addition of two or more currents would be carried out in exactly the same manner. Thus if the two alternators of Fig. 74 were connected in parallel, the combined current would in general not be equal to the arithmetical sum of the two, but to their vector sum. The latter would be obtained in the manner just described, taking AB and AC as currents instead of e.m.fs. Frequently we are concerned only with the relative values and phase angles of the different currents and e.m.fs. In this case, the vector diagram is drawn with different angles between the quantities, different e.m.fs., etc., and the changes can be studied in a general way. Sometimes numerical values of the results are required. We may obtain the result by scaling the values directly from the diagram. The accuracy of this method will, of course, depend upon the scale of the drawing and the care with which it is made. It will, in most cases, give results of sufficient accuracy for practical work.

If more accurate results are required, they can always be obtained by constructing the vector diagram and computing the ength of the lines representing the required quantities by the ordinary rules of trigonometry.

118. Effective Values of Current and E.M.F.—As we have seen, an alternating current or e.m.f. passes through a series of values ranging from zero to a certain maximum in both the positive and negative directions. It is necessary in speaking of the value of a current or an e.m.f. to determine just what is meant by its "value." This is defined by agreeing that a continuous current and an alternating current will be considered the same in value if their heating effects are the same when passed through the same resistance, *i.e.*, if the power lost is the same.

The heating effect of a continuous current, or of an alternating current at any given instant, is proportional to the square of the current at that instant. If we have a current, of whatever nature, which has a *varying* value, its *average* heating effect will be proportional to its *average square*. It is evident, therefore, that if we are to consider the power expended in heating the circuit as proportional to the square of the value of the current, that we shall have to consider this "value" as being the square root of the average square. This value is variously referred to as the effective value, the virtual value, the root mean square

value, or the true value. Since this is the value usually of interest, it is also commonly referred to simply as the value of the current. If we wish to consider the current at the highest point of the wave, we refer to this as the maximum value. Also, since in a circuit of constant resistance the power is equal to E^2/R, the above remarks apply equally well to the e.m.f.

We can readily derive the relation between the maximum value of a sine wave and the square root of the average square. Elementary trigonometry gives the relation

$$\sin^2 \theta + \cos^2 \theta = 1$$

If we consider that we pass through a complete cycle, the sine will pass through all values from $+1$ to -1. The cosine will pass through exactly the same values as the sine. Then

$$\text{average } \sin^2 \theta = \text{average } \cos^2 \theta = \frac{1}{2}$$

and therefore

$$\sqrt{\text{average } \sin^2} = \sqrt{\tfrac{1}{2}} = 0.707$$

If the maximum value of a sinusoidal wave of current or e.m.f. is, say, 100, the effective value will be 70.7. If the effective value is 100, the maximum value will be $100 \times \sqrt{2} = 141.4$.

PROBLEMS

47. An alternator having forty poles is revolving at the rate of 120 r.p.m. What is the frequency? In the case of an alternator having ten field poles, at what speed must the machine revolve in order that the frequency may be 60 cycles?

48. A circuit is carrying a 60-cycle alternating sinusoidal current whose maximum value is 10 amp. What is the value of the angular velocity? Write the equation of the current. Compute several values of the current at intervals of 0.001 sec., starting from the zero value of the current.

49. Represent the current in the foregoing circuit by means of a vector. Do the same using rectangular coordinates, and plotting the values obtained in the above example.

50. Two alternating currents whose maximum values are 100 and 150 amp. respectively differ in phase by 30°. What is the maximum value of the sum of the two currents? What is the effective value?

51. Two e.m.fs. differ in phase by 90°. The sum of the two is 100 volts and one of them is 75 volts. What is the other?

52. If there are three sinusoidal e.m.fs. of 110 volts (effective) each and differing in phase by 120°, what two values can be obtained for the sum of the three? What will be the maximum values?

CHAPTER XII

INDUCTANCE AND CAPACITANCE

119. Alternating- and Direct-currents Compared.—In the continuous-current circuit we always have the simple relation $I = E/R$. A very few experiments with alternating currents will disclose the fact that in many cases the current is far from the value indicated by this equation. This variation may be due to the presence of one or both of two factors, inductance or capacitance.

A further investigation into the action of alternating current will show the interesting fact that if we consider the circuit *at any given instant*, the current will be given by the same expression as in the case of the direct current, namely, $i = e/R$, in which the small letters are used to indicate instantaneous values. The e.m.f. e will, however, in general, *not* be the voltage supplied by the generator or other source of power, but will be the algebraic sum of the values of perhaps several e.m.fs. present in the circuit. These additional e.m.fs. may be due to the action of inductance or of capacitance. We shall now proceed to consider how these additional e.m.fs. are generated and their effect upon the circuit as a whole.

120. E.M.F. Due to Inductance.—As previously noted, an e.m.f. is generated whenever a conductor cuts lines of magnetic induction. The value of this e.m.f. is equal to the number of lines of induction cut per second if the cutting is uniform, or if it is not uniform, the number that would be cut if the cutting were to remain the same for the whole second. The number of lines which would be cut in a second is called the rate of change of the lines. This is represented mathematically by the expression

$$e = \frac{1}{10^8} \frac{d\varphi}{dt}$$

The factor 10^8 is introduced to change the unit of pressure from absolute units to volts. If there are several turns in series in the circuit, the e.m.f. generated is increased in proportion to the number of turns, giving the expression

$$e = \frac{N}{10^8} \frac{d\varphi}{dt}$$

As far as the final result is concerned, it makes no difference how the lines of induction are produced. Figure 76 shows a coil of wire or solenoid. Such a piece of apparatus is also called an inductor. If we thrust a permanent magnet into such a solenoid, we shall induce in the coil an e.m.f., and if the circuit is closed, a current. We might equally well have induced the e.m.f. by thrusting the coil over the magnet, or instead of the permanent magnet, an electromagnet might have been used. Moreover, in the latter case, instead of thrusting the electromagnet into the coil or the coil over the electromagnet, the same result might have been secured without any movement of the coil or magnet merely by opening or closing the circuit through the electromagnet. This arrangement would constitute a simple form of induction coil.

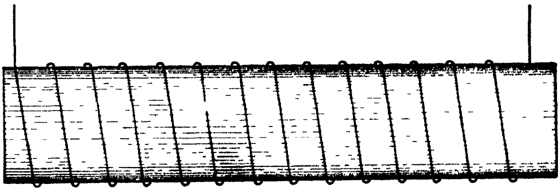

Fig. 76.

Still another form of induction is possible. If we pass current through the coil of Fig. 76 we shall set up in the interior of the solenoid, lines of magnetic induction. These lines, of course, return outside of the solenoid, thus completing the magnetic circuit. It is evident that while these lines *are being established* or while they *are dying down*, they will cut the wires of the solenoid, thus inducing an e.m.f. This e.m.f. is represented by the same expression as before.

121. Coefficient of Inductance.—It is, in general, inconvenient to make computations with lines of induction. This arises primarily from the fact that magnetic measurements are difficult to make. It is therefore preferable to reduce our formulæ to forms involving current instead of induction.

In such a coil of N turns as that of Fig. 76, the total magnetic induction, *provided there is no iron present,* is proportional to the current times the number of turns, or $\Phi = KNI$; or using instantaneous values, $\varphi = KNi$ where "K" is some constant. If iron is

present, this expression will still be approximately true, provided that the circuit is not saturated. Substituting this value of φ in the equation we get

$$e_L = \frac{N}{10^8}\frac{d\varphi}{dt} = \frac{N}{10^8}\frac{d(KNi)}{dt} = \frac{KN^2}{10^8}\frac{di}{dt} = L\frac{di}{dt}$$

where $L = \frac{KN^2}{10^8}$ is called the coefficient of self-inductance (or the inductance) of the solenoid. The unit of inductance is the henry. This value will be a true constant if the circuit contains no iron, but if iron is present it will in general, become somewhat smaller as the current increases. Even in this case it is usually regarded as a constant in theoretical investigations, an average value being taken.

122. Mechanical Analogy.—There is a very striking analogy between the coefficient of self-inductance of a circuit, and the mass of a body. As was just pointed out the e.m.f. required to change the current in a circuit may be expressed as

$$e_L = L\frac{di}{dt}$$

In mechanics, we have an exactly analogous equation. The force required to change the velocity of a body, is equal to the mass of the body times the acceleration. The acceleration is the rate of change of the velocity, and we may write,

$$f_M = Ma = M\frac{dv}{dt}$$

Here we have an exact analogy if for e.m.f., we substitute force; for velocity, current; and for inductance, mass.

This analogy is of help in giving an accurate mental picture of the actions taking place. It is much easier to understand a tangible phenomenon than an abstract one.

Continuing the mechanical analogy we may consider the relation of friction and resistanee, or electrical friction. In an electric circuit, we have

$$I = \frac{E}{R} \text{ or } E = RI$$

This equation is exact. We may write a similar equation for a body in uniform motion

$$V = \frac{F}{R} \text{ or } F = RV$$

where R is the resistance to motion or the friction. This equation however *is not exact*, but only an approximation. It illustrates excellently the fact already mentioned, that mechanical engineering problems are essentially more difficult than the corresponding electrical ones. Thus in one class of friction, that of bearings, it is generally stated that the force due to friction is constant, and does not increase at all with the velocity. A more careful investigation however shows many irregularities in the friction as the speed increases.

In another class of friction, that of a boat through the water, the statement is usually made that the friction is proportional to the square of the speed. It is, however, a common experience with boats of a certain form, that the friction at a certain critical speed will increase to a value far above that indicated by the formula. It is almost impossible to drive such vessels beyond a certain speed. With other hulls well adapted to high speeds, exactly the reverse is the case. Again we have a very complex phenomenon, contrasted with the simple electrical one.

In the consideration of many simple mechanical problems as for example, a railway car at ordinary speeds, or the motion of a fluid in a pipe, we shall be near enough for our purposes if we assume that the formula given is correct, or the force due to friction is proportional to the speed.

123. Starting a Mass or a Current.—With these assumptions, the problems illustrated in Figs. 77 and 78 can be solved by identical mathematical expressions. If we apply a steady force to a car at rest as shown in Fig. 78 it will gradually increase its speed until a velocity is reached such that the force due to resistance is equal to the propelling force. It will then continue to move at this speed, as long as the driving force is continued. During the period of acceleration, the velocities at different times will be represented by some such curve as that shown in Fig. 79. The equation of this curve could readily be determined by the use of differential equations. For the present purpose, it is unnecessary to do so.

The electric circuit presents an exact analogy to this. A current increases gradually instead of jumping at once to its final value, and follows the same time curve as in the case of the car. It is true, however, that this increase in current generally takes place so suddenly that it can not be observed unless special instruments are used. In some instances, however, as for example,

when a constant voltage is applied to the field magnet of a large machine the change in current can be observed with considerable accuracy on an ordinary ammeter. It may be perhaps 15

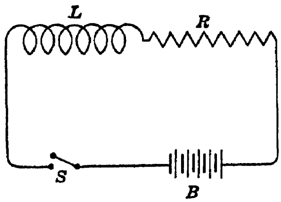

FIG. 77.

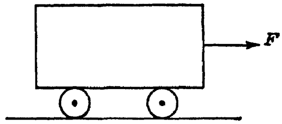

FIG. 78.

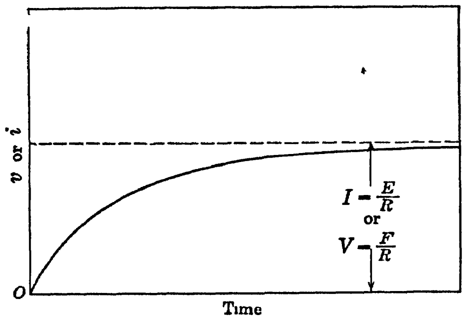

FIG. 79.

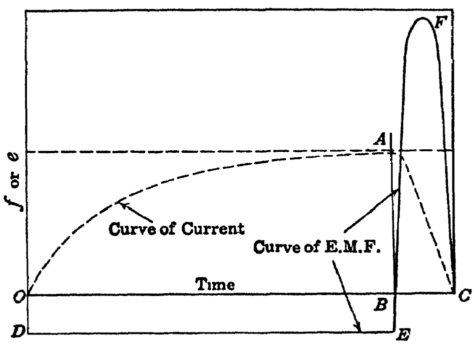

FIG. 80.

sec. before the needle of the ammeter ceases to move across the scale.

The analogy can be carried still further. If an attempt is made to stop the car suddenly, everyone is familiar with the result.

A very great force is developed, and this force is inversely proportional to the time required to stop the car. This force may be many times as great as the force applied to accelerate the car. These facts are represented in Fig. 80 where the velocity is shown as a dashed line. This line starts to descend rapidly at the point *A*, indicating that the motion is completely stopped in the time *BC*. During the period of acceleration, a force is applied from outside in the direction of the velocity. The car pushes backward with the same force as the applied force. The curve of *back* force is shown by the line *DE*, the force having the constant value *OD*. As soon, however, as an attempt is made to stop the body, it exerts force in the direction in which it is moving, and continues to exert this force as long as the retarding force is applied. The force during this interval is shown by the curve *E B F C*, and as shown, may rise to a far greater height than the applied force, but for a proportionally shorter time.

In the electric circuit, there will be a similar action. During the time that the current is rising in the coil, there will be exerted a back e.m.f. equal to the applied voltage. This back e.m.f. is due partly to the drop in the wire due to resistance, and partly to the tendency of such an inductive circuit to resist any change in the current. At the instant of break, this tendency exerts itself powerfully, that is, the current tends to continue flowing at the same value, and when forced to decrease by the opening of the circuit, sets up a large e.m.f. The result is a heavy spark across the break, since the induced e.m.f. is sufficient to force the current to flow for a short time across the gap through the air.

The amount of work stored in such a solenoid (due to building up the magnetic field) can be readily obtained. The work done during an interval dt is

$$dW = eidt$$

and since

$$e = L\frac{di}{dt}$$

we have

$$dW = Lidi$$

Integrating this with the limits $i = O$ and $i = I$ we obtain

$$W = L\int_O^I idi = \tfrac{1}{2}LI^2$$

In a similar manner, we could have derived an expression for the work stored in a moving body of mass M and would have obtained the corresponding expression

$$W = \frac{1}{2}MV^2$$

The phenomenon just considered is made use of in various mechanical appliances, such as the hydraulic ram and the pile driver. In the latter, the moderate force of gravity is increased to the tremendous force of the blow struck by the descending weight. There is a great increase of force, but it must be clearly noted that there is no increase of energy. Though the force of the blow is so great, it lasts such a short time that the actual work (*i.e.*, energy) expended in forcing the pile down is less than the work done in raising the weight.

The corresponding electrical device, the solenoid combined with a battery and switch, is used principally in one form of gas-engine ignition. The switch is located inside the cylinder of the engine and the points of the switch are caused to separate at the instant when it is desired to ignite the gas. A heavy spark is formed as the points separate, and sufficient heat is produced to ignite the mixture.

124. Field Discharge Switch.—If the switch in the field circuit of a dynamo or motor be opened while current is flowing in the field, a heavy arc will be observed. The size of this arc will depend upon the amount of energy stored in the field. In the case of a large machine this may be considerable, and the arc at the time of opening may be very destructive to the switch. Moreover, as just explained, there will be a great rise in voltage across the terminals of the switch. It may readily occur that with the field excited with current at 125 volts, the voltage when the switch is opened may rise to 1000 volts or more.

This high voltage is liable to puncture the insulation of the fields. This danger can be avoided by providing a side track, as it were, for the current. The connections for this are shown in Fig. 81. When the field is connected to the line, the resistor shown is not in circuit. As the switch is opened to disconnect the field from the line, the resistor is first connected across the terminals of the field, and a further movement of the switch disconnects both from the line. As soon as the connection to the line is broken, the current begins to decrease. It is, however, not forced to decrease to zero at once, but is allowed to flow

through the circuit of the resistor until its energy is dissipated. Such a device is known as a field discharge switch.

125. Resistance and Inductance.—We must now examine the effect of resistance and inductance in alternating-current circuits, leaving the consideration of capacitance until later. In taking up this problem use will be made of a mechanical analogy as in the preceding case. The comparison is exact, with the slight

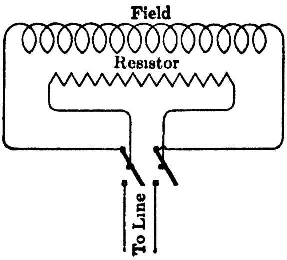

FIG. 81.

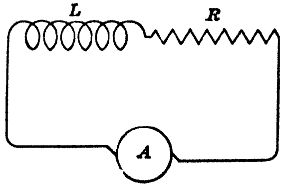

FIG. 82.

exception of the friction effect, and a complete understanding of the one case will give a clear conception of the other.

126. Mechanical Analogy.—Consider the electric circuit (comprising resistance and inductance) shown in Fig. 82. The inductance corresponds to the mass of a ponderable body, while the resistance corresponds to the friction. We could, therefore,

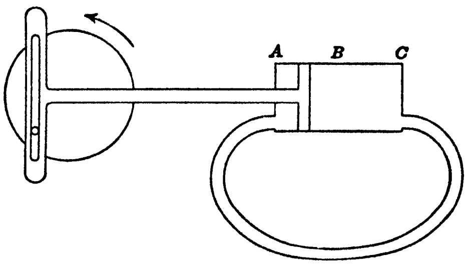

FIG. 83.

devise many mechanical analogies to this circuit. For instance, the car considered in the previous problem if moved rapidly back and forth on a level track would afford an illustration. The motion of the piston and connecting rod of a steam or gas engine is another familiar example. The motion, however, is not sinusoidal in this case, but becomes approximately so if the connecting rod is long relative to the stroke. The deductions which

will be arrived at presently can be applied to this case with the foregoing qualification.

For the present purpose, the hydraulic analogy shown in Fig. 83 may be considered. A piston, as shown, is supposed to be moved back and forth in a cylinder. The two ends of the cylinder are connected by means of a pipe. The piston derives its motion from a flywheel by means of a Scotch yoke. If the rotation of the flywheel is uniform, the piston and the contents of the cylinder and tube, if incompressible, will execute simple harmonic motion. We can then plot a curve like the one marked velocity or current in Fig. 84, in which the ordinates represent the velocity of the piston at the time represented by any abscissæ. The point A on the curve corresponds to the end A of the cylinder. Obviously, at this time, the velocity is zero. B represents the point at the middle of the stroke. The velocity is here a maximum. At C, the other end of the stroke, the velocity is again zero. As the piston starts to move back, the velocity is in the opposite direction, and the return motion is indicated in the figure by the portion CFE of the curve. Starting from E the cycle is again repeated.

127. Resistance without Inductance or Capacitance.—The force required to cause the movement described may be considered under various conditions. First, let us assume that the liquid used in the tube is so light or that the movement is so slow that the inertia of the liquid may be neglected in comparison with the friction. For example, we might use molasses for the liquid, make the connecting tube small and the motion slow. Under these circumstances, the inertia could evidently be neglected without material error. This case corresponds to an electric *circuit with resistance but with no inductance.* A bank of incandescent lamps, or a water rheostat, would approximate this condition.

In the case assumed, the only force acting is that due to friction. As we have shown, we may assume that the relation between the force and the velocity is expressed by the equation

$$f_R = vR$$

or in electrical units

$$e_R = iR$$

Since the velocity of the liquid is expressed by an equation such as

$$v = V \sin \omega t$$

or in the electric circuit

$$i = I \sin \omega t$$

it is evident that the expressions for the force will be respectively

$$f_R = RV \sin \omega t, \text{ and } e_R = RI \sin \omega t$$

We can therefore draw the force curves in either case as shown in Fig. 84. The force will be zero at the same time that the velocity or the current is zero, and will be a maximum when the above quantities are a maximum. In a case like this, we say that the *current and the e.m.f.* (or the velocity and the force) *are in phase.*

The power involved in the foregoing action is of great interest. Power is the product of force and velocity, or in electrical units, of current and e.m.f. Even though the force and velocity or the current and e.m.f. are variable, this relation holds *if we consider the power at any instant.* Thus using small letters to indicate instantaneous values,

$$p = vf \text{ or } p = ei$$

Thus in Fig. 84, if we consider any time such as that represented by the point G the power at this instant will be the product of the current or velocity, GI, times the force or e.m.f., GJ, giving some such value as GH. The maximum value of this product will evidently be at the time when the current and e.m.f. (or the velocity and the force) are a maximum. The power will also be zero whenever either one of the two factors is zero. The power will never be negative, since when one of the factors becomes negative, the other is negative also and the product of two negative quantities is positive. This will also be evident if we consider that at the flywheel rim the torque or turning moment will at all times have to be in the same direction, though it will drop to zero just at the instant when the piston is at the end of its stroke. We have then a condition in which the power is pulsating but always positive; that is, the mechanism or the electric generator always requires power to drive it except at an instant at the zero points, and never acts to return power to the flywheel in the one case, nor to the driving engine in the other.

128. Inductance without Resistance.—If, on the other hand, we consider that the cylinder and connecting tube in Fig. 83 are filled with some heavy liquid like mercury, that the passage is

short and large in section, and that the movement of the piston is rapid, the conditions will be entirely changed. In this case, we may consider that the friction is so small in comparison with the inertia, that it may be neglected. A moment's consideration will show that the force required to overcome the inertia of the piston will be greatest *when the piston is at rest.* To keep it in motion at a uniform speed, will require *no force at all,* under the assumption we have made, that the friction is zero. At the time A (see Fig. 85) we shall have to exert the maximum force to start the mass into motion. The required force will become less as the velocity increases, and will be zero when the piston is at the center of its stroke. This corresponds to the point B. Beyond this point, instead of a push on the piston, a pull will be required to bring the moving mass to rest. This we represent as a negative force, and the curve of force or e.m.f. consequently passes below the zero line at the point B. If the reader will imagine himself to be pushing a heavy lawn roller to and fro on a smooth level surface such as a stone sidewalk, he will readily understand the foregoing relations of force and velocity. In this case, we say the current or the velocity lags 90° behind the e.m.f. or the force. That the current is behind, will be apparent if we consider that as we pass from left to right, the e.m.f. attains its maximum value before the current.

129. Power in Inductive Circuit.—The curve of power can be constructed in the same manner as before. It will cross the zero axis whenever either the current or the e.m.f. is zero. In the present case, it is evident that one of the factors is sometimes negative, while the other is positive. Hence we shall have negative values of the power. In fact, if the curve of power is accurately constructed, the positive portions will be of exactly the same area as the negative ones, or *the net power is zero.*

The explanation of this apparently peculiar fact is that the liquid requires a push to set it in motion, but while it is slowing down, it on the other hand exerts a push on the piston. Thus, during the first quarter revolution, torque must be applied to the flywheel in the direction of rotation. During the next quarter turn, however, the inertia of the liquid will tend to keep up the motion and will return just as much work as was done upon it during the preceding quarter. The apparatus then acts during half the time as a pump, and during the other half as an hydraulic motor. In the electric circuit, a similar action takes place, the

dynamo machine acting as a generator for half the time and as a motor for the remaining time. It is thus alternately retarded and forced ahead, and the net power required is zero.

The above applies, of course, only in the assumed case of zero friction or zero electrical resistance. In practice, we can not have an apparatus without friction, or an electric circuit without resistance. Hence the condition stated can not be exactly realized, but a very near approximation can be made to it. The fact that the average power would be zero, might have been predicted at once, since if there is no friction, there would be no opportunity to dissipate any power. Similarly, in the electric circuit, if there is no resistance in the circuit shown, there is no chance for any loss of power, and consequently the net power must be zero.

130. Application to Steam Engine.—An excellent application of these principles is afforded in the case of the modern high-speed steam engine. It was at first thought essential by many engineers that the reciprocating parts of such engines should be made as light as possible in order to avoid vibration. This is an incorrect view of the matter. By making the piston, piston rod and connecting rod moderately heavy, it is possible to bring these parts to rest at the end of the stroke by means of the compression of the steam, purposely trapped in the cylinder by the closing of the exhaust valve slightly before the piston reaches the dead center. At the beginning of the succeeding power stroke, the heavy reciprocating parts serve to take the principal force of the impulse of the steam, thus relieving the crank pin from the sudden impulse. As the end of the stroke is reached, the force of the steam becomes less. The motion of the piston and other parts is however now being retarded, and they in consequence exert pressure on the crank pin, in addition to the force of the steam. In this manner, the turning moment of the engine is rendered much more uniform than would be the case if the reciprocating parts were light. It might also be well to point out that the foregoing demonstration shows that aside from the friction always present, the power expended in reciprocating a weight such as that of the piston of a steam engine is zero. It seems well to mention this point, since many people imagine that a large amount of power is wasted in this way in the reciprocating steam engine. This belief has no foundation in fact.

131. Circuits Having Both Resistance and Inductance.—As previously stated, it is impossible in practice to have electric circuits entirely free from resistance, or mechanical circuits free from friction. In practice, the condition which most frequently occurs in electrical circuits, is that shown in Fig. 82; where both resistance and inductance are present. To construct the curves for this circuit, we draw the current curve as before (see Fig. 86). It is now evident that we shall at any time have present two forces, one due to the resistance of the circuit, the other that necessary to overcome the effect of the inductance. We can plot these two curves separately as shown by the dashed curves in the figure. To get the curve of total applied e.m.f., we have only to add the corresponding values of the two component e.m.fs. at any given point. The smooth line drawn through these points will be the required curve.

To obtain the curve of power, we should proceed as before, obtaining the various points by multiplying together the corresponding values of the current curve and the curve of resultant e.m.f. This procedure gives the curve marked "Power." It will be seen that the power is partly positive and partly negative, the positive portion, however, being much greater than the negative portion. The net power will be obtained by subtracting the area of the negative portions from the area of the positive portions.

132. Vector Representation.—We can show the foregoing relations in a simpler manner by means of vector representation. Figures 84, 85 and 86 correspond respectively to Figs. 87, 88 and 89. In Fig. 87, the fact that the current and the e.m.f. are in the same phase is represented by drawing the vectors parallel to one another. For the case of a circuit containing inductance only (see Fig. 88), the current vector may first be drawn. The vector representing the e.m.f. will then be drawn so as to be 90° ahead of the current, or the current 90° behind the e.m.f. In the event of both inductance and resistance being present, we may first draw the current vector as shown in Fig. 89. The e.m.f. required to overcome resistance is drawn in phase with the current; that consumed by the inductance, 90° ahead of it. The total applied e.m.f. will be the resultant of these two e.m.fs. or the line OE.

133. Calculation of Power.—To find an expression for the power in Fig. 89 we may consider the actual e.m.f. OE as due to

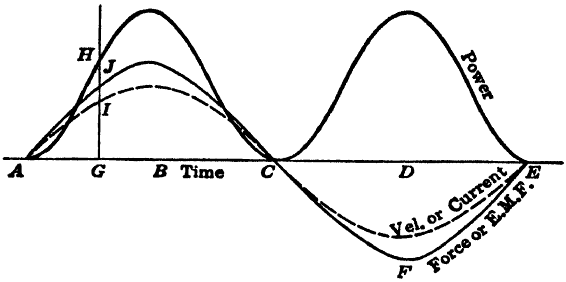

FIG 84

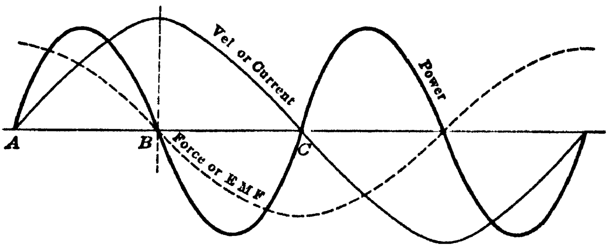

FIG. 85.

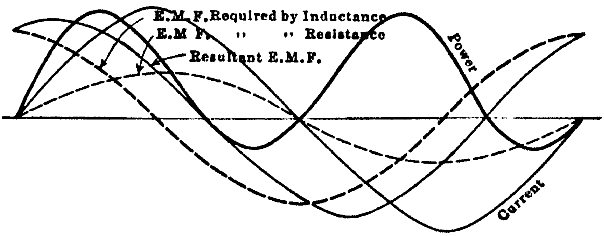

FIG. 86.

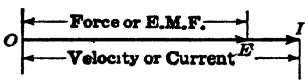

FIG. 87.

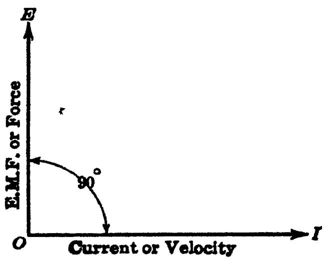

FIG. 88.

two components OE_L and OE_R. As we have shown, the component OE_L and the current OI would represent zero power, since they are at right angles. The total power is then the product of OI and OE_R or

$$P = \overline{OI} \cdot \overline{OE_R}$$

It is evident, however, that $\overline{OE_R} = \overline{OE} \cos \theta$. Making this substitution,

$$P = EI \cos \theta$$

or the power in an alternating-current circuit is the product of the effective values of the current and the e.m.f. times the cosine of the angle of lead or lag of the current. It must be remembered, however, that the expression angle of lag or lead has a definite meaning only in the case of sine waves of current and e.m.f. In the case of distorted waves we may assume an angle which would give the same result. This assumed angle has, however, no definite physical meaning.

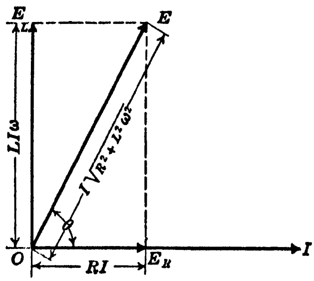

FIG. 89.

134. Mathematical Treatment.—The above facts can be deduced in a very simple manner by a mathematical treatment. It is, however, the opinion of the author that the physical interpretation, presented in a manner similar to that given above, should be thoroughly understood before an attempt is made to study the mathematical treatment.

Assuming a current represented by the formula,

$$i = I_m \sin \omega t$$

acting in a circuit where the inductance is zero and the resistance is R, we shall at any instant require an e.m.f. equal to Ri, or the expression for the e.m.f. at any instant will be

$$e_R = RI_m \sin \omega t$$

In the case of a circuit containing only inductance, the e.m.f. at any instant required to balance the back e.m.f. is given by the formula,

$$e_L = L\frac{di}{dt}$$

Substituting and performing the differentiation, we obtain

$$e_L = L\frac{d}{dt}(I_m \sin \omega t) = LI_m\omega \cos \omega t = LI_m\omega \sin(\omega t + 90°)$$

The instantaneous value of the required e.m.f. is equal to the maximum current times the coefficient of self-induction, times the angular velocity, times sin $(\omega t + 90°)$, and is advanced in phase by an angle of 90°.

The above e.m.fs. may be represented by means of vectors. The length of the vector may be either the maximum values of the quantities or the effective values, since the two are proportional. The latter are used in the diagrams. If maximum values are used the length of the vector will be $LI_m\omega$ or if effective values, $LI\omega$ where $I = I_m \div \sqrt{2}$. The total applied e.m.f. will be the vector sum of the two e.m.fs. Since the angle between the vectors is 90°, this resultant will evidently be (see Fig. 89)

$$E = I\sqrt{R^2 + L^2\omega^2}$$

or transposing,

$$I = \frac{E}{\sqrt{R^2 + L^2\omega^2}}$$

If E is the effective value of the e.m.f., I is the effective current. This is one of the most important equations in electrical engineering.

The value of the angle θ between the resultant e.m.f. and the current is given by the relation

$$\tan \theta = \frac{L\omega}{R}$$

If desired, we may also find the angle from the equations,

$$\cos \theta = \frac{R}{\sqrt{R^2 + L^2\omega^2}}$$

or

$$\sin \theta = \frac{L\omega}{\sqrt{R^2 + L^2\omega^2}}$$

135. Power.—The power at any instant is given by the expression

$$p = ei = I_m \sin \phi \cdot E_m \sin(\phi + \theta)$$

in which we have for convenience represented the angle ωt by the letter ϕ. To obtain the average power in the circuit, it is

necessary that we integrate this expression through an angle great enough to comprise one or more complete cycles of the power, and divide the result by the angle comprised in the cycle. We then have, integrating through the angle π,

$$P = \frac{E_m I_m}{\pi}\int_0^\pi \sin\phi \cdot \sin(\phi + \theta)d\phi =$$

$$\frac{E_m I_m}{\pi}\int_0^\pi (\sin^2\phi\cos\theta + \sin\phi \cdot \cos\phi\sin\theta)d\phi =$$

$$\frac{E_m I_m}{\pi}\left[\left(\frac{\phi}{2} - \frac{1}{4}\sin 2\phi\right)\cos\theta + \sin\theta \cdot \frac{1}{2}\sin^2\phi\right]_0^\pi = \frac{E_m I_m}{2}\cos\theta$$

The current and the e.m.f. in the above are the *maximum* values. To substitute effective values for maximum values we must multiply each by $\sqrt{2}$. This gives as the final result, if E and I are effective values,

$$P = EI\cos\theta$$

or the same result that we obtained by means of the graphic method.

136. Power Factor.—In a circuit carrying direct current the power is always the product of the volts and the amperes. In an alternating-current circuit the power can never be more than this, but is usually less. We define power factor by the expression,

$$\text{power factor} = \frac{\text{watts}}{\text{volts} \times \text{amperes}}$$

or

$$P.F. = \frac{P}{EI}$$

From the preceding article it will be seen that *in the special case of harmonic e.m.f. and current,*

$$P.F. = \frac{P}{EI} = \cos\theta.$$

It will be evident that the power factor can never be greater than one, but it may be anything from one to zero.

137. The Condenser.—A condenser consists of two conductors separated by a dielectric, that is, by a body which is not a conductor of electricity. For example, two plates of metal separated from one another by a layer of air or a sheet of glass make an excellent condenser for some purposes. If the two plates of such a condenser be connected to the two poles of a source of elec-

tricity as shown in Fig. 90, a momentary current will flow from the cell into the condenser. The current will last only for a moment, until a certain quantity of electricity has passed into the condenser. The condenser retains the electric charge. This charge can produce a current if we disconnect the terminals from the battery and connect them by means of a conductor. This current, like the charging current, will be only momentary.

An analogous hydraulic circuit is shown in Fig. 91. The pump P which is here assumed to be a centrifugal pump, operating at such a speed as to *produce a constant water pressure*, corresponds to the battery. The cell containing a flexible diaphragm corresponds to the condenser. When the circuit is first completed by connecting the diaphragm chamber to the pump, a momentary current of water will flow. This ceases almost at once, but a permanent *displacement* of the water remains. This is analogous to the charge of electricity. If the pump is disconnected and the water circuit completed by means of a pipe, a momentary current in the reverse direction will flow and the "charge" of water will disappear, that is, the diaphragm will return to its original position.

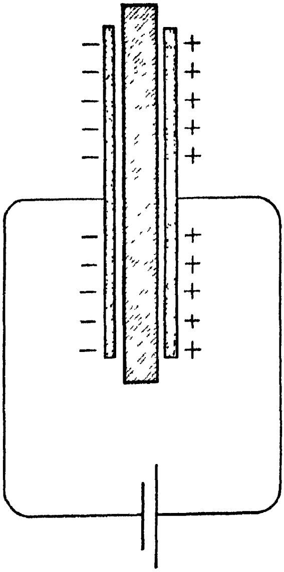

FIG. 90.

The quantity of electricity is the product of the current and the time during which the flow continues. If, as in the present instance, the flow is variable, the quantity is the integrated value of the product of the current and the time. It has been found that the quantity is proportional to the applied e.m.f., and we may write the equation

$$Q = CE$$

in which Q is the quantity, E is the applied e.m.f., and C is known as the capacitance (formerly called capacity) of the condenser. In the case of the hydraulic circuit we may write a similar equation

$$Q = CP$$

in which Q is the quantity of water displaced, C is the "capacitance" of the flexible diaphragm, and P is the pressure.

We may increase the capacitance of the condenser by increasing the area of the plates, by placing the plates nearer together, and by changing the dielectric used. For large capacitance when only moderate pressures are to be used, the distance between the plates is made very small and the plates are of large size. Since the plates are near together, the use of air insulation is impracticable. In any event, other substances cause the condenser to have a greater capacitance, and hence would be preferred. The dimensions of the condenser would also be too great if only two plates were used, and hence it is customary to pile up a number of sheets of the conductor, separating adjacent sheets by the dielectric used. This latter is usually mica or paraffined paper.

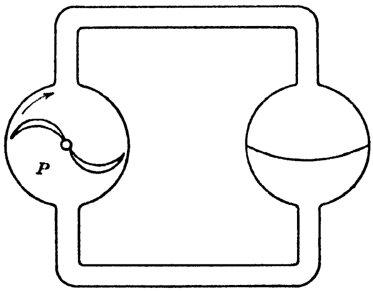

FIG. 91.

The unit of quantity of electricity is the coulomb. This is the quantity of electricity stored in a condenser when 1 amp. flows into it for 1 sec. If a condenser takes a charge of 1 coulomb when charged to a pressure of 1 volt, its capacitance is said to be 1 farad. A condenser of such a capacitance would be of extremely large size. Consequently, a unit of one millionth of the above value is preferable for ordinary use. This is known as the microfarad. A condenser of a capacitance of 1 microfarad and capable of withstanding 500 volts would have a volume of about 50 cu. in.

As is evident from the foregoing, a steady current either of electricity or of water can not long exist in a circuit in which a condenser is placed. In the case of an alternating current either of electricity or of water, the *quantity* of electricity or of water conveyed in the one direction or the other during one wave will in general not be very large, but the *current* may be large. This is particularly true if the frequency is high. In this case a condenser may offer but little apparent resistance to the passage of the current. In fact, as will be shown presently, if the circuit contains inductance or, in the case of the hydraulic analogue, if inertia is present, the introduction of the condenser or diaphragm may actually *increase* the flow of current.

138. Circuit Containing a Condenser Only.—To understand the action more fully, consider the electric circuit of Fig. 92 or the hydraulic circuit of Fig. 93. In the former we have an alternator supplying current to a condenser. In the latter the reciprocating piston forces an alternating current of water through the pipe. We may imagine that the water is without mass or friction, so the inertia and friction effects may be neglected. The diaphragm is at its central position and is exerting no pressure when the piston is at its middle point. The curves of pressure or e.m.f., quantity, and current for the two cases are shown in Fig. 94. Consider the time when the piston is at the end of the stroke. The diaphragm will be displaced to its greatest extent, and consequently the pressure on it will be at

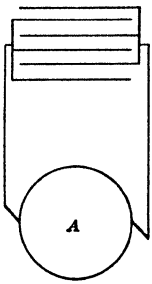

FIG. 92.

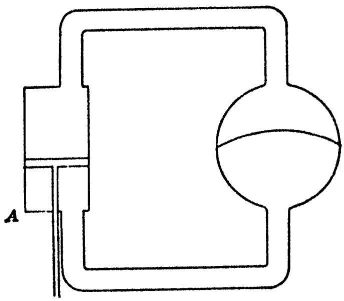

FIG. 93.

its maximum. This position of the piston corresponds to the point A on the diagram.

We may assume as before that the curve of current is a sine curve. The curve of quantity of water or electricity displaced will evidently coincide with the curve of displacement of the piston and of the pressure. A moment's consideration, however, will show that the *current* will be zero at the time when the quantity displaced is a maximum. This is evident in the case of the hydraulic system, since at this time the piston is at the end of the stroke and the flow or current must consequently be zero. As the pressure begins to decrease beyond A the water or electricity begins to flow out of the diaphragm chamber in the one case, or out of the condenser in the other. This flow will be *against* the direction of the *pressure*. Consequently, we must draw the current curve sloping downward from the point A.

At the time when the pressure or e.m.f. is zero the flow will evidently be greatest, since the piston is then moving with its greatest velocity, as it is at the center of its stroke. We can continue in this way to plot other points of the current curve, finally obtaining some such shape as shown. An inspection of this will show that the current reached its maximum value *before* the pressure. Moreover, since the displacement is 90°, we may conclude that in a condenser the current leads the applied e.m.f. by an angle of 90°. It will be recalled that this is just the reverse of the case of an inductance in which the current lags behind the applied e.m.f. by 90°.

We could readily draw the curves of power corresponding to the curves of Fig. 94. From what has gone before, it will be evident that the net power will be zero. This is so since all of the work done in deflecting the diaphragm of Fig. 93 will be

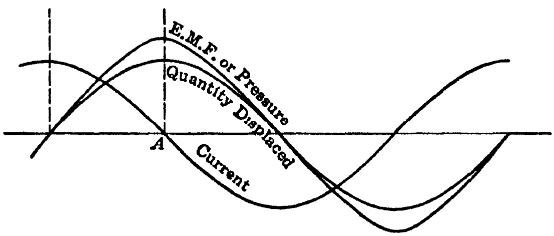

FIG. 94.

restored when the diaphragm assumes its normal position. Should there be some friction due to the movement of the diaphragm, or should it not assume exactly its original position when relieved from strain, some power would be lost. There is in every condenser some loss analogous to this effect. When a condenser is charged, there is an actual strain on the molecules of the dielectric. This causes a certain deformation and a consequent molecular friction. Consequently, there is some loss of power, but in general this is so small that it may be entirely neglected. This molecular friction in a condenser is known as dielectric hysteresis.

139. Capacitance of Transmission Lines.—In general, condensers are not used to any great extent in electrical engineering in connection with power machinery. They do find extensive use in telephone practice and other applications. In power machinery, the principal reason for studying their effects is that

these effects can be duplicated by the use of synchronous motors or generators. This will be explained in connection with these machines. It is also a fact that there is present in all long transmission lines a decided condenser effect. This is due to the fact that the two or more wires used in such a transmission constitute, with the surrounding air as dielectric, a condenser of considerable capacitance. It is true that the wires are far apart which tends to decrease the capacitance, but as the length is often more than 100 miles the total capacitance is considerable. Moreover, very high voltages, frequently of 100,000 volts or more, are used on such lines. At this pressure, the line current is correspondingly small and therefore the charging current will be a very considerable fraction of the full-load line current. A line of 200 miles operating at 100,000 volts will, for example, require a current about equal to the full-load current of a 2000-k.v.a. alternator. Hence, on such a line, it would be impossible to use units of less than the stated capacity since, even though there were no load on the line, the machine would be burned out in trying to supply the charging current to the line.

The above effects are even greater in cables adapted to be placed underground. In this case the conductors, in order to keep the cable of reasonable size, are close together. Consequently, the capacitance per mile is large. Fortunately, such cables are usually of moderate length only, and are rarely operated at pressures of more than 22,000 volts. Even so, trouble is sometimes encountered on account of condenser effects.

140. Circuits Containing Resistance, Inductance and Capacitance.—We have now to consider the more general case of a circuit containing resistance, capacitance and inductance. These three elements are found in all commercial circuits. It is true, as already pointed out, that the effect of the capacitance is usually relatively unimportant. In some cases, however, the capacitance of the line may be considerable, and even though this is not the case, synchronous machinery may be present at the end of the line, and by taking a leading current, cause essentially the same effects as would be caused by the presence of capacitance.

An electric circuit of this kind is shown in Fig. 95. Its mechanical analogue is represented in Fig. 93, if we assume that the liquid used has mass and also exerts friction upon the tubes. When only resistance and inductance were present in the circuit, we had two forces to consider: that to overcome friction or electri-

cal resistance, and that to overcome the self-induced e.m.f. or the inertia of the liquid. To these forces we have now added a third, the e.m.f. required to charge the condenser, or the force necessary to distort the diaphragm.

Figure 96 shows the curves corresponding to this case. As before, we start by drawing the curve of current. The applied e.m.f. required to overcome the resistance is shown by a sine wave in phase with the current. The e.m.f. required to overcome the inductance is 90° ahead of the current, while that to over-

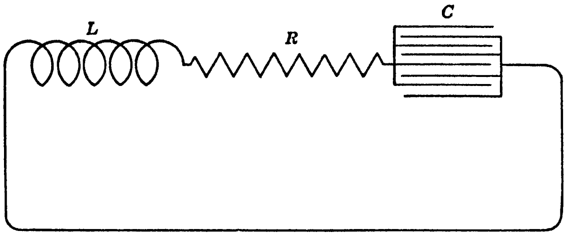

FIG. 95.

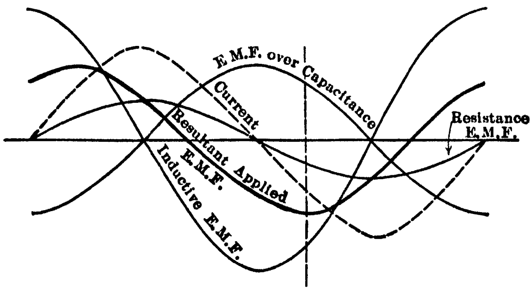

FIG 96.

come the capacitance is 90° behind the current. The resultant e.m.f. at any instant is the algebraic sum of the three-component e.m.fs. Thus at the time corresponding to the dashed vertical line we have:

Electromotive across the resistor	= − 2.5
Electromotive across the inductor	= − 11.3
Electromotive across the condenser	= + 6.1
Sum = instantaneous value of the resultant e.m.f.s.	= − 7.7

The above values are scaled from the curves.

141. Vector Representation.—These relations are more clearly

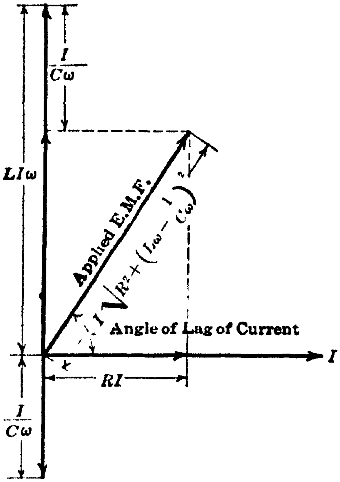

Fig. 97.

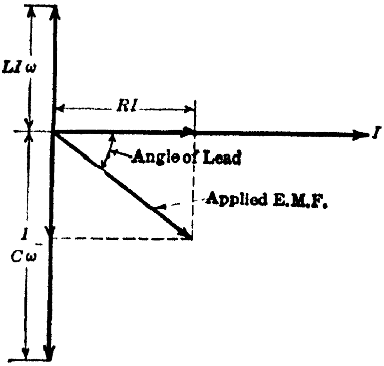

Fig. 98.

shown by the vector diagram of Fig. 97. Starting with the current as before, the e.m.f. consumed by resistance is drawn in phase with the current, and those consumed by inductance and capacitance respectively 90° ahead and 90° behind the current. The resultant of the e.m fs. over the reactor and over the condenser is readily obtained by taking their arithmetical difference. This difference, combined vectorially with the e.m.f. required by the resistance, gives the applied e.m.f. The angle of lag or lead of the current with respect to the e.m.f. will be as shown. Figure 98 shows a case where the drop over the condenser is greater than that over the inductor, causing the current to lead the resultant e.m.f. In Fig. 99 the capacity and inductive e.m.fs. are equal, and the current neither leads nor lags.

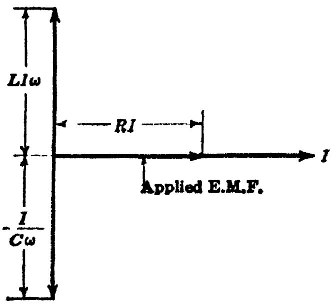

Fig. 99.

142. Mathematical Treatment.—We can readily deduce the same facts mathematically. Let

$$i = I_m \sin \omega t$$

then the maximum e.m.f. across the condenser will be

$$E_{MC} = Q/C = \frac{\int_0^{I_m} idt}{C} = -\frac{I_m}{C\omega} \cos \omega t$$

or using effective values

$$E_C = \frac{I}{C\omega} = \frac{I}{2\pi fC}$$

from which we see that the current in a condenser is 90° ahead of the e.m.f., and is equal in value to the current divided by the product of the capacitance and the angular velocity. The angular velocity is the angle swept out in 1 sec. by the revolving vector. Since a complete revolution is equal to 2π radians and since there are f revolutions per second, we have

$$\omega = 2\pi f$$

This will explain the substitution in the above equations.

Since the e.m.f. over the condenser and that over the inductor are exactly opposed to one another, it will be seen that we may offset the effect of one of these e.m.fs. in a circuit, by the use of the other. Thus if the current in a circuit lags behind the e.m.f., the introduction of a condenser of the proper capacitance will cause the current to be in phase with the e.m.f. In practice, this is most frequently done by the employment of a so-called synchronous condenser. This is merely a synchronous motor used for power-factor correction. This subject is fully discussed in Art. 209.

The way in which capacitance and inductance offset one another will perhaps be most readily seen from a consideration of the mechanical analogy of Fig. 93. The forces required to overcome inertia are greatest at the beginning of the stroke. At this time, however, the displacement of the water and that of the diaphragm are also greatest. Consequently the force due to the diaphragm is also greatest, and this force is in such a direction as to tend to start the liquid in the direction in which it is about to move. Consequently it is easy to see that the one force may partially or entirely offset the other.

We are now in a position to derive the equation of a circuit containing resistance, inductance and capacitance in series.

Since the e.m.fs. over the inductance and capacitance differ by 180°, it is evident that the resultant e.m.f. of these two will be simply their difference or

$$\left(L\omega I - \frac{I}{C\omega}\right) = I\left(L\omega - \frac{1}{C\omega}\right)$$

This resultant must be compounded with the e.m.f. due to the resistance ($E_R = RI$), and since this latter differs 90° in phase from each of the others, the resultant will be found by extracting the square root of the sum of the squares of the other two or

$$E = I\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$$

This may also be put in the form

$$I = \frac{E}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$$

As before, E and I may be regarded as either maximum or effective values. Effective values are usually employed.

This can be readily reduced to correspond to the cases when any one of the three elements is lacking. Thus if there is no inductance in the circuit, the value of L is zero and we get the form

$$I = \frac{E}{\sqrt{R^2 + \left(\frac{1}{C\omega}\right)^2}}$$

If there is no condenser in the circuit it might seem that we should substitute zero for C. This is not the case, since as we increase the value of the capacitance the less becomes its effect. Thus if the capacitance becomes infinite, there will be no drop of potential over it since

$$E = \frac{I}{C\omega}$$

and hence it will be without effect. Therefore, with an infinite capacitance in the line, the conditions would be the same as though the connection were made with a solid conductor instead of through the condenser. Therefore, instead of substituting zero for the capacitance in case no condenser is present, we should substitute the value infinity.

This rather peculiar result will be better understood from a

consideration of the hydraulic analogy. Thus in the latter, if the diaphragm is made of infinite size, it will be perfectly flexible and will therefore exert no force, and will be entirely without effect on the circuit.

Making the substitution discussed above, we get in the case of the circuit without a condenser the equation

$$I = \frac{E}{\sqrt{R_2 + L^2\omega^2}}$$

This is the same as the expression we have already derived in Art. 134.

143. Resistance, Reactance and Impedance.—We have shown that the general expression for the current in a circuit containing resistance, reactance and capacitance is

$$I = \frac{E}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$$

This is frequently written

$$I = \frac{E}{\sqrt{R^2 + X^2}} = \frac{E}{Z} \text{ where } X = L\omega - \frac{1}{C\omega}$$

R is the resistance, X is called the reactance and the expression

$$Z = \sqrt{R^2 + X^2}$$

is the impedance. The portion $L\omega$ of the reactance is called the inductive reactance and $\frac{1}{C\omega}$ is the condensive reactance. The unit of reactance is the ohm.

144. Resonance.—One particular adjustment of the inductive and the condensive reactance in a circuit demands particular attention. It will be seen from the equation

$$I = \frac{E}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$$

that we may readily adjust the capacitance or the inductance to such a value that

$$\left(L\omega - \frac{1}{C\omega}\right) = 0$$

In this case, the current is given by the same equation as would be used in the case of a continuous-current circuit, namely

$$I = \frac{E}{R}$$

with this adjustment, the circuit is said to be in *resonance.*

In practice, this condition of resonance is of importance because it may lead to the development of dangerously high potentials at various points of a system. Thus in Fig. 99, the applied e.m.f. may be 220 volts.

$$E = RI = 220 \text{ volts.}$$

The e.m.f. over the inductance is

$$E_L = LI\omega,$$

and that over the capacitance

$$E_C = -\frac{I}{C\omega}$$

If the circuit is in exact resonance, E_L and E_C will be equal. It may readily occur that either of them may be far larger than

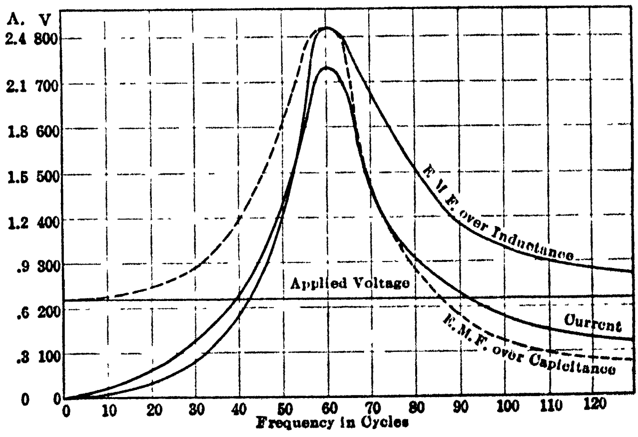

FIG. 100.

the applied e.m.f. This will be particularly the case if the resistance is low, and the frequency is high.

Resonance can also be obtained with fixed values of the capacitance and inductance, by varying the frequency. In Fig. 100 are shown the curves obtained in a test of an actual circuit. The

inductance was constant and had a value of 1.0 henry, and the capacitance was likewise constant at 7.0 microfarads. The applied e.m.f. was 220 volts, and the resistance, 100 ohms. Resonance was obtained at 60 cycles, and it will be seen that the e.m.f. over the inductor and that over the condenser were equal and were 3.77 times the applied e.m.f. or 830 volts.

145. Oscillatory Discharges.—Considering again Fig. 93, suppose that the piston is removed from the cylinder, and that the diaphragm is displaced to one side and then released. It is evident that if the friction of the liquid in the tube is reasonably low, the liquid will oscillate for some time instead of coming to rest at once. The curve of current is shown in Fig. 101.

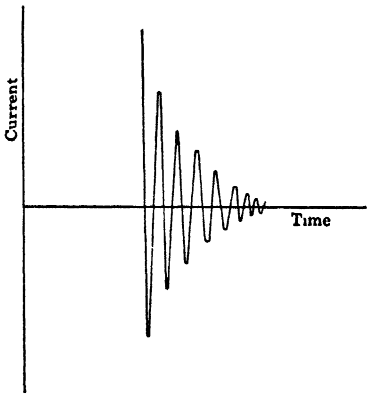

FIG. 101.

This condition is analogous to the case of a condenser, inductor and resistor in series without an applied e.m.f., *i.e.*, with the combination short-circuited. The connection is shown in Fig. 102. In either the electric or the liquid circuit, we have shown that with an applied alternating force the greatest current will be obtained when we have the condition of resonance. With the short-circuited electric circuit, or the hydraulic circuit without the piston, the system is not restricted to any particular frequency, and will tend to oscillate at that frequency which will give resonance. Hence we shall have the condition of resonance, namely,

$$\left(L\omega - \frac{1}{C\omega}\right) = 0$$

Substituting for ω its value $2\pi f$ and solving for f, the frequency, we get

$$f = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{LC}}$$

If we desire that the frequency in such a circuit be high, we may readily accomplish our object by making both L and C small. This is the case in wireless telegraphy. The oscillations

in the sending circuit are obtained by means of a circuit like that of Fig. 103, in which the condenser is charged by a transformer and allowed to discharge across a gap and through the primary of a high-

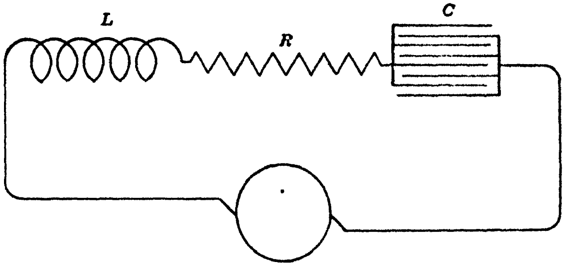

Fig. 102.

tension, high-frequency transformer. For wireless telegraphy, a frequency of about 700,000 cycles per second is customary. Since electric waves travel with the velocity of light (186,000

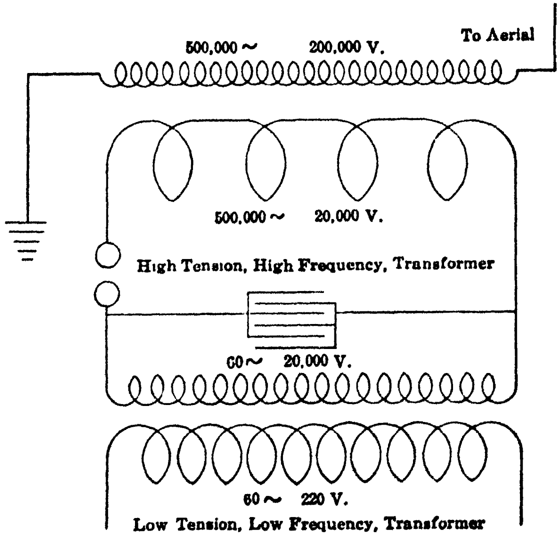

Fig. 103.

miles per second), it will be seen that this corresponds to a wave length of approximately ¼ mile.

PROBLEMS

(Note: In the following, unless the contrary is expressly stated, all currents and e.m.fs. are supposed to be harmonic.)

53. An alternating e m.f. of 100 volts at a frequency of 60 cycles per second is applied to a resistanceless inductor of 0.04 henry. Calculate the current.

54. With the same voltage and the same inductance as in the above problem, what would be the necessary frequency in order that the current may be 100 amp.?

55. In a certain circuit the resistance is zero, the applied e.m.f. is 220 volts, the frequency is 25 cycles and the current is 25 amp. What is the inductance?

56. The voltage of a circuit is 440 and the current is 0 01 amp. What is the impedance? If the resistance is zero, what is the reactance?

57. A certain inductive resistance has an inductance of 0 01 henry and a resistance of 2 ohms. If the voltage is 220 volts and the frequency 60 cycles, what is the current? What is the power factor? What is the angle of lag?

58. If in the case of the above inductor a direct current of the same value were passed through it, what would be the e m f. required? What would be the power? How does this compare with the power expended in the alternating-current circuit?

59. In a certain circuit the power factor is 0 6. $R = 3$, $L = 0.03$; what is the frequency? What would be the frequency for unity power factor? For zero power factor?

60. A certain condenser has a capacitance of 2 microfarads. If it is connected directly across a 220-volt, 60-cycle circuit what will be the current? What would be the current when connected across a direct-current circuit?

61. A condenser connected across 60-cycle mains takes a current of 1 amp., the voltage being 110. What is the capacitance of the condenser?

62. A condenser of a capacitance of 1 microfarad is connected in series with a non-inductive resistance of 1000 ohms. A potential of 250 volts at a frequency of 100 cycles is applied to the terminals. What is the current? What is the drop across the condenser? What is the drop across the resistance? What is the power factor? What is the power?

63. A certain a-c. voltmeter has a resistance of 1500 ohms and zero inductance. When connected directly across a 60-cycle circuit it reads 150 volts. When it is connected in series with a condenser and the two are connected across the line the reading is 75 volts What is the current flowing through the voltmeter and the condenser? Assuming that the line voltage remains 150, draw a vector diagram and compute the value of the e.m f. across the condenser. Confirm this by measurement from the diagram. From the above compute the capacitance of the condenser. (Note: The preceding furnishes a very convenient means of measuring the capacitance of a condenser, using only a voltmeter. The frequency of commercial lines is usually known nearly enough and the resistance of the voltmeter is usually given with the instrument.)

64. An harmonic electric current of 100 amp. flows through a circuit consisting of a resistor of 1 ohm, an inductor of 0.002 henry, and a condenser of 0.00004 farad. The frequency is 60 cycles per second. Draw a vector diagram and compute the voltages across the three elements of the circuit Also compute the voltage across the three in series. What is the

power factor in each of the three? What is the power factor of the whole circuit?

65. In the case of the foregoing circuit, at what frequency would resonance occur? Answer the questions of Problem 64 for the frequency of resonance.

66. A voltage of 100 at a frequency of 60 cycles is applied to a circuit consisting of a non-inductive resistor of 5 ohms, a reactor of 0.005 henry, and a condenser of 0 00002 farad in series What is the current? The voltage across each of the three elements of the circuit? The power factor of the circuit?

67. In the above, what value of the capacitance will be necessary in order that the power factor may be unity?

68. What is the reactance of 0 24 henry at a frequency of 60 cycles per second? What is the reactance of a 10-microfarad condenser at the same frequency? What is the reactance of the two in series?

69. A reactance coil is tested at 60 cycles with the following results: Volts 100, amperes 5, watts 173 Draw the vector diagram. What is the power factor? The angle of lag? What is the resistance? The reactance? The impedance? The inductance?

70. An harmonic e m f of 2200 volts produces a current of 100 amp in a circuit, the current lagging 25° behind the e m.f. Calculate the resistance, reactance and impedance

71. A number of incandescent lamps taking 100 amp. of current are connected to an alternating-current line. An inductive circuit of negligible resistance is connected to the same line and takes a current of 15 amp. What is the total current in the line?

72. An inductive resistance is connected across a 220-volt 60-cycle line and takes a current of 10 amp. at a power factor of 0 7. A condenser is then connected across the line in parallel with the inductive resistance. What must be the capacitance of the condenser in order that the power factor may be 0 85, the current being lagging? What for unity power factor?

73. In a wireless telegraph system the capacitance of the aerial is 1×10^{-6} microfarads and the inductance of the circuit is 0 05 henry. What is the frequency of the oscillations in this circuit? What is the wave length?

CHAPTER XIII

ALTERNATING-CURRENT MEASURING INSTRUMENTS

146. Action of Direct-current Instruments on Alternating-current Circuits.—In a previous chapter we have discussed some forms of continuous-current instruments. It will be noted that the instruments described in this chapter, while primarily designed for use on alternating-current circuits, will nevertheless in most cases operate with satisfaction on direct-current circuits. The converse is not necessarily true. Thus, an instrument of the D'Arsonval type if used in connection with an alternating current of commercial frequency will give no deflection. It will have a tendency to deflect first in one direction and then in the other, but the duration of the impulses is so short that the pointer has not time to move an appreciable distance and as far as the eye can detect does not move at all. The plunger type of instrument, if the moving plunger is of soft iron, may be used, since the plunger will be drawn into the solenoid no matter in which direction the current is passing.

147. The Electrodynamometer Type.—An instrument of this type is shown in Fig. 104. The construction is essentially the same whether used as a voltmeter, ammeter or wattmeter. This type of instrument is common in the highest grade and most accurate forms of alternating-current instruments. In general it contains two coils, one fixed and one movable. The fixed coil is usually the larger and the movable coil is arranged to rotate within it. The axes of the two coils are at a considerable angle when the instrument is carrying no current. For use as an ammeter or as a voltmeter the two coils are connected in series. On passing current through the instrument the coils tend to turn to such a position that their axes are parallel. This motion is resisted by a spring. To understand this action we may think of the stationary coil as taking the place of the permanent magnet in the D'Arsonval type of instrument. The movable coil tends to turn so as to place its axis parallel to the magnetic lines. It is evident that the coil will turn in the same direction no matter

what the direction of the current, since the current is reversed in both coils at the same time. Moreover, the force acting on the coil will at all times be proportional to the *square* of the current, since the same current passes through both coils. The deflection of the pointer will therefore be dependent upon the average square of the current, or the instrument will indicate correctly irrespective of the wave shape.

The indications will also be entirely independent of the frequency when the instrument is used as an ammeter. When

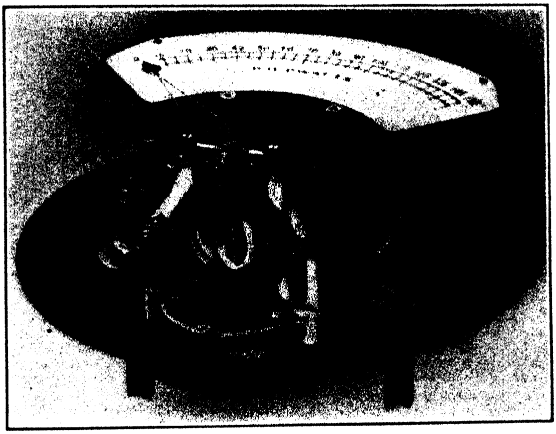

FIG. 104.

employed as a voltmeter, the current passing through the instrument is given by the expression

$$I = \frac{E}{\sqrt{R^2 + (2\pi f L)^2}}$$

The frequency enters into this expression. In order that its effect may be negligible, it is necessary that the inductance L should be made very small in comparison with the resistance R. Since a large resistance is necessarily used, and since the inductance of the coils is small, it is not difficult to do this. Commercial instruments give readings practically independent of the frequency for any of the frequencies in common use.

The illustration (Fig. 104) will give an idea of the internal construction of the instrument.

148. The Wattmeter.—If connected as shown in Fig. 105, the electrodynamometer type of instrument becomes a wattmeter.

The stationary coil is wound with comparatively coarse wire and the whole current passes through it. The movable coil is wound with fine wire and has a large non-inductive resistance connected in series with it. This coil is connected as a shunt across the line. The stationary coil has therefore passing through it the total current of the circuit. The movable coil carries a current proportional to the voltage of the circuit and in phase with the voltage. If the current and the e.m.f. in the main circuit are in phase, the currents in the two coils are also in phase and the deflection of the coil is a maximum for the given currents. If, on the other hand, the current and the e.m.f. differ in phase by 90°, the current in the movable coil will be passing through zero at the instant when the current in the other coil is a maximum. An instant before this takes place the tendency to deflect will be in one direction and an instant after, in the opposite direction, since the current has reversed in only one of the coils. The time during which the coil is urged in the one direction is equal to the time during which it is urged in the opposite direction, and consequently the net tendency to turn is zero, or no power is indicated. This is as it should be since with 90° lag the power is zero. Whatever the lag of the current, the tendency to turn at any instant is proportional to the instantaneous power, that is, to ei, and consequently the amount of power is correctly indicated.

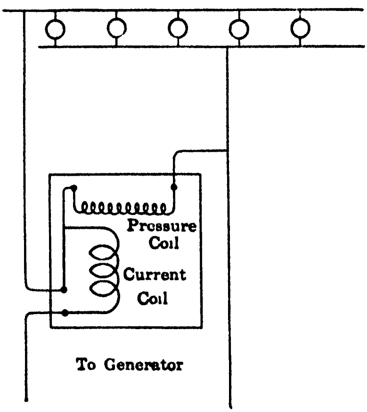

FIG. 105.

149. Hot Wire Instruments.—The passage of current through a wire causes it to heat and consequently to become longer. We may use this property to measure the strength of the current. One simple method of doing this is shown in Fig. 106. The current to be measured is passed through the wire W. Attached to the center of this wire is a light flexible cord T. This passes around the roller attached to the pointer and is kept in tension by means of the spring S. When current passes, W becomes longer, and the spring acting on the roller through the cord T causes

the pointer to move across the scale. It will be apparent that the movement of the pointer will be great for a comparatively small lengthening of the wire.

Hot wire instruments may be used either as ammeters or as voltmeters, a suitable resistance being used in the latter case. They may be employed upon either continuous- or alternating-current circuits. They are not affected by external magnetic fields, wave shape, or, within reasonable limits, by frequency. The principal difficulty in the design of these instruments seems to be to avoid the effects of the expansion of the plate, upon which the mechanism is mounted, when the room temperature changes.

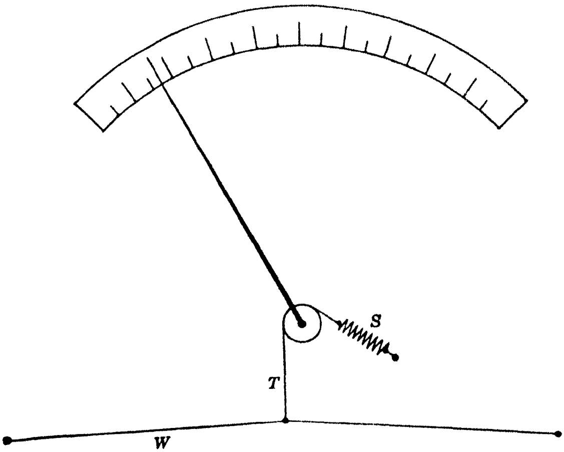

FIG. 106.

This causes the pointer to move from the zero point. By making the base of a material having the same coefficient of expansion as the wire the zero may be made reasonably stable.

Hot wire instruments, particularly in the case of ammeters, having thick wires, are naturally somewhat sluggish, that is, it requires some time for the wire to heat up to its final temperature. This may be a disadvantage in certain applications, but it may also be advantageous in case we wish to measure a current which is rapidly fluctuating. The current in the armature of a lightly loaded synchronous motor is frequently unsteady and sometimes the hot wire instrument is the only one which will give a readable deflection.

150. The Spark Gap.—Very high voltages are frequently best measured by means of a spark gap. Two sharp needles or two spheres of a definite diameter are used. The distance between the needle points when the current jumps is a measure of the voltage across the gap. Very careful measurements have been made of the properties of spark gaps and it is possible with the aid of a curve to estimate quite closely the voltage of the circuit. It must, however, be remembered that the breakdown of the gap is dependent upon the *maximum* value of the voltage and not upon the effective value.

151. The Electrostatic Voltmeter.—If two insulated metal plates are at different potentials they will attract one another with a force proportional to the square of the difference of potential. This force may be measured by noting the deflection of a spring or otherwise, and will be a measure of the e.m.f. Either direct or alternating e.m.fs. may be measured. The instruments are particularly adapted to voltages of 1000 or over although they are also built for lower voltages.

It will be noted that both of the last two instruments described are inherently voltmeters, while all the others are inherently ammeters.

152. The Oscillograph.—In the study of alternating-current phenomena it is important to be able to follow the fluctuations of the current or the voltage as it passes through its cycle. Since a complete cycle requires usually only $1/25$ or $1/60$ of a second, it will be apparent that this is difficult to accomplish. If it required, say, an hour to complete a cycle, the fluctuations of the current or the e.m.f. could be readily followed by means of a continuous-current instrument, the zero being preferably in the middle of the scale. With a frequency of 1 cycle per second, the pointer would begin to lag somewhat behind the current or e.m.f. and at, say, 25 cycles per second there would be little or no deflection, since the negative impulses would equal the positive, and since the pointer would not have time to move between them.

If, however, the pointer and other moving parts be made very light, they will be capable of keeping up with the variations of the current at a higher frequency, and by going to extreme lightness, we may construct an instrument capable of following accurately the variations of a current of any commercial frequency. Figure 107 illustrates the construction employed. A very fine wire is held in the shape of a loop between the poles of

a magnet. Across the loop is cemented a light mirror. A beam of light reflected from the mirror is used as the pointer. When current passes through the loop, one side is pushed forward by the action of the current in the magnetic field while the other side is pressed backward. As a consequence the mirror is twisted through a small angle and the beam of light is deflected.

The movements of the beam of light may be recorded by allowing it to trace its path upon a falling photographic plate. As an alternative we may use a film wrapped upon a rotating cylinder. Figures 69, 70, 71, and 72 show curves obtained in this way.

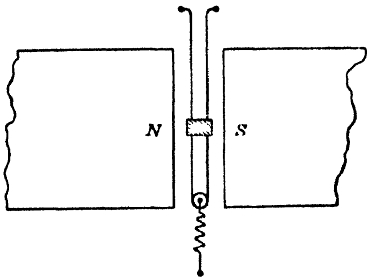

FIG 107.

We may also examine the shape of the wave without the necessity of photographing it by allowing the beam of light to fall upon a mirror rocked through a small angle by a synchronous motor. The motor should be driven from the circuit to be investigated, and the motion of the synchronous mirror should be such as to deflect the beam of light at right angles to the deflection due to the current. A shutter, also actuated by the synchronous motor, should cut off the light during the return of the beam. Under these conditions, the spot of light will trace out the same curve repeatedly, and since the eye can not follow the rapid movement of the spot of light, the curve will appear to be stationary upon the screen. It can then be readily examined or traced for preservation.

CHAPTER XIV

SINGLE-PHASE AND POLYPHASE SYSTEMS

153. Alternating-Current Generators.—Alternating-current generators, motors and transmission lines are usually single, two or three phase, although systems with a greater number of phases are occasionally used. Figure 108 shows the essential elements of a single-phase synchronous generator or motor. This machine has a stationary armature and a field revolving within it. It is much easier to insulate a stationary armature, and the fact that it is not necessary to use slip rings to convey the current from the

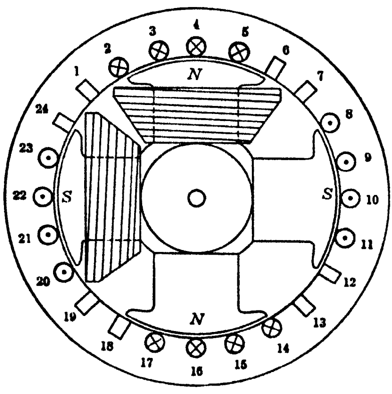

FIG. 108.

armature to the outside circuit makes the machine cheaper to construct. For these reasons the older type of revolving armature alternator is practically obsolete.

The armature in this case is provided with 24 slots or 6 slots per pole. In practice the number of slots per pole may vary from 3 to 18 or more. Of the 24 slots only 16 have conductors in them, the remainder being left vacant. The old type of alternator, in which only 1 slot per pole was used, is obsolete. The idea underlying the connection of the conductors is exceedingly simple although it frequently leads to somewhat complex dia-

grams. *If the conductors are so connected that the passage of continuous current through the winding gives a series of bands of current, all the currents in a band flowing in the same direction, the connection will be correct.* One way of doing this is shown in Fig. 109. If we start at the terminal marked +, and follow through the winding to the terminal marked −, it will be seen that the arrows indicate correctly the direction of a continuous current passing through the winding. There is first a band of four conductors with the current in the same direction in all of them, then a band with all the currents in the opposite direction, and so on.

The field is excited with continuous current. When the relative positions of the field and armature are those shown in Fig. 108, with clockwise rotation, an e.m.f. will be induced in conductors 2, 3, 4, and 5 and in 14, 15, 16, and 17 directed from the observer, with the direction of rotation as shown. In conductors

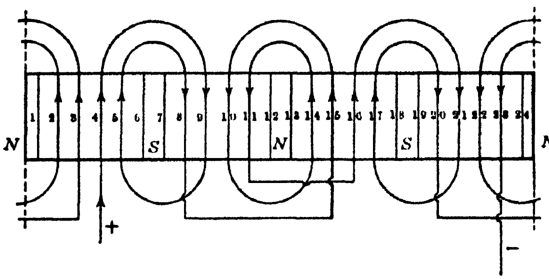

FIG. 109.

8, 9, 10, and 11, and 20, 21, 22, and 23, the e.m.f. will be in the opposite direction. These conductors are to be connected in series with one another in such a manner that their e.m.fs. will all add together, and all assist the current to flow. This will be the case if the rule already stated is adhered to, as will be apparent from Fig. 109.

In the alternator just described one-third of the slots have been left vacant. There is nothing to prevent us from placing coils in these slots and connecting these coils in series with the rest of the winding. The gain in e.m.f. from these coils would however be small. Thus a coil in slots 6 and 7, would embrace only a small portion of the flux from a pole and would generate only a small e.m.f. Omission of the coils in slots 6 and 7, 12 and 13, 18 and 19, and 24 and 1 as shown in Fig. 109 reduces the generated voltage 13.4 per cent., but saves approximately 30 per cent. of

the copper used on the armature. In addition to increasing the cost, the use of these coils would result in a lowering of the efficiency, and in a poorer regulation. For this reason they are rarely used in a single-phase alternator.

As stated, the field of an alternator is excited by means of continuous current. This is usually supplied by a small direct-current generator. If only one or two alternators are to be installed in a given station, the exciters may be belted or direct-connected to the main generators. In larger stations, it is customary to drive the exciters from individual prime movers (usually steam engines or water wheels), or by means of alternating-current motors, taking their current from the main generators.

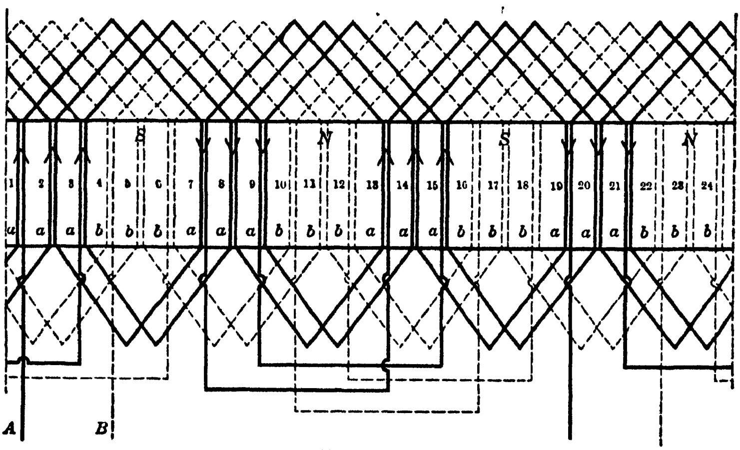

FIG. 110.

In addition, in large stations a storage battery is usually provided as insurance against failure of exciting current, and to supply current for the station lights in case of a complete shutdown.

154. The Two-phase Generator.—We have pointed out that in the single-phase generator it is advisable to use only about two-thirds of the slots, the remaining third being vacant. By thus leaving out one-third of the coils we lose about 13 per cent. in voltage and save nearly 30 per cent. in the amount of armature copper. We may go a step farther and use only half of the slots. By doing this we lose 30 per cent. in voltage and save 50 per cent. in armature copper. A winding of this character is shown by the full lines of Fig. 110. This winding is of a different type from that of Fig. 109. There are two coil sides in each slot instead of

one, and the coils are all alike. This is advantageous from the manufacturing standpoint and at the same time, makes it easier for the user to carry spare coils in case of a breakdown. By tracing through the winding it will be evident that we have followed the same principle as in Fig. 109, namely, that when current passes through the armature a number of bands of current, having alternately opposite directions, are formed.

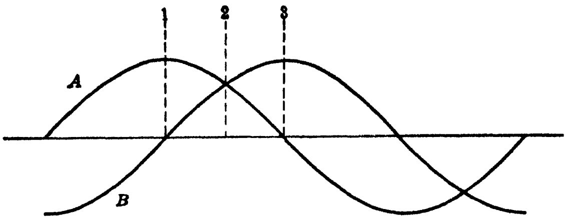

FIG. 111

In a single-phase machine we would usually make use of a greater percentage of the slots than one-half, although we could use only one-half of them with comparatively little loss. However, if we have only half of the slots occupied it would at once appear that we could readily wind a second winding in the vacant slots and thus double the capacity of our alternator at the expense of doubling the armature copper, everything else remaining the same. The capacity of the generator would be 41 per cent. greater than that of the same machine wound single phase, with all the slots used.

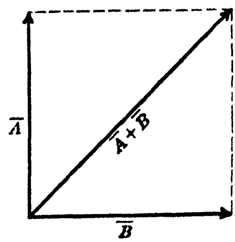

FIG. 112.

With this connection, the e.m.fs. of the two phases differ by 90°, that is, when one e.m.f. is a maximum the other is zero and *vice versa*. The e.m.f. of the *A* phase is a maximum when the center of the pole is opposite the band of conductors marked *A*. At this same time the *B* band of conductors is opposite the space between two poles and consequently no voltage is generated in the *B* winding. The two e.m.fs. may be represented by the curves of Fig. 111, or by the vectors *A* and *B* of Fig. 112.

This figure also serves to explain why more capacity can be obtained from the machine connected two phase than when connected single phase. If we should connect the two windings in

series the combination would constitute a single-phase winding. The e.m.f. would be the vector sum of the two separate e.m.fs. and would be represented by the line marked $A + B$ in Fig. 112. If we take A and B as each being equal to 1, $A + B$ will be equal to $\sqrt{2} = 1.41$. We may assume without serious error that each winding will carry the same current whether connected single or two phase, or the capacities will be proportional to the voltages. The rating of the single-phase machine may then be taken as 1.41 while the rating of the two-phase machine would be 2 or 41 per cent. greater.

155. Electromotive Force of an Alternator.—In an alternator the flux through a coil is represented at any instant by the expression,

$$\varphi = \Phi \sin \omega t$$

Where Φ is the maximum value of the flux passing through one armature coil. The instantaneous e.m.f. induced in the armature winding is given by

$$e = -\frac{N}{10^8}\frac{d\varphi}{dt} = -\frac{N\Phi\omega}{10^8}\cos \omega t$$

where N is the number of turns (*i.e.*, half the number of conductors) converted in series. The maximum value of the e.m.f. is $E_M = \frac{N\Phi\omega}{10^8}$, and the effective value is

$$E = \frac{E_M}{\sqrt{2}} = \frac{2\pi f N\Phi}{\sqrt{2} \times 10^8} = 4.44\, fN\Phi \div 10^8$$

With a distributed winding as ordinarily used the maximum e.m.f. does not occur in all the coils at the same time. To allow for this we multiply by a factor called the breadth coefficient. This varies slightly with different windings but is very nearly 0.95 for three-phase windings and 0.90 for two phase.

156. Method of Connecting Load.—The connection of a two-phase alternator to its load is shown in Fig. 113. The two circuits are usually entirely independent of one another, although in some cases they are interconnected as will be shown presently. To supply lights we should run two wires to each group exactly as though they were to be operated from a single-phase generator. In order that the voltages may not differ too much it is desirable that the loads on the two circuits be nearly the same. Thus if a system were being operated single phase and we wished to change

to two phase, it would merely be necessary to divide the load into two fairly equal parts and connect the parts to the two phases. Small single-phase motors might be operated from one phase, but in general, the motors would be wound for two-phase operation, and it would therefore be necessary to run all four wires to each of them.

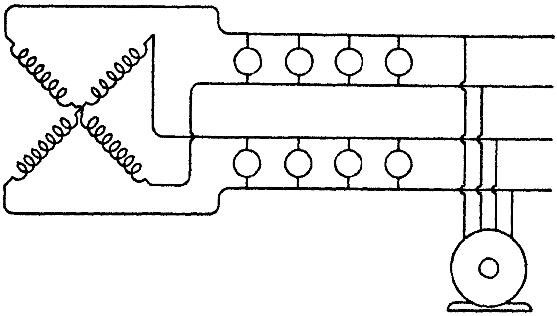

FIG. 113.

In the foregoing we have shown the phases as entirely insulated from one another. There is however nothing to prevent us from connecting any *one* point on either winding to any *one* point on the other. Since there would be only the one connection, there would be no circuit between the two windings and no current could flow through the connection. The operation of each winding would therefore be the same as before.

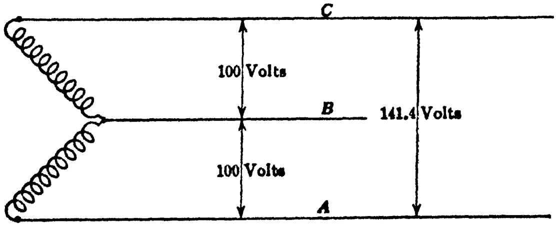

FIG. 114.

Thus we might connect the ends of the windings together as shown in Fig. 114. If each winding developed 100 volts we should have this pressure between wires *A* and *B* and between *B* and *C*, while between *A* and *C* there would be 141 volts. This connection may be used in special cases but in general it possesses no particular advantage and is rarely used.

A more common connection is that of Fig. 115, in which the middle points of the two windings are connected. In this case if each winding generates 100 volts, we can make four additional connections giving in each case 70.7 volts or half of that between *A* and *C* in Fig. 114.

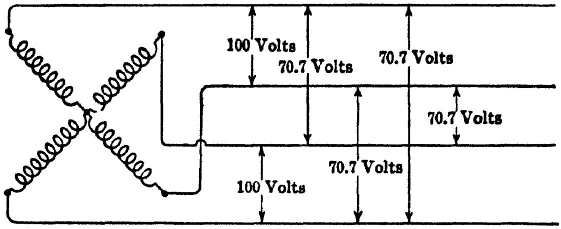

Fig. 115.

157. Three-phase Systems.—It has been shown that by using the two-phase system we can obtain a considerably greater output from a given generator than would be possible with single-phase operation. By using a three-phase connection it is pos-

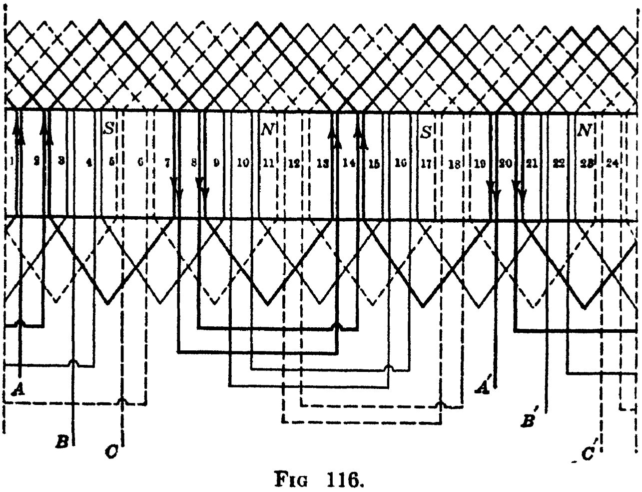

Fig 116.

sible to increase the power output about 5 per cent. above that of a two-phase generator. There is also a distinct advantage in transmission. Three wires in a three-phase system will transmit a certain amount of power at a given voltage with the same loss as would be present in four wires of the same size in a two-

phase transmission. These and other advantages have caused the three-phase system to be preferred in most cases. It may be considered the standard method of alternating-current generation and distribution.

A diagram of a three-phase winding is shown in Fig. 116. One phase is represented by means of heavy lines, the second by a light

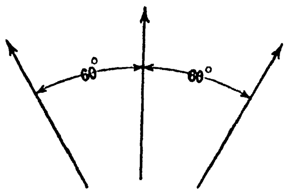

FIG. 117

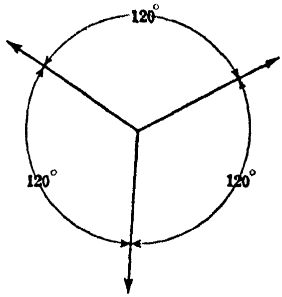

FIG. 118

line and the third by means of a broken line. Since each band spans a distance of one-third of a pole pitch, each winding will differ in phase from its neighbor by 60°. This relation is shown in Fig 117. This connection gives an unsymmetrical e m.f., and to obtain a three-phase connection it is necessary to reverse the connections of the middle phase. We then have the relation

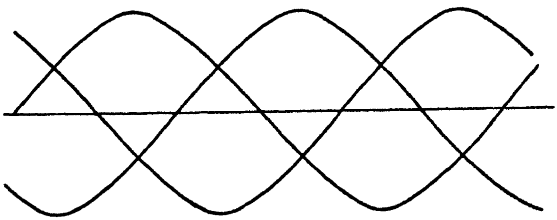

FIG. 119.

shown in Fig. 118, or a difference of phase of 120°. The corresponding curves plotted in rectangular co-ordinates are shown in Fig. 119.

158. Advantages of Three Phase over Single Phase.—To show the advantage of the three-phase connection over the single phase we may connect the three phases in series to form a single-phase winding. There are two possible ways of doing this.

One would be represented by the vector diagram of Fig. 120. The sum of A and B is equal in magnitude to either A or B or to C. Adding C in the phase shown just neutralizes the sum of A and B or the total sum is zero. By reversing the connection of C, however, we obtain a relation represented by Fig. 121 in which the total voltage is exactly twice the voltage of any one phase. Operated three phase we get the full voltage of each phase. We may therefore conclude that if the output of a single-phase generator in which all of the slots are used is represented by 2, the output of the same machine wound three phase would be 3, or an advantage of 50 per cent. in favor of the three-phase machine.

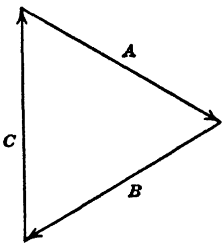

FIG. 120.

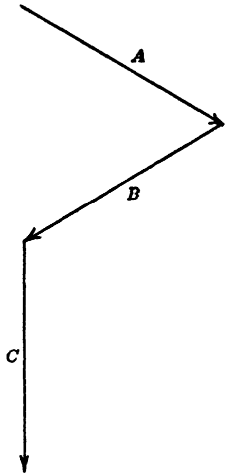

FIG. 121.

As we have previously shown, we would, in general, in a single-phase machine use only about two-thirds of the slots, the remainder being left vacant. We can readily show the exact loss of capacity involved by means of a vector diagram. If we should use only two of the three windings of a three-phase machine in a single-phase machine we could connect them in such a manner as to give a vector diagram the same as that of Fig. 121 if C were omitted. The voltage would then be that of one phase only or might be represented by 1. This would obviously be undesirable, and the machine can be very much improved by reversing the connections of the B phase giving the vector diagram shown in Fig. 122. It will be seen that the angles between A and B and the resultant are in both cases equal to 30°. The resultant will be given by the expression

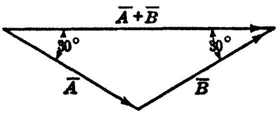

FIG. 122.

$$\bar{A} + \bar{B} = 2A \cos 30° = 2 \times \frac{\sqrt{3}}{2} A = 1.73A$$

Since if all three windings are used we obtain a voltage of $2A$, there is an increase of approximately 15 per cent. in voltage if

three windings instead of two are used. As previously pointed out, three windings require 50 per cent. more copper, and involve 50 per cent. more RI^2 loss. The increase in voltage is hardly enough to pay for the disadvantage and consequently only about two-thirds of the slots are ordinarily used, in a single-phase generator or motor.

159. Three-phase Connections.—The most obvious method

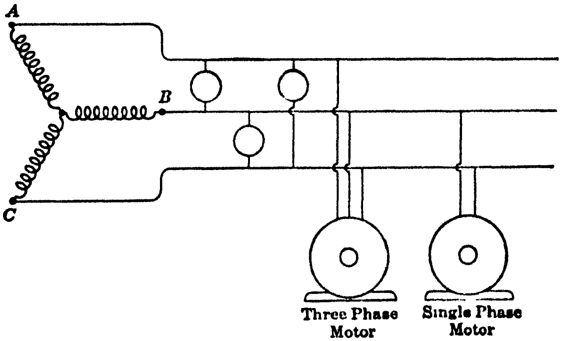

FIG. 123.

of connecting a three-phase alternator to its load would be to run three independent circuits of two wires each. This is rarely or never done except in the case of rotary converters, since it involves the use of six wires, six pole switches, six pole cut-outs, etc. Such an arrangement would be known as a six-phase system.

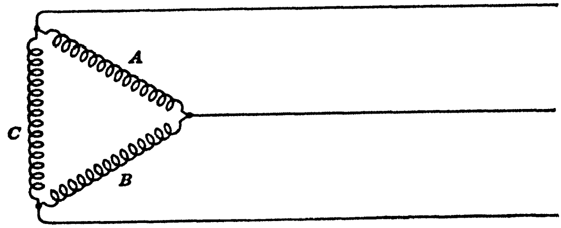

FIG. 124.

Two methods of connection are in general use. These are illustrated in Figs. 123 and 124 and are known respectively as the star or "Y" connection and the delta or mesh connection. The corresponding vector diagrams are given in Figs. 125 and 126. With either of these connections it will be evident that all the voltages from A to B, from B to C, and from C to A will be

the same in magnitude but will differ 120° in phase. We can therefore connect single-phase apparatus across any one of these three circuits as shown in Fig. 123. Apparatus requiring three-phase current such as induction motors, may be connected to all three lines.

Fig. 125 requires a little further explanation in order that it may be entirely clear. The vectors A, B and C have all been drawn as being directed outward from the center, that is, the voltage A is considered as being positive when its direction is from the neutral point outward and likewise with the other e.m.fs. If we consider a current passing from the point c to the point a, it will be evident that a positive direction of the e.m.f. in the winding A will help the passage of the current while a positive e.m.f. in C will oppose it. At the instant shown we may consider that A has its maximum positive value while C is nega-

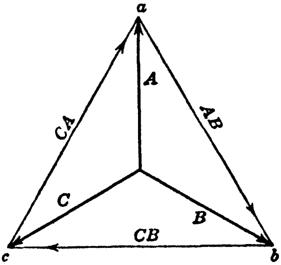

Fig. 125.

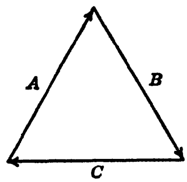

Fig. 126

tive. Both e.m.fs. will therefore help the passage of the current. In considering the effect of the e.m.f., C, upon the current it will therefore be necessary to consider it as being negative. The vector CA is therefore the vector sum of A and $-C$. Likewise, AB is the vector sum of B and $-A$, and BC the vector sum of C and $-B$.

160. Voltage and Current Relations.—With the star connection (Fig. 123) the current in any one of the three generator windings is obviously the same as the current in the corresponding line wire, since the two are directly connected in series. The voltage between any two line wires, however, is not the voltage of one of the generator windings. If we represent the line voltage, CA by E, and the voltage of one generator phase, A by E_g, it will be evident from Fig. 125 that

$$E = 2E_g \cos 30° = \sqrt{3}E_g = 1.73\ E_g$$

With the mesh connection of Fig. 124, it will be seen that the line voltage is the same as that of one generator winding. The line current will, however, be the vector sum of the currents in the two generator windings connected to the line wire. If the two currents are the same, the process of finding their resultant will be the same as that of finding the resultant of the two voltages of Fig. 123, since they are at the same angle to one another as the two voltages. Designating the line current by I and the generator current per phase by I_g we have

$$I = \sqrt{3}I_g = 1.73I_g$$

161. Power in Balanced Three-phase Circuits.—If we consider one of the windings in Fig. 123 or Fig. 124, it will be evident that the power in this winding is equal to the current times the e.m.f. times the power factor. The total power of the generator will be the sum of the power of the three phases. This will be true for either balanced or unbalanced loads. For balanced loads we may write for the star connection

$$P = 3E_gI \cos \theta = \sqrt{3}EI \cos \theta$$

where P is the power output of the three phases and $\cos \theta$ is the power factor. A simple substitution will give the same result for the mesh connection.

162. Substitution of a Three-phase Alternator for a Single-phase Machine.—When a piece of apparatus is connected to a line so as to take current from it, the voltage of the line will in general be changed, and will usually be lowered. It is therefore advisable that the current taken from the three circuits be the same, both in quantity and in the angle of lead or lag, so that the three voltages may remain balanced. However, it is by no means necessary that this be done, provided a small unbalancing of the three voltages is not objectionable. Thus all the load might be connected between the lines A and B. The winding C would then carry no current and the machine would operate as a single-phase alternator; or we might connect all the lighting load across A and B, running the third wire C only to whatever three-phase motors might be connected to the system. The unbalancing of the voltage on the motors, while somewhat undesirable, might not be serious. This expedient is sometimes used when it is desirable to add a motor load to an established single-phase system. A three-phase generator may be purchased, the lights

operated as before from one phase, and all three wires run to such motors as may be installed.

163. Rotating Magnetic Field in the Armature of the Alternator.—Figure 108 shows a section of a revolving field alternator. In discussing this machine it was assumed that all of the magnetic field was produced by the action of the field magnet. This is not the case, as the armature current has a considerable effect upon the magnetism. In the single-phase alternator the armature current has a tendency to cause the field magnetism to pulsate. In the two- or three-phase machine, the effect is practically constant and tends either to weaken or to strengthen the magnetism which would be present due to the field alone.

Referring to either Fig. 108 or Fig. 109, it will be seen that if a continuous current be passed through the winding of the armature in the direction shown, four poles will be formed on the armature surface. The strength of these poles will be greatest in the positions indicated and will gradually diminish in strength to the positions half way between the points marked. There will thus be a gradual shading off from one pole to the next.

164. Action with Single-phase Alternating Current.—If a single-phase alternating current be passed through the winding described, poles will be formed as with the continuous current, but these poles will die out and reverse in phase with the current. In other words, the field is stationary in space and pulsating in value. If the field magnet is within the armature and the machine itself is generating the current which passes through the armature, the same considerations will hold. The effect of the armature current upon the field magnetism is, then, to cause it to pulsate somewhat in value. If the current delivered by the alternator is neither lagging nor leading, there will be but little effect upon the average value of the magnetism since the armature exerts its strongest effect midway between the field poles. If the current is lagging it will have a powerful tendency to weaken the field, and conversely if it is leading it will strengthen the field. This effect, known as armature reaction, will be more fully treated in the chapter upon the synchronous machine.

165. Action with Two-phase Alternating Current.—If but one winding only of the two-phase armature of Fig. 110 is considered, it will be seen that an alternating current passed through it would have the same effect as that just described. If an alter-

nating current were passed through the other winding, the effect would be the same except that the poles would be shifted to a position midway between those due to the first winding. If a *single-phase* current were passed through both of these windings in series, the two sets of poles would unite to form a resultant set of poles midway between the other two sets.

If, however, currents differing in phase by 90° (that is, so related that when one current is a maximum the other is zero) are passed through the two phases, the action is quite different. Thus in Fig. 110, at the time when the current in the *A* phase is a maximum in the direction shown the maximum points of the north poles will be opposite slots 11 and 23, and those of the south poles opposite 5 and 17. A quarter of a period later the current in the *A* phase will be zero and that in the *B* phase a maximum. The poles will then be opposite the slots 2, 8, 14, and 20, or in other words, the poles have moved three slots or one-half of a pole pitch to the right.

If the condition at a time intermediate between the two times noted above is considered, namely, when the two currents are equal as shown at time 2 of Fig. 111, it will be evident that both windings will have an effect and resultant poles will be formed midway between those due to each winding alone.

From the foregoing it appears that the poles will travel along the face of the armature at a practically uniform rate of speed and that the strength of the poles will be practically constant. The speed will be such that a distance equal to twice the pole pitch will be passed over during 1 cycle. Since the poles of the field magnet also travel at the same rate, the field due to the armature and that due to the field magnet will *maintain the same relative position,* as long as the conditions do not change.

166. Action with a Three-phase Current.—In a like manner with the aid of Figs. 116 and 120 one can trace the magnetizing action of a three-phase current upon the armature. It will be found that at all times at least two of the phases are carrying current and most of the time all of the phases are active. The speed of the field will be the same as before, and consequently the same as the speed of the field magnet. This fact would be true for any number of phases.

167. The Synchronous Motor.—The foregoing considerations lead to seeing in a simple manner how an alternator may act as a motor. Assume that the armature of the alternator repre-

sented in Fig. 108 is wound for either two- or three-phase operation and that the corresponding two- or three-phase current is passed through it. A revolving magnetic field will be set up as previously indicated. Rotate the field magnet by external power at the same speed as the revolving magnetic field, and remove the driving power. Even though there is no current in the field windings, the revolving magnetism of the armature will attract the poles of the field as any magnet attracts soft iron, and will carry the poles around with it. The machine will then act as a motor and will continue to revolve at *exactly* this same speed as long as the frequency of the current supplied remains the same. Variations of voltage, current, changes in the load, etc., will have absolutely no effect upon the speed, provided that the load doesnot become so great or the voltage so low that the machine is unable to carry the load. If this is the case, the motor will stop entirely.

The passage of current through the field coils will strengthen the attraction between the field and armature and enable the machine to carry more load without "dropping out of step," but will have absolutely no effect upon the speed. With no current in the field winding the power factor will be very low and the current will lag far behind the e.m.f. The passage of a current through the field will bring the armature current more nearly into phase with the e.m.f., and by adjusting the field strength to just the proper value, the lag can be reduced to zero. A stronger field current than this will result in a leading current. These facts will be more fully treated in Chap. XVI.

168. Measurement of Power in Polyphase Circuits.—The construction of the wattmeter has been discussed in Chap. XIII, and its connection on a single-phase circuit is illustrated in Fig. 105. It is obvious that the total power of a two-phase circuit can be measured by connecting one wattmeter in each of the phases as shown in Fig. 127. The total power will be the sum of the two readings. In case two wattmeters are not available, it would be necessary to measure the power of one phase and then shift the wattmeter to the other phase. The readings might, of course, be far from the truth if the power should change between the two readings.

The connection shown in Fig. 127 will give the total power correctly only in case the phases are not interconnected in both the load and the generator. If the generator windings are connected to one another at their middle point and the loads on the

two phases are similarly connected, current can flow out on wire 2 and back on 3 without passing through the current coils of the wattmeters at all. Consequently the power developed by this current would not be indicated on the instruments. A circuit of this character would be more properly called a four-phase

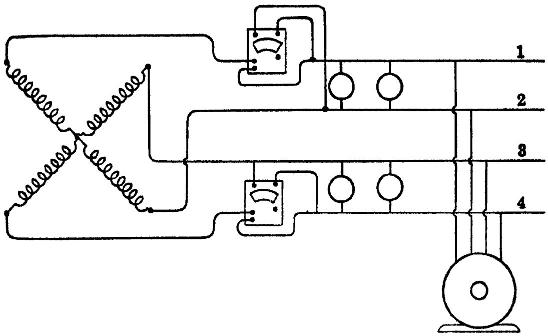

FIG. 127.

circuit than a two-phase circuit. Such interconnection on both the generator and the load is not common.

169. Measurement of Power in Three-phase Circuits.—If power is supplied from a star-connected three-phase generator or from three transformers connected in star, the most obvious method of measuring the power is to employ three wattmeters as shown in Fig.

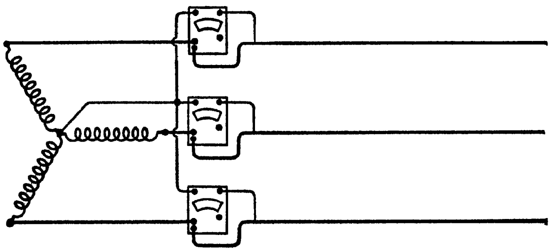

FIG. 128.

128. The current coil of each is connected in one of the line wires, and all the pressure coils are connected to the neutral point at one end and to their respective line wires at the other. Each wattmeter measures the power developed by one of the three windings, and the sum of the three readings will be the total power. Moreover, if the three phases are balanced the readings of all the wattmeters will be the same. This method is not much

used except in laboratory work, since it requires the use of three wattmeters and since it is rarely the case that the neutral point is readily accessible. Occasionally a small three-phase reactor is used to provide an "artificial neutral," or two non-inductive resistors having the same resistance as the pressure coil of the wattmeter may be used with it for the same purpose.

170. The Two-wattmeter Method.—Referring to the mesh connection of Fig. 124, and the corresponding vector diagram, Fig. 126, it will be seen that the sum of the two e.m.fs., A and B, is equal to the third e.m.f. C. It is then quite evident that the third winding C might be omitted and the machine would continue to operate as a three-phase generator with only the two windings as indicated in Fig. 129.

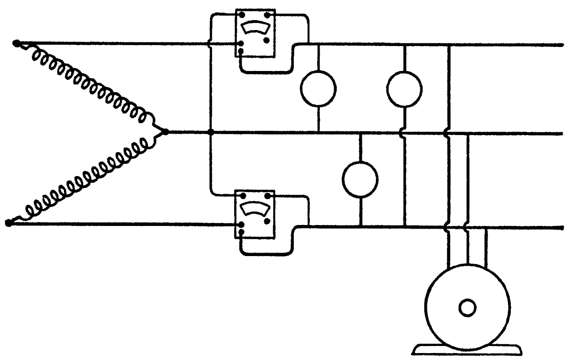

FIG. 129.

The capacity of the machine would be reduced and there would be a tendency for the voltages of the three phases to differ. On this account such a winding is not used on generators. Two transformers are, however, frequently so connected, since it is cheaper to provide two large transformers than three small ones.

It is then evident that a true three-phase current can be obtained from two windings connected as shown in Fig. 129. It is likewise evident that two wattmeters connected as in this figure will measure correctly the total power of the circuit, since each of them will measure the power of its respective winding. If the two windings of Fig. 129 are replaced by three windings, connected either in star or in mesh, the currents in the line wires and the voltages between them *would not be changed*, since these are determined *by the connected load.* The power would be the same and the readings of the wattmeters likewise the same.

Consequently the two wattmeters will measure correctly the power of the three-phase circuit.

Without going fully into the reasons, it should be pointed out that the two wattmeters will not necessarily read alike even though the circuit be balanced. If the circuit is balanced and the power factor is unity, the vectors A, B, and C in Fig. 125 may be taken as representing both the currents in the three lines and the voltages of the three generator windings from the neutral point to the terminals. They are, however, not the voltages between lines which are represented both in magnitude and direction by the three vectors AB, CA, and CB. If the two currents flowing in the two wattmeter coils are C and B, the voltages applied across the pressure coils are AB and CA. In the one case the current lags 30°; in the other it leads by the same angle. If the circuit is balanced the two readings will be the same.

Suppose, however, that the current lags 30°. In the wattmeter in which the current was 30° ahead of the e.m.f., the two will now be in phase, and in the other instrument in which the lag was 30°, it will now be 60°. The reading of the first wattmeter will now be greater than before while that of the second will be far less. If the lag of the current in the circuit as a whole becomes 60°, there will be a lag in one meter of $60° - 30° = 30°$, while in the other it will be $60° + 30° = 90°$. The reading of this second meter will then be zero. With still greater lag of the current the reading of the second meter will reverse and its indication must then be subtracted from that of the first. It should also be noted that a lag of 60° corresponds to a power factor of 0.5, and this is then the power factor of the circuit, when the reading of one meter is zero.

The two-wattmeter method as described is correct for unbalanced loads, for any power factor and for distorted waves of current or e.m.f.

171. Polyphase Wattmeters.—In many cases it is a great convenience as well as an aid to accuracy to be able to determine the power in a polyphase circuit with one reading. This is particularly true in the case of switchboard instruments. An instrument to do this can be readily constructed by mounting two wattmeter movements on one spindle. The torques of the two will then be automatically added or subtracted as the case may require, and the total indication of the instrument will be proportional to the total power of the circuit. The same principle may be applied to watthour meters. Polyphase meters

may be operated on single-phase circuits by connecting in only one side of the meter, or both sets of coils may be connected in series or in parallel.

172. Power Factor of Unbalanced Polyphase Circuits.—In a two- or three-phase circuit the voltages, currents and power factors may all be different. There are then two or three different power factors so that the term power factor as applied to the whole circuit loses its significance. It is the common practice when the unbalancing is not too great to take the average current and the average voltage, and compute the power factor as though the circuit were balanced. This will give results good enough for ordinary commercial purposes. However, it is not at all impossible with this method to obtain power factors greater than unity, a result obviously absurd.

173. Line Regulation.—Every transmission line has resistance, inductance and capacitance. The last is due to the fact that the two wires comprising a line are at different potentials and are separated by an insulator; namely, the air, or in the case of a cable, whatever insulating material is used. Since the conductors are quite a distance apart (except in the case of cables) the condenser effect is small unless the line is very long or the voltage high. In the following discussion it will be neglected.

Both the capacitance and the inductance of a line are *distributed* along the line, whereas in ordinary computations it is assumed that they are "lumped" at one or more points. If all of these factors are taken into account, the problem of line regulation becomes very complicated. In the following discussion it is assumed that the capacitance may be entirely neglected and that the inductance is "lumped" at one point of the line. The results will be commercially correct for short lines operating at moderate voltages.

174. Regulation at 100 Per Cent. Power Factor.—Fig. 130 shows the vector diagram of a transmission line when the connected load is non-inductive. E represents the voltage at the receiving end of the line, and I, the current in phase with it. To the terminal voltage must be added a voltage RI to overcome the ohmic drop in the line and a voltage, $LI\omega$, to overcome the inductive drop. The first is in phase with the line current and the latter 90° ahead of it. The voltage which must be applied to the sending end of the line is represented by E_0. It will be seen that the applied voltage is larger than the voltage at the receiving end and is somewhat ahead of it in phase.

175. Regulation with Lagging Current.—The vector diagram for lagging current is shown in Fig. 131. The same principles are applied. It will be apparent that if E is the same as in the

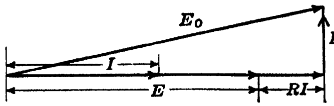

Fig. 130.

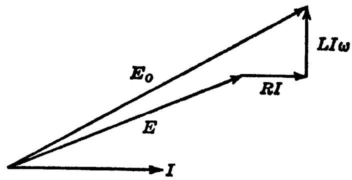

Fig. 131.

first case the applied voltage must be much greater, or the drop is far greater.

176. Regulation with Leading Current.—Fig. 132 shows the method of finding the regulation when the current is leading. If the RI drop is small and the $LI\omega$ drop large, it will be evident that E_o may be smaller than E, that is, the voltage is greater at the receiving end than at the sending end. It should not be inferred that the *power* is greater at the receiving end. This would be impossible, and it will always be found that the product of E_o and I times the cosine of the angle between them is greater than EI times the cosine of the angle between them.

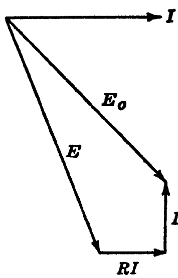

Fig. 132.

It will be clear now that the regulation of a line depends not only upon the constants of the line itself, but also upon the kind of load that is supplied from it. If the load consists of incandescent lamps with a power factor of nearly 100 per cent. the drop will be small and the regulation good. If the line supplies a load of induction motors taking a lagging current, the regulation will be much poorer. If on the other hand the load consists of an over-excited synchronous motor or other load capable of taking a leading current the voltage at the far end of the line may be greater than the sending voltage, or the regulation may be negative. The power loss in the line will be the same in all cases for the *same current.* For the same power the current, and therefore the loss, will be least if the power factor is 100 per cent.

The above method can be applied to each phase of a two-phase circuit. In the case of a three-phase line E_o and E should be

taken as the voltages from the neutral point to the line and the same method applied.

PROBLEMS

74. An alternator has ten poles and revolves at a speed of 720 r.p.m. What is the frequency?

75. At what speed must a two-pole alternator revolve in order that the frequency may be 60 cycles per second? What is the highest possible speed of a 60-cycle alternator? Of a 25-cycle alternator?

76. In the case of a three-phase star-connected armature the voltage from the neutral point to each terminal was 1328 volts, the current per line wire was 100 amp., and the receiving circuit was non-inductive, so that the lag was zero. What was the voltage between line wires? What was the power output of the generator?

77. In measuring the power output of a three-phase generator, three wattmeters were connected as shown in Fig. 128. The reading of each was 1000 watts, the voltage between line wires was 230 volts, and the current in each line wire was 12 amp. What was the power factor?

78. Two wattmeters connected as shown in Fig. 129 to a three-phase alternator gave readings of 2000 watts and 1200 watts respectively. The current in each line was 7 amp. and the line voltage was 440. What was the power output? What was the power factor?

79. A single-phase transmission line 20 miles long and operating at 60 cycles consists of two No. 6 conductors having a resistance of 0.395 ohm per 1000 ft. of each wire or a resistance of 0 79 ohm per 1000 ft. of line. The inductance is 0.0077 henry per 1000 ft. of line. The voltage at the end of the line is 22,000 and the current is 10 amp. at unity power factor. What is the power at the end of the line? What is the power loss in the line? What is the power at the beginning of the line? What is the efficiency of the line?

80. In the above line what is the resistance drop? What is the drop due to reactance? Draw the vector diagram and determine the voltage at the beginning of the line. What is the power factor at the beginning of the line?

81. Solve the preceding problem for a power factor of 0.6 lagging current at the end of the line.

82. Solve the preceding problem for a power factor of 0.6 leading current at the end of the line.

83. In testing a certain single-phase transmission line the far end of the line was short-circuited and 60-cycle current was then passed through the line. Readings were taken as follows: Amperes 50, volts 106, watts 2700. What was the resistance of the line? What was the reactance? The impedance? The inductance?

84. In the above line the power at the end was 100 kw. The voltage was 2200, the current 45.5 amp. What was the efficiency of the line?

85. In the above line, what was the voltage at the beginning of the line? What was the regulation on unity power factor?

86. In the foregoing, assuming that the current and voltage were the same at the end of the line, but that the power was zero, what was the voltage at the beginning of the line? What was the regulation?

CHAPTER XV

THE TRANSFORMER

177. Transformation of Continuous Current.—To transform a continuous current from one voltage to another, is a difficult and expensive process. For example, a current of 100 amp. at 1000 volts, corresponding to 100 kw. might be converted to a current of nearly 1000 amp. at 100 volts. The power would again be nearly 100 kw., the difference being due to the power lost in the transformation. To carry out this transformation, the simplest process would be to employ a direct-current motor operating at 1000 volts, and driving a direct-current generator furnishing current at 100 volts. This would necessitate the use of machinery of a total capacity of 200 kw. The machinery would be expensive since it contains rotating parts, and would cost say $4000. It would require the constant presence of an attendant and the process would moreover be rather inefficient. The loss in the case cited would be about 20 kw., giving an efficiency of 80 per cent. For these reasons, continuous currents are rarely transformed in voltage.

On the other hand, alternating current can be readily transformed from one voltage to another. The cost of a suitable transformer for the conditions described would be about $500. Its efficiency would be say 98 per cent. Since it has no moving parts, the only attention required would be inspection at infrequent intervals. Such transformers are therefore used freely in alternating-current practice.

178. General Construction of Transformer.—A view of the exterior of a lighting transformer is shown in Fig. 133, and the interior of the same transformer is shown in Fig. 134. Essentially the transformer consists of a core of laminated iron forming a closed magnetic circuit, and usually two coils of wire wound around this magnetic circuit. This is illustrated in Fig. 135. In practice, the coils are not wound on opposite sides of the rectangle as shown, but in general, each coil is divided and one-half is wound on one side and half on the other. Thus the coils are

wound one on top of the other, and the opportunity for magnetic leakage is decreased.

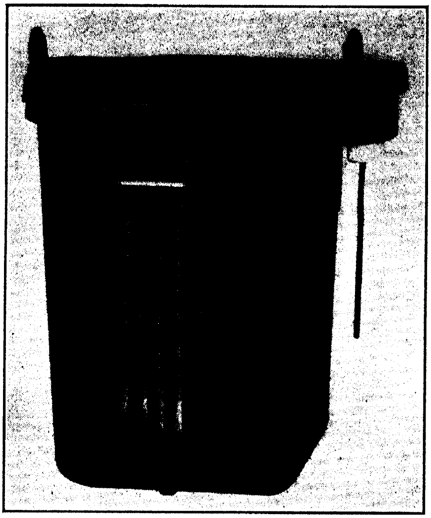

FIG. 133.

179. Elementary Theory.—Consider now that there is an harmonically varying flux in the laminated iron core of Fig. 135.

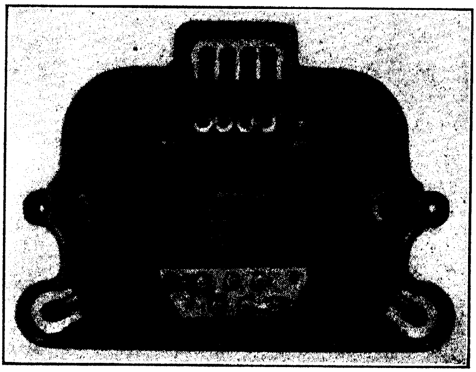

FIG. 134.

For the present this flux may be considered as produced in any convenient manner, say by means of an alternating current in a third coil on the core. This flux will induce a certain e.m.f. in

each turn of both the secondary and the primary, and this e.m.f. will be the *same per turn* in each, provided there is no magnetic leakage, that is, if all of the flux which passes through one of the coils passes through the other also. The total e.m.f. induced in either of the coils will be the e.m.f. per turn, multiplied by the number of turns in the coil. This gives at once the first important law of the transformer, namely, that the e.m.fs. *induced* in the primary and secondary are proportional to the number of turns in the respective windings. Moreover the *induced* e.m.fs. will be in the same direction around the core.

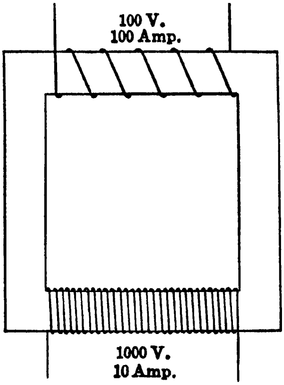

FIG. 135.

In practice, the magnetic flux of the transformer is supplied not by means of a third winding, but by means of a current circulating in one of the two main windings. This winding is called the primary. If no current is taken from the secondary, the current which will flow in the primary when it is connected across the mains will be very small compared to the full-load current of the transformer. This arises from the fact that few ampere turns are required to magnetize the core, since it is a closed magnetic circuit, and is built of iron with a low magnetic reluctance. In general, about 2 per cent. of the full-load current is required to magnetize the core.

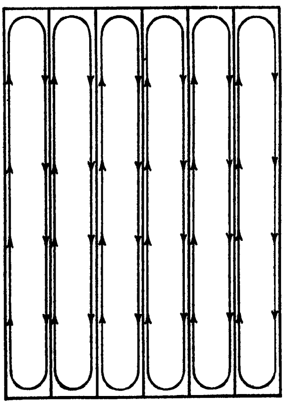
FIG. 136.

180. Core Loss.—While operating in this manner on open circuit, there is a loss in the transformer called the core loss. This loss is composed of two parts, that due to eddy currents and that due to hysteresis. The former is caused by currents circulating in the iron of the core. From a cross-section of a few laminations as shown in Fig. 136 it will be apparent that if the flux is in a direction perpendicular to the paper, there is an opportunity for currents to cir-

culate in the core as shown. The section of a lamination will have induced in it an e.m.f. just as though it were a turn of wire of the same dimensions. This e.m.f., it is true, is small, since only a small amount of flux passes through any one lamination, but on the other hand, the resistance is small also, and consequently a considerable current may be set up. This current may, however, be reduced to as small an amount as necessary by decreasing the thickness of the laminations. The sheets are therefore made as thin as considerations of economy in construction will allow.

The exact cause of hysteresis loss is not known. It is supposed, however, that when a piece of iron is magnetized, the molecules of the iron turn somewhat upon their axes, so as to point along the lines of the magnetic induction. When the magnetism is reversed, the molecules turn more or less so as to point in the opposite direction. In turning in this manner, there is apparently developed an internal friction, analogous to that noticed when a piece of steel is bent rapidly backward and forward. This loss increases in direct proportion to the number of reversals per second, and also increases with the magnetic density, although not in direct proportion to it. The core loss will be from as high as 4 per cent. of full-load capacity in small transformers, to as low as 0.5 per cent. in large ones.

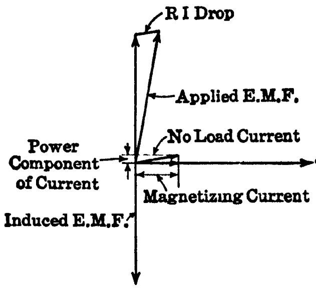

FIG. 137.

181. Vector Diagram of Unloaded Transformer.—It is now possible to draw the vector diagram of an unloaded transformer. The start is made with the magnetizing current (see Fig. 137). In the diagram this is drawn as a horizontal line. It might, of course, have been drawn in any direction, since the diagram as a whole is supposed to be rotating around the center point once for each cycle of the current. The magnetism is assumed for the present to be proportional at each instant to the magnetizing current. This would be strictly true if the core were composed of air instead of iron. With an iron core, on account of the saturation of the iron, and the core loss, this is not strictly true, but it is near enough for the present purpose.

The variation of flux may be represented by an equation of the form

$$\phi = \Phi \sin \omega t$$

At any instant, the induced e.m.f. will be equal to $- N\frac{d\phi}{dt}$ where N is the number of turns on the primary, or performing the differentiation,

$$- e = N\frac{d\phi}{dt} = \Phi N \omega \cos \omega t$$

From this it appears that the induced e.m.f. has a *maximum* value equal to

$$N\Phi\omega = 2\pi f N \Phi$$

or changing to effective values, by dividing by $\sqrt{2}$, and dividing by 10^8 to reduce to volts,

$$E = 4.44 f N \Phi \div 10^8$$

From the equation, it is seen that the induced e.m.f. is 90° behind the flux and consequently also 90° behind the magnetizing current. It will therefore be represented by a vector drawn as shown in the diagram. The e.m.f. which *must be applied to overcome this induced e.m.f.* will be represented by an exactly equal and opposite vector.

Considering the diagram as explained so far, it will be noted that the current, and the applied e.m.f. are at right angles, and therefore the power is zero. As stated, however, there is a core loss in the transformer. To supply this loss, it is necessary to have a component of the current in phase with the applied e.m.f. This component will therefore be represented by a vertical line as shown. The total current will be the resultant of these two currents, and is called the no-load current or the leakage current.

Perhaps the explanation of the foregoing phenomenon would have been more simple though not so complete if we had started with the no-load current, and explained that on account of the magnetic friction or hysteresis, the magnetism will lag behind the current by a small angle as shown.

In addition to the above, a small e.m.f will be required to overcome the resistance of the primary coil. As noted before, the no-load current is very small, and consequently the drop due to it is entirely negligible. For completeness it may, however, be represented by means of a short vector drawn parallel

to the vector representing the current. The applied e.m.f. will then be a vector equal to the sum of the e.m.f. required to overcome the self-induced e.m.f. and the e.m.f. required to force the current through the resistance of the primary. In the figure the latter vector is shown much exaggerated.

In all of the foregoing discussion, the secondary coil has been assumed to be on open circuit. There has therefore been no current in it, and it has been entirely without effect. All of the explanation will therefore apply to the case of a single coil wound on an iron core. In this case, the apparatus would be called a choke coil, or simply an inductor. If no iron had been present, the diagram would have been modified by the fact that the no-load current would be identical with the magnetizing current.

182. Transformers under Load.—As soon as a current-consuming apparatus is connected to the secondary of a transformer, the conditions change. The first point to be noted is that the flux through the core remains approximately constant. This follows from the fact that the induced e.m.f. in the primary of the transformer is at all times *nearly equal to the applied e.m.f.* Since this induced e.m.f. is constant, the flux which produces it will be constant also. Likewise the core loss, being dependent upon the magnetic flux, will be constant irrespective of the load. It is therefore proper to consider the line marked "no-load current" as being constant.

In considering the loaded transformer, it will be simpler to assume a transformer with a one to one ratio, that is, with the same number of turns in the secondary as in the primary. Any deductions made by considering such a transformer will apply equally well to a transformer of a different ratio. With this understanding, the e.m.f. induced in the secondary will be represented by the same line as that showing the induced voltage in the primary.

When current is taken from the secondary, the angular relation of the current to the e.m.f. will be dependent upon the *nature of the receiving circuit.* If this circuit contains resistance and inductance, the current will lag. On the other hand, if capacitance is present, the current may be ahead of the e.m.f. In Fig. 138 it is assumed that the former is the case, and the current has been drawn lagging behind the induced e.m.f. by an angle θ_s. The magnetizing force, acting upon the core of the transformer, is the resultant of *all* of the currents acting. As soon as current

passes in the secondary, it as well as the primary current will tend to magnetize the core. In fact, in most cases, the secondary current will be far stronger than the no-load current in the primary. From the direction in which it is drawn, it will be apparent that the secondary alone would tend to exert a strong demag-

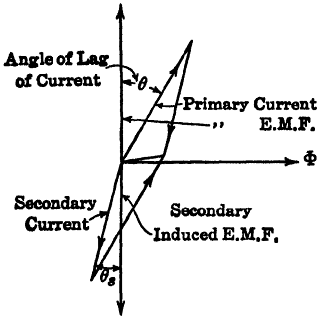

FIG. 138.

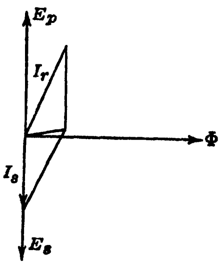

FIG. 139.

netizing effect upon the core. As soon as this takes place, the back induced e.m.f. of the primary will be reduced and the apparatus will take more current from the main through the primary. The value of this additional current will be just enough. to offset the demagnetizing effect of the current in the secondary The *resultant* of the primary and the secondary current will then be just equal to the required no-load current. The construction of the parallelogram, giving the resultant of these two currents is shown in Fig. 138.

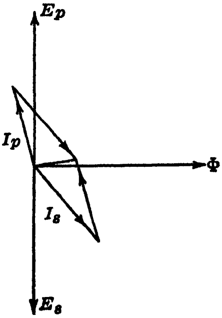

FIG. 140.

Figure 139 illustrates a case in which the current instead of lagging, is in phase with the secondary e.m.f. The primary current lags somewhat behind the primary e.m.f., or in other words, the primary and the secondary currents do not differ by exactly 180°. In Fig. 140 is shown a case where the current in the secondary is leading. The primary current in this event usually leads also, although it might happen that it would not if the secondary lead were small. The no-load current in all of these diagrams is exaggerated so that the deviation of the primary and secondary currents from exact opposition

is also exaggerated. In practice, the primary and secondary currents are nearly in opposition. In fact, transformers are frequently used to transform to a smaller current when a large alternating current is to be measured by means of an ammeter or wattmeter, capable of carrying only a small current. Unless such measurements are to be very exact, it is usually assumed that the primary and secondary currents are in the inverse ratio of the turns, and that they are in exact phase opposition.

183. Leakage Flux.—When a transformer is under load, the greater part of the magnetic flux passes through both the primary and the secondary coils. It will be seen, however that there is an opportunity for some flux to surround one of the coils without passing through the other. In practice, every effort is made to reduce the amount of this leakage flux, but some is always present.

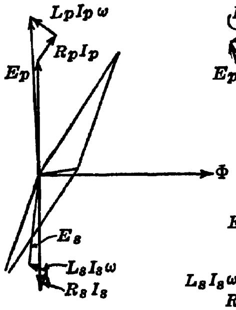

FIG. 141.

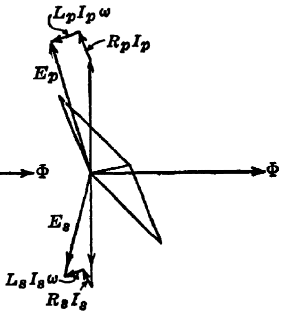

FIG. 142.

The main flux is nearly in phase with the *resultant* of the primary and the secondary currents. The leakage fluxes on the other hand, pass through only one of the two coils. They are therefore nearly in phase with the currents in their respective coils. Each of these fluxes therefore induces an e.m.f. in its respective coil, lagging nearly 90° behind the current in the coil. In the case of the secondary, this e.m.f. is added vectorially to the terminal e.m.f. In the primary, an additional e.m.f. equal and opposite to this induced e.m.f. must be added vectorially to overcome it. In addition to this, it is necessary to add vectorially to the primary the e.m.f. required to overcome the resistance of the primary winding, and to subtract a similar quantity from the secondary voltage. These latter two vectors will be in phase with the respective currents. The complete construction of the

vector diagram for the case of a lagging current is shown in Fig. 141, and Fig. 142 shows the same for a leading current. In the former case, the voltage at the secondary terminals is less than that at the primary terminals, while with the leading current, the secondary voltage is greater than that of the primary. Stated generally, it may be said that the secondary voltage may be either somewhat less or somewhat more than would be indicated by the ratio of the number of turns on the primary and on the secondary.

184. Regulation.—The foregoing discussion introduces the subject of transformer regulation. Good regulation in a transformer means that the secondary voltage varies but little as the load is changed, the primary voltage being constant. Good regulation is highly desirable, particularly when transformers are used for house to house distribution. The attempt is made at the central station to keep the primary voltage as nearly constant as possible. The inherent regulation of the transformers is relied upon to keep the secondary voltage also constant.

Technically, the regulation is defined as the rise in voltage of the secondary when full load is thrown off the transformer, divided by the full-load voltage, the frequency and primary voltage remaining constant. The foregoing will show that this regulation will be dependent upon the nature of the load carried. If the current is leading, the regulation may even be negative. The term regulation is therefore meaningless, unless the conditions are known. Ordinarily, the regulation at unity power factor is the important one, particularly for lighting transformers. Unless otherwise specified, this would be understood.

If transformers are to be used to supply current to induction motors, the current will always be lagging. Under these circumstances, there is a strong argument in favor of specifying the regulation at zero power factor with lagging current. The advantage of this is that the regulation is specified for the worst possible condition, and in practice better results than this can be counted upon.

185. Constant-current Transformers.—In continuous-current practice, the constant-current system of distribution is nearly obsolete. For street lighting by means of alternating current, using either arc lamps or incandescent lamps, it has however great advantages. Because such circuits are out of doors, the allowable voltage is much higher than for interior illumination.

By operating the lamps in series, the current to be carried is much reduced, and the investment in copper correspondingly reduced. Such high voltage circuits are not allowed inside of buildings on account of the insurance regulations, but are in almost universal use for outside work.

A special transformer is required to transform from constant potential at which the current is generated to constant current.

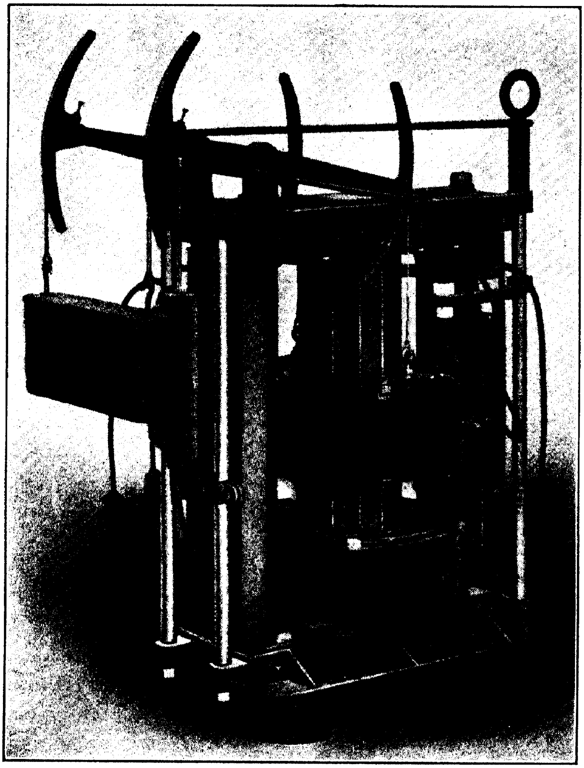

Fig. 143.

A transformer for this purpose is shown in Fig. 143. It has two coils, one of which is free to move away from or toward the other. The weight of the moving coil is partially counterbalanced by the weights shown. When the machine is in operation, there is a repulsion between the two coils, because the currents are in opposite directions in the two. This repulsion together with the

weights used, is enough to hold the moving coil suspended in the proper position. If for any reason the current increases, the repulsion of the two coils increases and the coils are forced further apart. This in turn results in an increased magnetic leakage. Less of the useful flux of the transformer, therefore, passes through the secondary, and at the same time the leakage flux is increased in both primary and secondary. The reduction of the useful flux results in a reduction of the secondary voltage, and consequently the current is reduced. By properly shaping the curves of the arms, the current may be made to remain the same whatever the position of the moving coil. The whole transformer is usually immersed in oil on account of the better cooling and the improved insulation.

With some types of arc lamps, a direct current is preferable or necessary, or it may be that the constant-current transformer is to be used to feed a circuit formerly supplied by a direct-current machine. In this event, it is customary to rectify the secondary current by means of a mercury arc rectifier. These have a very high efficiency at the high voltage used. They are commonly completely immersed in oil to secure better insulation and cooling. (See Art. 244.)

186. Instrument Transformers.—Transformers are frequently used in connection with measuring instruments. For pressures up to about 600 volts, voltmeters adapted to be directly connected across the line are commonly used. For voltages of 1100 or more this becomes rather impracticable, since a large resistance would be required for such a voltmeter. Moreover it is undesirable to have a voltage of this or greater magnitude present on the front of a switchboard on account of the danger to life. The difficulty is readily met by using a small transformer to reduce the voltage to a value usually near 100 volts. The scale of the instrument may be calibrated to read directly the line pressure instead of the secondary pressure.

For similar reasons current transformers are frequently used. The current to be measured may be so great that it would be impracticable to construct an instrument capable of carrying it, or the voltage may be so high that it is desirable to have the ammeter insulated from the line. Transformers for this purpose are usually wound to have a full-load secondary current of 5 amp. The scale may as before be calibrated to read the primary current directly.

In using the pressure transformer little error is introduced. Moreover, practically all alternating-current systems are operated at constant potential, and the voltmeters are required to read only one value of the voltage with accuracy. Any possible error at this one point may be completely corrected for by calibrating the voltmeter and transformer as a unit.

An ammeter is required to operate over a larger range, and the error in a current transformer is likely to be larger than in the case of the pressure transformer. The error may be corrected for as before by calibrating the transformer and ammeter as a unit. It is, however, rarely necessary to do this.

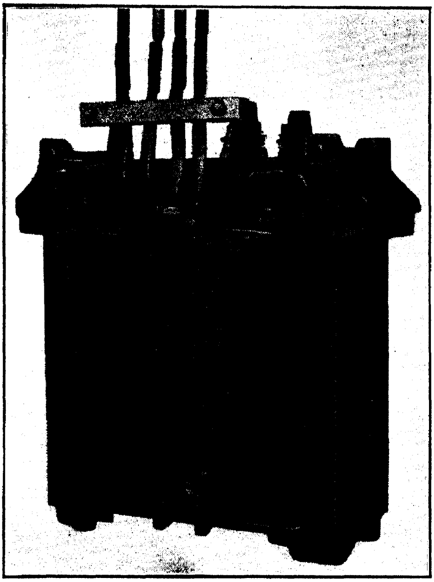

FIG. 144.

The only measuring instruments whose accuracy is at all seriously affected by the use of instrument transformers, are wattmeters and watthour meters. These are frequently used in connection with both potential and current transformers. Unfortunately, in addition to the ratio error just mentioned, both of these types of transformers introduce a small phase error. This becomes of some moment, especially if the power factor of the load to be measured is low. The indications of wattmeters provided with either potential or current transformers should there-

fore be accepted with some caution, particularly if the load is small and the power factor low.

187. Types of Transformers.—Three general types of transformers are in use, the core type, the shell type, and a combination of the two preceding called the cruciform type. The core type has been described and is illustrated in Figs. 133 and 134. If the coils and the core in the core type of transformer are inter-

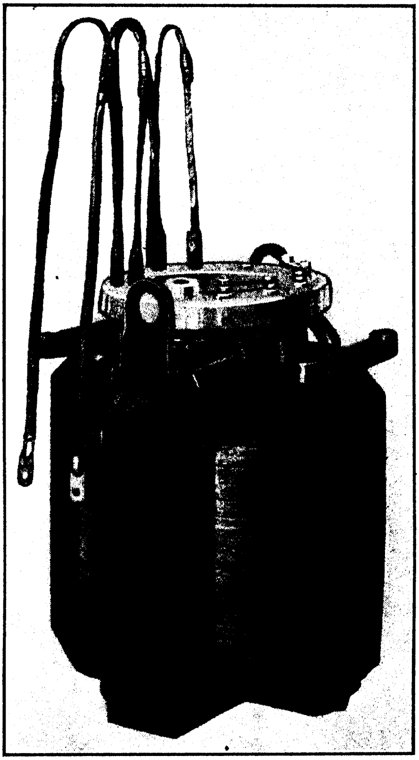

FIG. 145.

changed the shell type illustrated in Fig. 144 is obtained. It may be considered that the core type has one core and two coils, while the shell type has two cores and one coil. The flux after passing through the coils divides into two parts and returns on the outside.

The cruciform type of transformer, shown in Fig. 145, is a modification of the shell type. The return magnetic circuit, instead

of being divided into two parts, consists of four separate limbs. This possesses the advantage over the shell type that the amount of iron required is not so great. This is apparent since the average distance to be traveled by the returning lines of induction is slightly less.

188. Cooling of Transformers.—The cooling of small transformers presents no particular difficulty. The core and coils are practically always enclosed in an iron case filled with insulating oil. The oil, in addition to its insulating properties, insures that all parts of the transformer remain at practically the same temperature. If any point becomes hotter than the average, the oil in its vicinity becomes heated and rises. Its place is taken by cooler oil, and thus there is a tendency to equalize the temperature.

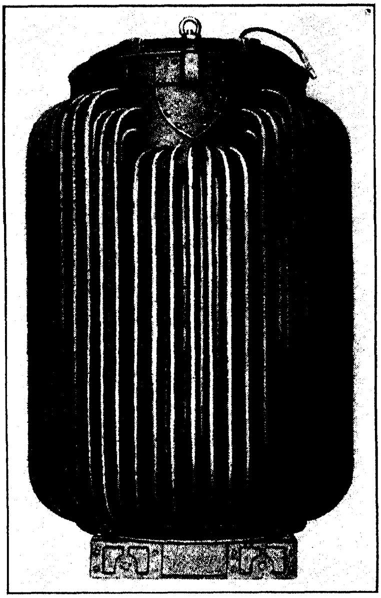

FIG. 146.

As transformers increase in size, the problem of disposing of the heat becomes more serious. The losses of a transformer may be assumed to be approximately proportional to the volume or weight of the apparatus. This will be the case if the current densities and the magnetic densities are the same in the smaller and in the larger sizes. If a number of transformers all having the *same relative proportions* are constructed, their weights and consequently their volumes and losses will be in proportion to the cube of their dimensions. Their outside surface and, therefore, their relative heat radiating abilities will increase only in proportion to the square of their dimensions. Thus comparing two transformers whose dimensions are as two to one, the larger will weigh eight times as much as the smaller, and will in consequence have a loss eight times as great. It will, however, have only four times the radiating surface and the temperature rise above the

room temperature will, therefore, be twice as great as in the smaller transformer. The problem of cooling the larger sizes of transformers is, therefore, a much more serious one than in the case of the smaller sizes. The same problem is met with, but to a lesser extent, in the design of all classes of electrical machinery. In dynamos not so much difficulty is experienced, since in the larger machines there is not the same incentive to employ the same relative shape as in the smaller ones.

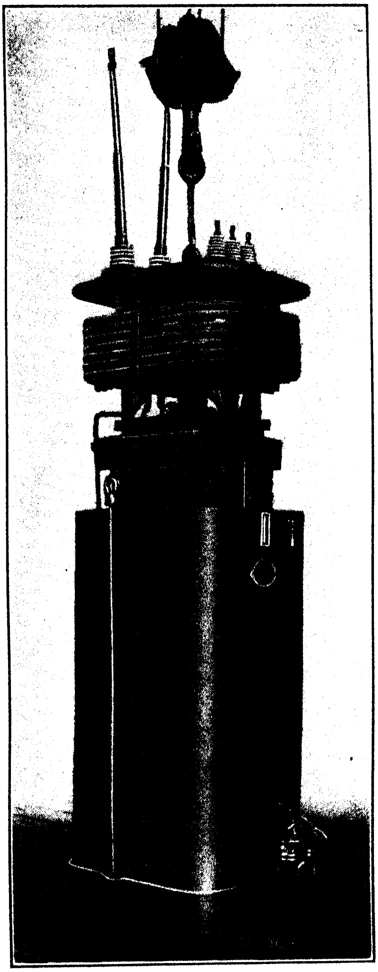

FIG. 147.

One expedient to overcome this difficulty is the provision of a corrugated surface for the containing case. The corrugations are sometimes made as deep as 4 in. or more. Soon, however, a point is reached where further deepening of the corrugations is without much effect, and some other method must be adopted to increase the cooling surface. One successful way of doing this is illustrated in Fig. 146. As will be seen from the illustration, numerous tubes are provided extending from the bottom to the top of the case. The oil in the tubes becomes cooler than that in the interior, and descends, thus setting up a circulation. The greatly increased radiating surface is sufficient to dissipate all of the heat generated without undue rise of temperature.

Another effective method makes use of a coiled pipe immersed in the oil near the top of the containing case. Cold water is

forced through this coil, and this cools the oil in its vicinity, causing it to descend, and thus set up a circulation. This method is naturally largely used in transformers in water-power plants, as a plentiful supply of water under pressure is always at hand.

In the last-mentioned method there is some possibility that the water will freeze and burst the coil. In this event, the windings of the transformer would probably be injured. In some installations this danger is avoided by pumping the oil itself from the cases of the transformers and leading it through a system of air-cooled tubes or through a radiator in which it is cooled. A water-cooled transformer is shown in Fig. 147.

Transformers intended for comparatively moderate pressures, generally not in excess of 40,000 volts, frequently are not immersed in oil and are cooled by air blown through them by means of a motor-driven fan. A trench is provided over which all of the transformers are so placed that the air from the trench can escape only by passing through the transformers. Numerous ventilating ducts are provided in the latter, and every effort is made to provide a free passage for the air.

189. Losses and Efficiency of Transformers.—There are two losses in a transformer, the fixed loss and the copper loss. The former comprises the hysteresis and eddy-current losses and was treated in Art. 180. As there explained, these losses are practically constant, no matter what the load.

The copper loss, on the other hand, is proportional to the square of the current and consequently to the square of the load.

The fixed loss is readily measured by connecting a wattmeter in the primary circuit of the transformer, the secondary being on open circuit. The applied voltage must be the rated voltage To get the copper loss we measure the resistance of primary and secondary by voltmeter and ammeter or otherwise. The full-load current is readily ascertained from the name plate or by a simple calculation.

For example, assume a transformer rated 100 k.v.a., 2200–220 volts, 60 cycles. The full-load primary current is:

$$100{,}000 \div 2200 = 45.5 \text{ amp.}$$

The secondary current may be taken as ten times as great or 455 amp. This, while not strictly exact, is near enough for the purpose. The fixed loss is found to be 1.3 kw., the resistance of

the primary is 0.33 ohm; that of the secondary 0.0038 ohm. The calculations to determine full-load efficiency are as follows:

$$\begin{array}{llll}
\text{Fixed loss} & & = & 1300 \text{ watts} \\
I^2R \text{ primary} & = 45.5^2 \times 0.33 & = & 684 \text{ watts} \\
I^2R \text{ secondary} & = 455^2 \times 0.0038 & = & 787 \text{ watts} \\
\hline
\quad \text{Total loss} & & = & 2771 \text{ watts or} \\
 & & & 2.77 \text{ kw.} \\
\text{Output (at 100 \% power factor)} & & = & 100.00 \text{ kw.} \\
\text{Input} & = \text{output} + \text{losses} & = & 102.77 \text{ k.w.} \\
\text{Efficiency} & = \dfrac{\text{output}}{\text{input}} & = & 97.34 \text{ per cent.}
\end{array}$$

In a similar manner the efficiency for any other load may be computed.

190. Connection of Transformers.—Single Phase.—As a matter of convenience, transformers are usually provided with

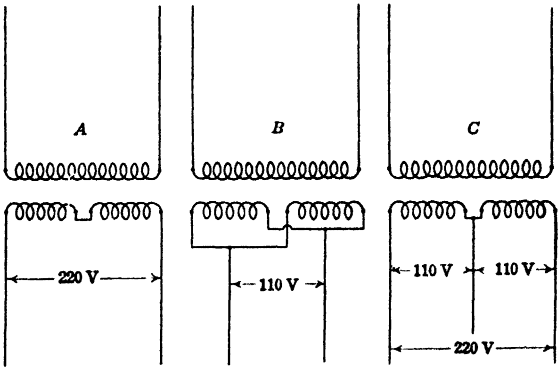

Fig. 148.

two secondary windings. These have the same number of turns and consequently generate the same voltage, frequently 110 volts. The two secondaries may be connected in any one of the three ways shown in Fig. 148. As shown at A, the two secondaries are connected in series and the terminal voltage is double that of one coil. If a voltage of 110 is desired the connection B is used. The current-carrying capacity of the transformer is double that of the transformer connected as at A since the two coils are in parallel. Since the voltage is halved, the capacity is the same. The connection C is the same as the A except that the neutral wire is connected, giving a three-wire system.

191. Two-phase Connections.—The ordinary method of connecting transformers to a two-phase circuit is shown under A in Fig. 149. Each transformer is connected to one of the phases in the same way that a transformer would be connected to a single-phase line. There is usually no connection between the transformers. The three-wire connection shown under B of the same figure is occasionally used. The voltage across the two outside wires in this case is equal to the voltage of either transformer multiplied by $\sqrt{2}$.

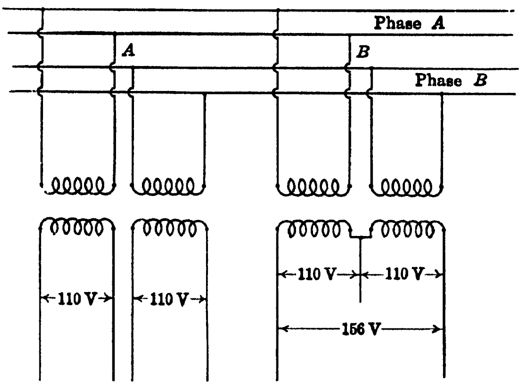

Fig. 149.

192. Three-phase Connections.—To change the voltage of a three-phase circuit, three transformers are usually employed, although two may be used as is pointed out later. Perhaps the commonest arrangement is that shown under A in Fig. 150. The three transformers are connected in delta on both the primary and the secondary sides. It may be assumed that the ratio of transformation is ten to one and that the primary voltage is 2200. It is evident that the secondary voltage of each transformer will be 220 volts and since they are connected in delta, the line voltage will also be 220.

In B of Fig. 150 are shown three transformers connected in star on both the primary and secondary. The voltage across each primary winding will not be the line voltage, but this voltage divided by $\sqrt{3}$. Thus with the same assumptions as before, the voltage across the terminals of each transformer will be $2200 \div \sqrt{3} = 1270$ volts. Since the ratio of the transformers is ten to one, the voltage across each secondary winding will be 127 volts.

Since the secondaries are also connected in star, the line voltage will be $127 \times \sqrt{3} = 220$ volts.

With this connection unless the three transformers are identical, the voltage drops across the windings will not be the same

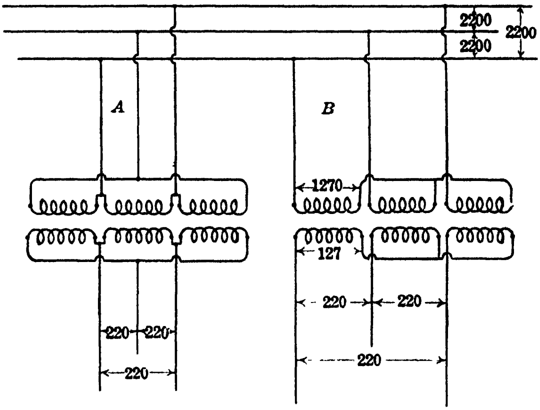

FIG. 150

and one or more of the transformers will be operating at a higher voltage than that for which it was designed. This connection is, therefore, little used

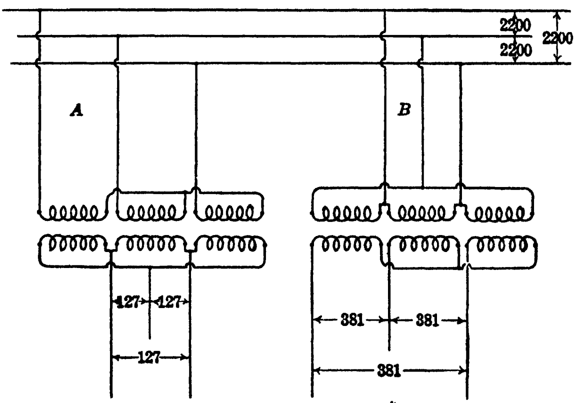

FIG. 151

In *A* of Fig. 151, the transformers are connected in star on the primary side and in delta on the secondary. Each primary winding has impressed upon it a voltage of 1270. Since the

secondaries are connected in delta, the line voltage is the same as the transformer voltage, or 127.

In *B* of the same figure the primary is in delta and the secondary in star. The secondary voltage of each transformer is 220 volts and since they are connected in star the line voltage is $220 \times \sqrt{3} = 381$ volts.

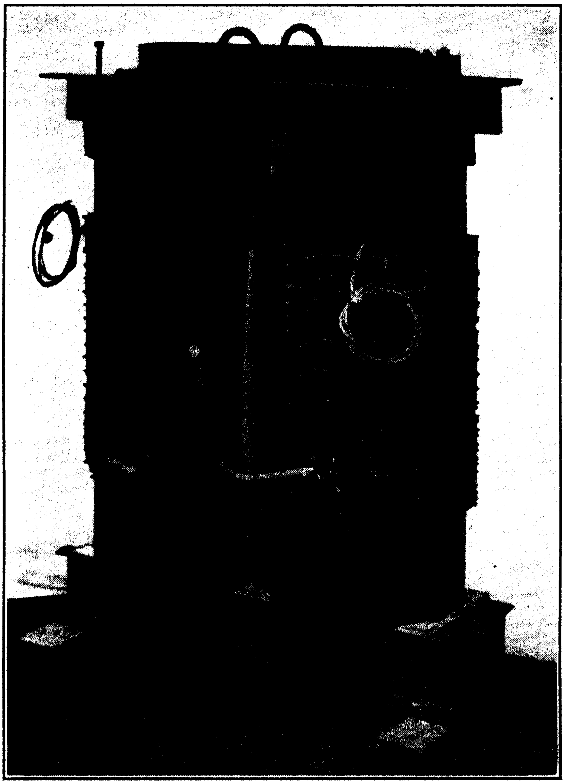

Fig. 152.

Connections of the type indicated in Fig. 151 are frequently used in long distance transmission lines. When a line is first installed the load may be light and the transformers at both ends of the line may be connected in delta on both sides. Later, when the load has increased to such an extent as to warrant it, the high voltage sides may be reconnected in star. This will increase the line voltage 73 per cent., leaving the sending and

receiving voltage the same. On the other hand, these connections are rarely used for general distribution systems as they give rise to odd voltages unless special windings are used.

193. Three-phase Transformers.—Instead of using three separate transformers connected as shown in the previous diagrams to transform a three-phase current, it is entirely possible to combine the transformers in one structure. Such a transformer is shown in Fig. 152. Each phase is wound upon one of the three legs. The three fluxes in the legs will differ in phase by 120°. The flux in the yokes connecting the legs together will be the resultant of the three fluxes.

By building a transformer in this manner, a large saving in labor and material is effected. The transformer is somewhat

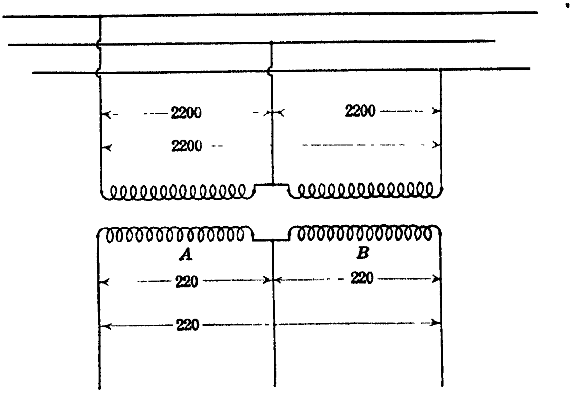

FIG 153.

cheaper and slightly more efficient. The floor space required is considerably reduced. Perhaps the greatest disadvantage of the three-phase transformer is the fact that in case of trouble the whole unit must be taken out of service. With three separate transformers it is usually necessary to remove only one of the three. The amount of capital tied up in spare units is thus somewhat less. Moreover, if the three units are connected in delta on both sides, it is possible to operate with only two of the three at a decreased load. This is explained in the following secton.

194. The Open-delta Connection.—Two transformers connected as shown in Fig. 153, may be used to transform the voltage of a three-phase line. It is evident that the voltage across the

secondary of transformer A will be 220 (assuming that the ratio of the transformer winding is ten to one), and the same will be true of B. The voltages of A and B will differ in phase by 120°. Therefore the voltage across the two outside wires in the figure will be the vector sum of two voltages of 220 volts each differing in phase 120°. This sum will also be 220, and there is a three-phase voltage at the terminals of the secondary.

This connection is sometimes convenient when a small amount of power is to be transformed. It would hardly be used in large work since even with 100 per cent. power factor on the secondary the voltages and currents in the transformers will not be in phase. The power capacity of the transformers is therefore reduced.

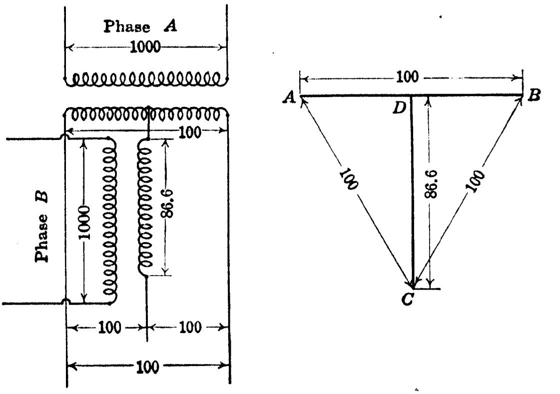

FIG. 154.

The voltage regulation is also poor since the current may be lagging in one transformer and at the same time leading in the other.

195. Transformation of the Number of Phases.—At the present time no method exists for transforming from single phase to polyphase or *vice versa* without the use of rotating apparatus. Of course, a transformer can be connected to a polyphase line and take single-phase current from it, but the current in the line will also be single phase.

It is, however, a simple matter to change from any number of phases greater than one to any other number greater than one. In Fig. 154 is illustrated the method of changing from two to three phases or *vice versa*. Two transformers are used. The primaries are identical but the secondary of one has only 86.6 per cent. as many turns as that of the other. The transformer with

the greater number of secondary turns has a connection brought out from the middle of its secondary winding. The connections are as shown at the left of Fig. 154.

The action will be readily understood from the vector diagram at the right of the figure. Assuming that the two-phase line is the primary, the secondary voltages will differ 90° in phase, and since one winding is connected to the center of the other, the diagram will be as shown. Assuming that the voltage from A to B is 100, that from C to D will be 86.6, since the turns are assumed to be in the ratio of 100 to 86.6. The voltage from A to D or from D to B will be 50. The voltage from A to C or from C to B will be the resultant of the two voltages and will be equal to

$$\sqrt{(86.6^2 + 50^2)} = 100$$

Since the three voltages AB, BC, and CA are the same, a three-phase system results. The connection will work equally well to transform from three to two phase. This is known as the Scott connection.

PROBLEMS

87. A certain transformer is rated 10 kv-a., primary voltage 2200, secondary voltage 220 When connected to 2200-volt mains of the proper frequency the input is 80 watts, the output being zero. The resistance of the primary is 2 95 ohms, that of the secondary 0 0306 ohm Calculate the efficiency at ¼, ½, ¾, full load and 1¼ load Also calculate the maximum efficiency. This will occur when the copper loss and the iron loss are equal

88. The above transformer is used on a distributing system and is connected to the primary mains at all times. It is used an average of 3 hr per day at full load. Determine the value of the power lost in iron loss and in copper loss in 1 year if it costs 1 cent per kilowatt-hour to generate the power.

89. Three transformers are used to step up the voltage of a three-phase line The ratio of the turns on the primary to those on the secondary is 1 to 10. The transformers are connected in delta on the primary and in "Y" on the secondary. If the primary voltage is 6600, what is the secondary voltage?

90. Using the same transformers, what would be the secondary voltage if the transformers were connected in star on the primary and in delta on the secondary?

CHAPTER XVI

SYNCHRONOUS GENERATORS AND MOTORS

196. General Construction.—A perspective view of a synchronous machine is shown in Fig. 155 and separate views of the armature and field are shown in Figs. 156 and 157. This is a modern type in which the field is always the revolving element. Figure 158 illustrates an older type in which the armature re-

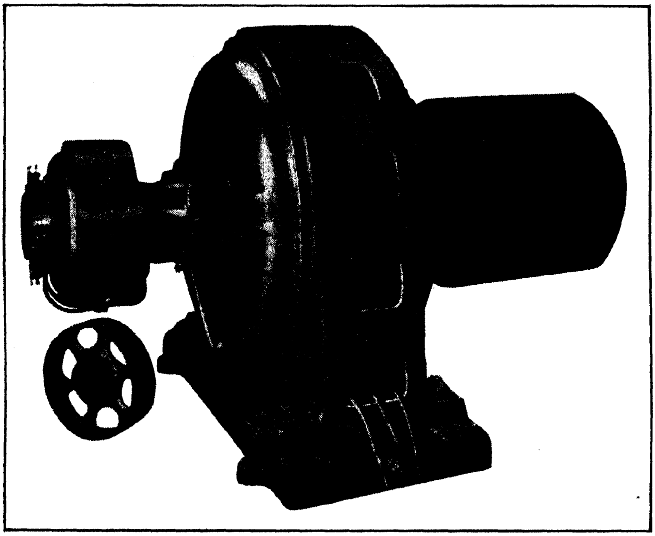

FIG. 155.

volves and the field is at rest. This form is rarely built at the present time. The principal reasons for this have already been given.

Any synchronous machine may be used either as a generator or as a motor. In fact, this is true of all dynamo-electric machines, whether operated by alternating or direct currents. All that is required to change generator action into motor action is

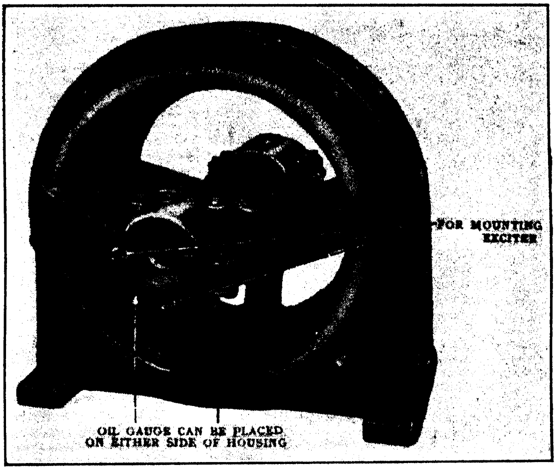

FIG. 156.

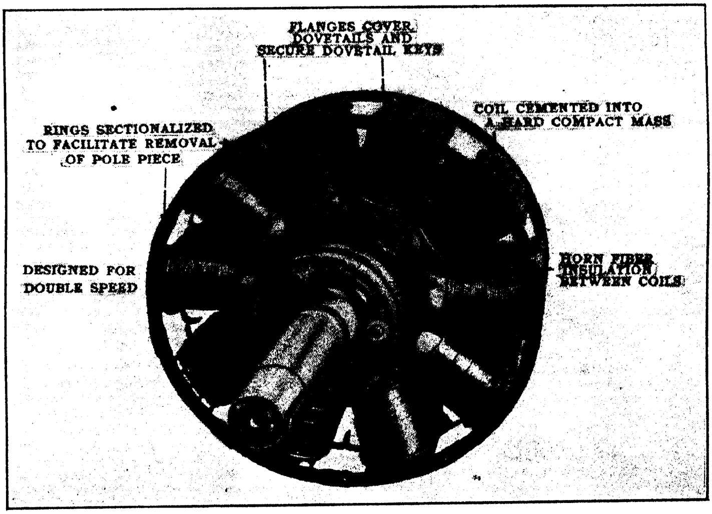

FIG. 157.

that the current in the armature should be reversed, the direction of the flux from the field poles remaining the same.

In Fig. 159 is shown a section of a synchronous machine. *N* and *S* represent the field poles. These may be of either solid

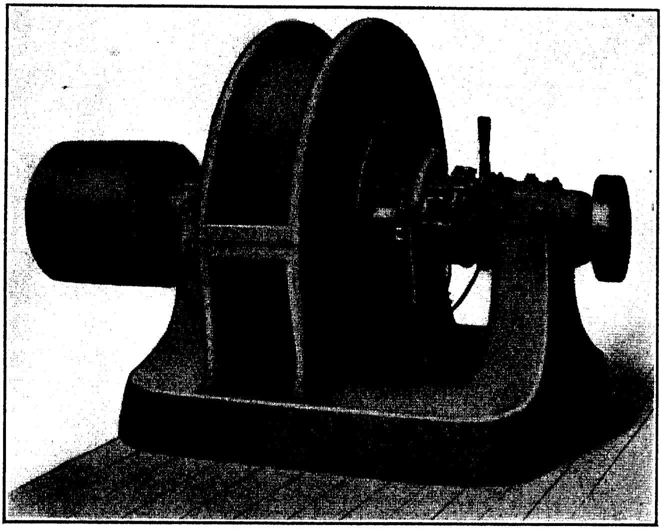

FIG. 158.

or laminated construction. The practical difference is not great, each construction having advantages from certain standpoints. The armature is represented as a broken rectangle above the

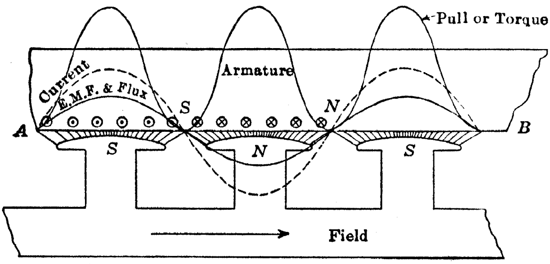

FIG. 159.

field poles. It is always constructed of laminated iron, as otherwise the heat due to eddy-current losses would be so great as to destroy the machine. The conductors are placed in slots parallel

to or nearly parallel to the pole faces, or as shown in the diagram, perpendicular to the plane of the paper.

197. Action as a Generator.—In considering the elementary action of the machine as a generator, assume that the number of these conductors is *very great,* and that *each is connected to its own receiving circuit,* and let these circuits be exactly alike. For the present, assume that they are all non-inductive.

It will be evident that the e.m.f. induced will be greatest in those conductors which are directly under the center of a pole face, and zero in those midway between the poles. At intermediate points, the e.m.f. will be in the same direction as that of the conductors under the center of the pole, but will be of lesser value.

Fig. 160.

If opposite each point of the pole face, we plot a point whose distance from the line AB is proportional to the e.m.f. induced in the conductor and join all of these points by means of a smooth line, a curve of the general shape of that marked e.m.f., in Fig. 159, is obtained. As the field moves with respect to the armature,

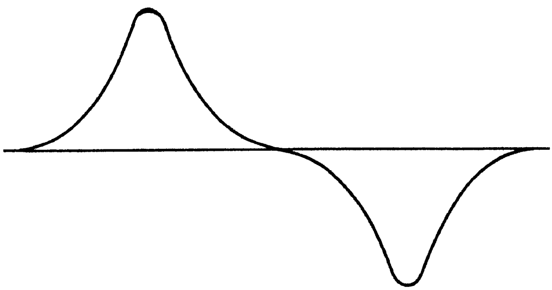

Fig. 161

this curve will retain the same position with respect to the field poles. Thus if the armature stands still and the field revolves, the curve will also revolve at the same rate. If, on the contrary, the armature is the revolving part, the curve will be stationary in space. This curve may be called the curve of *space distribution of e.m.f.* It should be carefully distinguished from the curve of *time variation of e.m.f*; although here, they are of the same shape, but this would not in general be true.

198. Space Curve of E.M.F.—The curve of e.m.f. distribution may assume various shapes. Figure 160 shows its approximate shape when the synchronous machine is so constructed that the air gap is of the same length at all parts of the pole face. This results in a nearly uniform value of the flux at all parts of the pole face and consequently in a nearly uniform value of the e.m.f. as long as the conductor is moving through this uniform flux. In the spaces between the poles there will be little flux, and the e.m.f. induced will be correspondingly feeble. Figure 161 illustrates the type of wave induced in case the pole is very narrow and the spaces between the poles correspondingly large.

Figure 162 shows a sine wave of e.m.f. distribution. To obtain this shape, the flux must also have a sinusoidal distribution. To accomplish this, the pole faces are rounded somewhat

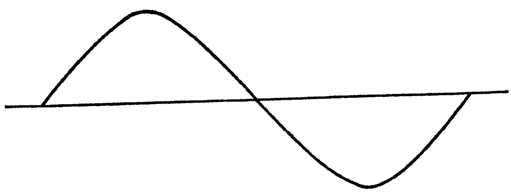

FIG. 162.

as shown in Fig. 159. This causes the flux to be strongest at the center of the pole faces and to become gradually weaker as the point midway between the poles is reached. It should, however, not be assumed that it is necessary to have this distribution of the flux in order to have a sine wave of *terminal e.m.f.* in the machine. It is entirely possible, by combining several non-harmonic waves of e.m.f., to produce a resultant sine wave. In general, it is desirable that the machine should have a wave of e.m.f. distribution like that of Fig. 159, but it is even more necessary that the terminal wave of e.m.f. should be sinusoidal. However, if the curve of e.m.f. distribution is sinusoidal, the curve of terminal e.m.f. will *always* be sinusoidal, irrespective of the connections employed, since two or more sine waves always add together to form another sine wave.

199. Space Curve of Flux and Current.—Since the motion of the conductors through the flux is constant, the e.m.f. at any point is proportional to the flux at that point. The same curve

(taken with the proper ordinates) which shows the space distribution of e.m.f. will therefore serve as the space curve of flux.

Assuming now that a sine wave of e.m.f. distribution has been obtained, and that each conductor is connected to its own receiving circuit, a curve of current distribution may be plotted in addition to the curve of e.m.f. distribution. If all the receiving circuits are equal and in addition are non-inductive, the curve of current distribution will be sinusoidal and in phase with the curve of e.m.f. distribution as indicated by the dashed line of Fig. 159. The current then will rise and fall in exact synchronism with the movement of the field poles or of the flux. As a short and convenient means of designation these three elements of the machine may be called the flux sheet, the e.m.f. sheet, and the current sheet. These three sheets may be regarded as being all in the same phase and as having the same shape. If the machine is of the revolving field variety, all three sheets revolve around the armature at synchronous speed. If on the other hand, the field magnet is stationary, the three curves remain stationary in space, but are of course as before in motion with respect to the armature.

200. Torque in a Synchronous Machine.—The next step is to examine the production of *torque* in a machine of this character. It will be remembered that one of the fundamental facts of electricity is that a conductor carrying current across a magnetic field in such a direction as to be perpendicular to the direction of the lines of induction, is subjected to a force expressed by the following equation:

$$F = \mathcal{B}lI$$

where F is the force in dynes, B the flux per square centimeter, and L is the length of the conductor in centimeters, and I is the current in absolute units. Expressed in the customary English units of pounds, inches, flux per square inch, and amperes, the equation becomes,

$$F = \frac{8.85}{10^8}\,\mathcal{B}lI$$

The curve of space distribution of pull (or torque) is obtained by multiplying the product of the current and the flux density at any given point by a constant. This curve is indicated in Fig. 159. Since the flux and the current change directions at the same points the pull is always in the same direction.

Since the relative positions of the flux sheet and the current sheet do not change, it is evident that the *torque will be constant* and consequently the power input and output of the machine will also be constant. In other words, the pull changes *in space* but not *in time*.

In the foregoing case, when the machine is acting as a generator, the torque is in such a direction as to oppose the motion of the machine. If the direction of all of the currents is reversed, the torque will also be reversed, and will be in the direction of the motion. The machine will then be a synchronous motor. Since such operation generally involves the presence of at least two synchronous machines, one as generator, the other as motor, the case is somewhat complicated. The additional factors which must be considered will be treated in the subsequent pages.

201. Effect of Power Factor on Torque.—Return again to the conception of the machine as a synchronous generator, but

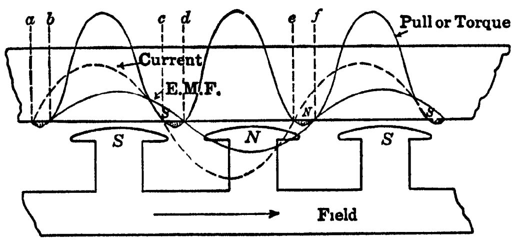

FIG. 163.

instead of assuming the receiving circuits non-inductive, let them be replaced by circuits having inductance so as to cause the current to lag behind the e.m.f. Since it has been assumed that all the receiving circuits are alike, each current will lag the same amount, and the net result will be that the curve of current distribution will be moved bodily to the left if the direction of motion of the field is from left to right. This condition is shown in Fig. 163.

If the current and flux are of the same maximum values as before, the torque required to keep the generator in motion will now be less. In Figs. 159, 163, and 164, the curve of e.m.f. may also be taken as the curve of flux, since the two are proportional. In Fig. 159, the flux and the current are in all cases in the same relative direction. With the convention adopted, when the curves are on the same side of the zero line, it is assumed that the

torque is positive, *i.e.*, that the machine is acting as a generator. In Fig. 163, however, the action is not the same at all points of the periphery. Thus in the portion *ab*, the flux and the current are in opposite directions or on this portion of the periphery motor action exists and the pull on the field is in the direction from left to right. In the portion *bc*, on the contrary, the pull is from right to left, or generator action exists. The interval *cd* is again the seat of motor action, *de*, of generator action and so on. The net torque of the machine will be the torque of all of the sections located in the positions corresponding to *bc*, minus the torque of all such sections as *ab*. The torque required to keep the machine in motion is therefore reduced. Yet the *torque of the whole machine is constant* during the whole time. The fact is that at certain parts of the periphery there is a pull in the direction of the motion, and over a larger part of the periphery, a pull opposed to the motion. The pull is then on the whole in opposition to the motion, or the machine is a generator, but the pull is less than would be the case if the current and the flux (and consequently the e.m.f.) were in the same phase.

As previously explained, if the current and the e.m.f. are in the same phase, which is equivalent to saying that the flux distribution curve and the current distribution curve are in the same phase as shown in Fig. 159 the power is represented by the equation

$$P = EI$$

If on the other hand, the current and e.m.f. are not in phase, the expression becomes

$$P = pEI$$

in which p is a number not greater than 1 and is known as the power factor. It is often difficult to understand how a large current and a large e.m.f. may exist in a circuit at the same time that the power is little or nothing. The foregoing explains how this happens in a generator supplying an inductive circuit.

202. The Case of Zero Power Factor.—Fig. 164 shows the conditions when the lag of the current behind the e.m.f. is 90°. It will be seen that the portion *ab* of the curves is now exactly equal to the portion *bc*. The positive and the negative parts are therefore equal and the net torque and consequently the net power is zero. A further lag of the current would result in the negative portion of the torque becoming greater than the positive or the

machine would be acting as a motor. It would not be possible by using inductance to obtain a circuit in which the lag was 90° or more, since there would always be some loss in the circuit which would cause the power to be positive. It is, however, entirely possible to obtain such a condition when two machines are operating in parallel on the same load. If the throttle of the steam engine connected to one of the generators be gradually closed, the power given to that machine will be gradually reduced. If this be carried far enough, the power as indicated by a wattmeter will decrease and finally reach zero. Closing the throttle further will cause the power to reverse, or the machine will operate as a motor. At the time when the power indicated by the wattmeter is zero,

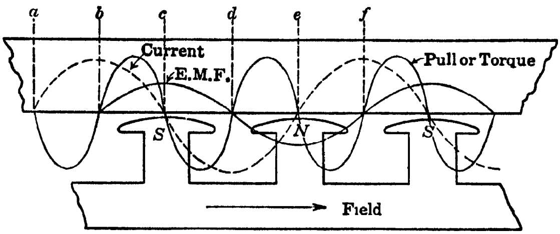

FIG. 164.

current will in general still be flowing. The power factor is then zero. The details of this action will be better understood when the action of the machine as a synchronous motor has been more fully explained.

203. Influence of the Number of Phases.—Heretofore it has been considered that the number of conductors per pole and the number of phases was infinite or at least very great, and that each was connected to its own receiving circuit. In actual synchronous machines, the number of conductors per phase is comparatively small, and these are connected to form a small number of phases. Usually the conductors are separated into either one, two or three groups per pole. These conductors are then connected to form a single-, two- or three-phase winding. Of these, the three-phase winding is the one generally used. The two-phase winding was common a few years ago, but is rarely seen now, while the single-phase winding is very rarely used except for machines furnished to plants where it is desirable that additional machines should conform to the earlier type of equipment.

Figure 165 shows a section of a three-phase machine. There are six conductors per pole, or two per pole per phase. In practice, except in the case of very small machines, there are usually three or four conductors per pole per phase, although two are by no means uncommon. All the conductors of each phase are connected in series by connecting one of the *a* conductors under a north pole to one of the *a* conductors under the adjoining south pole. There are several means of doing this, some of which were explained in Chap. XIV.

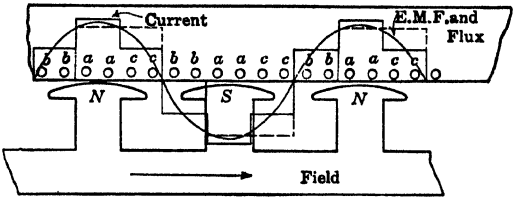

FIG. 165.

It will be seen that the curve of e.m.f. distribution will be a sine curve as before. The curve of current distribution or the current sheet will, however, be different. This is apparent on account of the fact that all of the conductors in one phase must have the same current flowing in them, since all are connected in series. The result is that the current sheet assumes the stepped appearance of Fig. 165. The relation of the three currents *in*

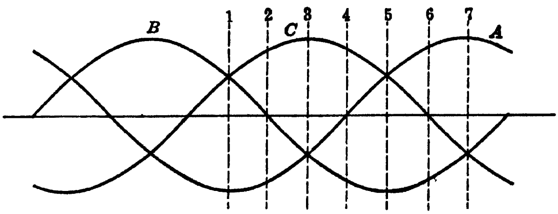

FIG. 166.

time phase will be as shown in Fig. 166. The current sheet of Fig. 165 is shown at the time marked "1" in Fig. 166. An instant later, at the time marked "2," the shape of the current wave will be different since now the current in phase B has become zero while A has decreased somewhat and at the same time C has increased. The sum of the three currents is, however, the same as before. The shape of the current wave at this time will

be as shown by the dotted line in Fig. 165. An instant later, at the time marked "3" the shape will be the same as at the time "1" but removed still farther to the right. At intermediate times, the shape of the current sheet will be between that corresponding to the time "1" and that of "2." *The current sheet as a whole moves steadily to the right, but with a slight change of shape as it progresses.*

This discussion is also directly applicable to the induction motor. The windings of the stator of an induction machine and of a synchronous machine are identical, except as modified by the fact that the induction machine is frequently wound for a lower voltage than the synchronous machine. A rotating magnetic field is set up by the rotating current sheet in the case of both machines. The action of this will be considered later. It may be well to repeat that the stator current may have either a magnetizing or a demagnetizing effect upon the main field, depending upon whether it is a lagging or a leading current. When it is in phase with the e.m.f. its magnetizing effect is nearly zero.

204. Synchronous Machines in Parallel.—In practice, it is seldom possible to operate one synchronous machine alone on a line. This is often due to the fact that it is impossible, or at least impracticable, to build machines of a capacity great enough to take care of the entire output of a station as in the case of the large power houses at Niagara Falls. In any event, it is considered advisable to have the capacity of the station divided into several units, so that the station may operate more efficiently at light loads, and so that it may be easier to have reserve units in case of breakdown. It might, however, be possible to divide the load between the various machines, so that each would operate independently of the others, but this is highly undesirable, and unnecessary.

In order that machines should operate in parallel to supply power to the same circuit, it is necessary that the e.m.fs. of all of them should rise and fall practically together, that is, the frequency of all must be the same. This in turn means that the machines must operate at *exactly* the same speed if they have the same number of poles, or if the numbers of poles are different, at speeds exactly in inverse proportion to their respective numbers of poles. This relation must be continued as long as the machines are in parallel, hence it is quite common to have

several machines operate for days at a time, no one of them either gaining or losing even a fraction of a revolution on the others. The action is as though the machines were connected together by means of invisible gear wheels. Indeed, if these gear wheels are regarded as being slightly flexible, an accurate mechanical analogy to the action of two or more synchronous machines operating in parallel is obtained.

When machines are operating together in parallel in this manner, they are held in synchronism by the inherent electrical action of the machines. If one alternator tries to get ahead, due to an increased admission of steam to the driving engine or to other causes, it automatically takes more load and is restrained from increasing its speed, at least permanently. It is true it does revolve faster for an instant until it is slightly ahead of the others,

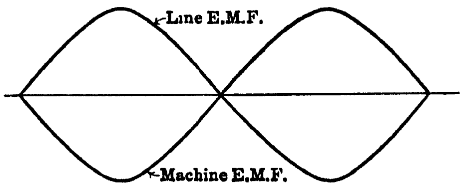

FIG. 167.

but the farther ahead it gets, the more the load is increased. The amount of advance is always very small, being only a fraction of a pole span. On the other hand, if the load is reduced, the machine falls back slightly in phase, thus reducing its load and restoring the balance. Even though the driving power is entirely removed, the machine will not stop, but will continue to revolve as a motor at the same speed as before.

205. Relations of E.M.F. and Current.—In Fig. 167 the curve marked "Line e.m.f." represents the curve of applied voltage from the line. This may be the e.m.f. of one machine or it may be the resultant applied e.m.f. due to several machines. The back e.m.f. of the machine under consideration is marked "machine e.m.f.," and is represented as being exactly equal and opposite to the line e.m.f. The two e.m.f. waves may or may not be sine waves. It results in a somewhat simpler diagram, however, if they are taken as sine waves, but the same method of analysis is applicable to any form of wave.

With the waves equal and opposite, as shown, they will be in

perfect balance at all times and there will be no tendency for current to flow through the machine, and the machine will develop no power.

Suppose now that the driving force be removed from the machine considered, say by closing the throttle of the steam engine driving it. The first tendency of the machine will, of course, be to stop. It will actually drop back a little in step until its e.m.f. is no longer directly opposed to that of the line. This is shown in Fig. 168. The two e.m.fs. will now no longer balance one another. If their *difference* at each point is plotted, the curve of difference will be as shown in the curve marked "Resultant e.m.f." This is nearly 90° different in phase from either of the original curves. It will be a sine wave if the machine e.m.f. and that of the line are sinusoidal.

This unbalanced e.m.f. will, of course, set up a current between the two machines. In such a circuit the reactance will, in general, be far greater than the resistance and the current will, therefore, lag nearly 90° behind the *resultant* e.m.f. In Fig. 168 it is shown as lagging approximately 80°. Since the resultant e.m.f. is already nearly 90° out of phase with the line e.m.f. and the machine e.m.f., and since the current lags nearly 90° behind the *resultant e.m.f.*, the current will be brought nearly into phase, and phase opposition, respectively with the line and the machine e.m.f. It, therefore, represents power in connection with both of these e.m.fs. Since the current flows in general in opposition to the machine e.m.f., the machine is acting as a motor. The machine or machines supplying the line are, of course, acting at the same time as generators. What happens then is that the machine will drop back in phase (*not in speed*) until sufficient current flows to supply enough torque to maintain the motion. It will then continue to operate as a motor.

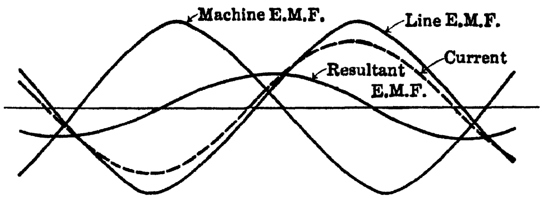

FIG. 168

If, on the other hand, the power supplied to the machine under

consideration had been increased instead of reduced, the machine would have advanced somewhat in phase, the resultant e.m.f. and the corresponding current would have been reversed, and the machine instead of acting as a motor would have become a generator. It is thus possible to increase or diminish the power output of a synchronous machine, or even cause it to act as a motor by changing in the corresponding manner the *power input* to the machine. All this is accomplished *without any change in the field current,* the generated voltage or the speed of the machine.

206. Effect of Change of Field Current.—A change in the field current of a synchronous machine results in only a slight change in the power output of the machine. This is a rather surprising result to one accustomed only to the action of continuous-current machinery. It is true that changing the field current

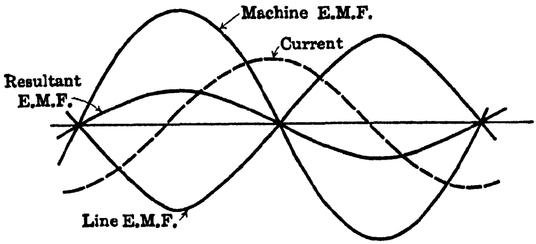

FIG 169.

of a synchronous machine does result in a change of the *current* delivered by the machine, but the power remains practically the same. Assuming that the machine and the line e.m.fs. are equal and opposite, as shown in Fig. 167, increase the field current and the generated voltage of the machine under consideration. The result will be an unbalanced voltage as shown in Fig. 169 in phase with the machine e.m.f. This will cause a current to flow, and this current, as before, will lag nearly 90° behind the resultant e.m.f. The current is then nearly 90° in phase from both the applied and the machine voltage and consequently represents little power. It will be seen that with the machine voltage higher than the line voltage, the resulting current is *leading the line e.m.f.* It is often desirable to introduce leading current into a system in order to offset lagging current due to induction motors or to other causes. If the machine voltage had been lowered instead of

raised, the resultant e.m.f. and current would have been reversed and the current would have been lagging instead of leading.

207. Effect of Regulation of Prime Mover.—In the above, it has just been demonstrated that changing the field current of a synchronous machine produces little or no effect upon its power output. That this is so could have been readily shown by a study of the action of the prime mover, driving the synchronous machine. The operation of the governor of any prime mover is dependent upon *the speed* of the machine. In general, in order that a steam engine, gas engine, water turbine, or other source of power should develop more output, it is necessary that its *speed be reduced.* This causes the governor balls to drop slightly, and these in turn actuate suitable mechanism to allow the admission of more steam, gas or water as the case may be. But in the case of a prime mover driving a synchronous machine, there can be no change in speed as long as the speed of the other machines supplying the line remains the same. Since varying the field strength will not change the speed, the output of the prime mover will not change, and in consequence the mechanical input of the synchronous machine can not change at all and the output only very slightly.

When the load upon a station containing several synchronous machines in parallel is increased, the case is somewhat different. The *speed of all of the machines* will decrease the same amount, and if the governors are perfectly adjusted, the admission of power to all of the machines will increase in the proper proportion, leaving all the machines loaded to the same percentage of their rated capacity. If the governors are so set that some of the units increase their output more than others for the same reduction in speed, the increase of load will not be divided equally among the machines, and those whose prime movers regulate more closely will take more than their proportion of the load. In certain cases it is desirable that this should be the case. Thus if one or more reciprocating units are operating in parallel with one or more steam turbines, it is generally considered desirable that the former should operate at as near their rated load as possible and that the turbines should take the fluctuations. This is desirable since the efficiency of a turbine does not decrease so much for under- or overload as does that of a reciprocating engine. The result mentioned can be secured by setting the turbine governors to regulate more closely than those of the reciprocating engines.

The only way of changing the load on one of a number of synchronous machines is to *change the power input to its prime mover*. This is generally done by changing the stiffness of a spring comprising part of the governor mechanism. It is customary in the case of steam turbines to mount a small motor on the frame of the turbine, and connect this with the governor spring in such a manner that the latter may be compressed or extended by means of the motor. The output of the generator can then be readily controlled by means of a small double-throw switch on the switchboard, this small switch being so connected as to stop, start or reverse the small motor.

208. Treatment by Means of Vectors.—The results described could have been deduced by means of vectors instead of the

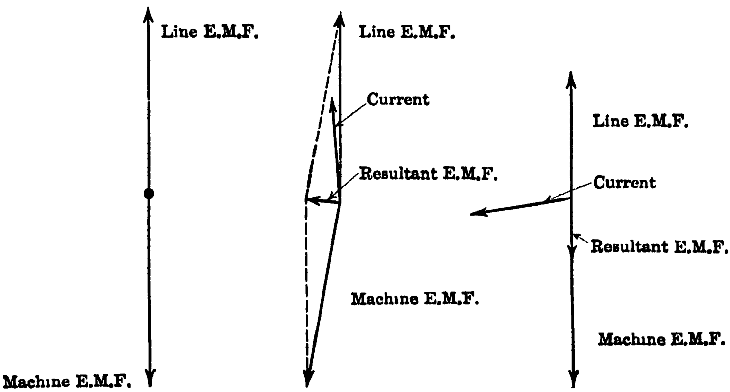

FIG. 170. FIG. 171. FIG. 172.

curves used in the preceding figures. Indeed the method would have been much simpler, although essentially the same as that employed. The vector method, however, is not nearly so clear to most as a consideration of the curves of current and e.m.f. as just presented. It must also be kept in mind that this method of analysis is applicable to any form of wave, while the vector method of treatment is applicable to sine waves only. With these qualifications in mind, however, the vector method presents a powerful method of analysis, and possesses the great advantage that the diagrams are in general so simple that it is easy to carry a picture of them in the mind.

Figures 170, 171, 172, are the vector diagrams corresponding to Figs. 167, 168, and 169. In Fig. 170, the line and the machine

e.m.fs. are equal and opposite, giving as before a zero resultant. In Fig. 171, the two e.m.fs. are equal but the driving force has been removed from the machine, allowing it to drop back slightly. The resultant e.m.f. will have the direction as shown, and the current will as before lag behind this *resultant* by an angle of almost 90°. This brings the current almost into phase with the line e.m.f. and into phase opposition to the machine e.m.f. The machine will then operate as a motor. Figure 172 shows what happens when the field current is increased so that the machine e.m.f. is greater than that of the line. The resultant e.m.f. is in phase with that of the machine, and the current, since it lags

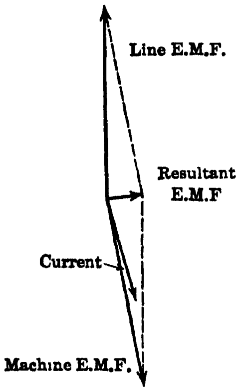

FIG. 173.

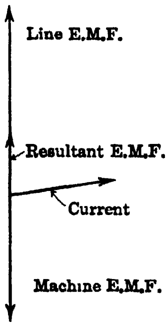

FIG. 174.

almost 90°, is nearly in quadrature with both the line and the machine e.m.f. The power developed is then nearly zero.

In Figs. 173 and 174, are shown the vector diagrams respectively of a synchronous machine acting as a generator, and of a machine with its generated e.m.f. less than that of the line, and taking a small amount of power as a motor. The construction of these will be readily understood. In fact, the only change is the substitution of the word "line," for "machine" and *vice versa.* It should be noted that in the latter case, the current consumed by the synchronous machine is lagging.

209. The Synchronous Condenser.—The ability of the synchronous machine to take either a leading or a lagging current by using a relatively strong or a weak field current, is frequently used in practice to improve the power factor of a circuit. A machine employed for this purpose is called

a synchronous condenser. Thus a transmission line may be employed to transmit power from a waterfall to a distant point where the power would in most cases be used largely to drive induction motors. The induction motor always takes current lagging behind the e.m.f., the lag being particularly great when the load is light. A synchronous motor may be readily used to correct the power factor. Thus Fig. 175 shows the line e.m.f. and the current taken by an induction motor. An overexcited synchronous motor operating without load may be made to take a current leading by nearly 90° as indicated in the diagram. The combined current will be as shown by the line marked "resultant current." This it will be seen is materially less than the current required by the induction motor alone. Hence the use of such a synchronous condenser will in many cases reduce the line current and consequently the line loss, and at the same time improve the regulation. Moreover, the synchronous machine need not be used for its condenser effect alone, but may be made to carry load in addition to correcting the power factor. It can be shown that such a machine will be most economical considering both functions, if the wattless and the power components of its current are made equal, *i.e.*, if its power factor is made 70.7 per cent. It will then carry approximately 70 per cent. of its rated load as a motor, and at the same time, consume 70 per cent. of the wattless current it could carry as a synchronous condenser alone.

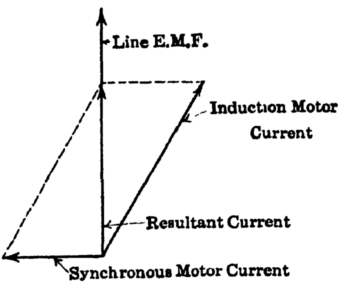

FIG. 175.

210. Operation with Distorted Waves.—It will be seen from a consideration of the vector diagrams that by a proper regulation of the field current, the current and the e.m.f. of a synchronous machine can always be brought into the same phase and consequently that the power factor can always be made 100 per cent. It must, however, be kept clearly in mind that the *vector* diagrams apply *only to sine waves*, and are meaningless, except in an approximate sense, in the case of other waves. The diagrams, in which the waves are drawn out as in Figs. 167, 168, and 169, are applicable to waves of any shape. Thus in Fig. 176, we have a line e.m.f. of a sine shape but the back e.m.f. of

the machine is distorted. No adjustment of the field strength would cause these two e.m.fs. to balance one another. The resultant e.m.f. in the case shown is a sine wave of three times the fundamental frequency. It would set up a current lagging nearly 90° behind itself, and likewise of triple frequency. The power factor would in this case be far from unity, since no matter how light the load, there would be a large current circulating between the machines. This effect is most noticeable at light load, since at larger loads, its effect is somewhat masked by the large load current in phase with the e.m.f. The writer has even seen cases where the *addition* of a considerable load resulted in a marked decrease in the minimum current which could be obtained by field adjustment. This was probably due to a change of the wave shape of the motor under load

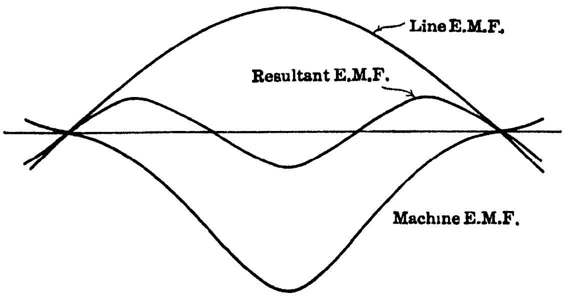

FIG. 176.

211. Hunting.—In the case of a number of alternators operating in parallel, it will sometimes be noticeable that while the load and current from the whole station are constant, the current and power outputs of some individual machines are very unsteady. The current and power will rise together then drop back to minimum current and perhaps zero power, the current will then rise again while the power will perhaps reverse and become negative. The machine is then, of course, operating for the moment as a motor. This action is called hunting. If the hunting is less violent, there will be merely a rise and fall in the current and power, the latter never reversing. This pulsation of power and current will frequently occur with a definite frequency, the time of 1 cycle being generally from 1 sec. to about 10 sec. The same variation of power and current will frequently occur when a machine is operating as a synchronous motor.

The cause of this phenomenon is usually something external to the motor or generator, and some machines are more sensitive to such external causes than are others. In general the external cause is a pulsation in the power supply as, for instance, the variation of torque during the revolution of a single-cylinder steam engine or, to a still greater degree, of that of a single-cylinder gas engine. In this latter case, power is applied during only one stroke in four, and even then not at a uniform rate. When a power impulse does occur, the synchronous machine to which the gas engine is connected is forced ahead in phase, so that the vector diagram becomes like that of Fig. 173. Immediately thereafter the driving force is removed as the engine starts on its idle strokes and the synchronous machine may begin to act as a motor, with a vector diagram like that of Fig. 171. When the machine surges forward it is liable to go too far; this increases the generator action to such an extent that the machine is retarded greatly, it then swings back, again going too far and so on. The action may become cumulative, and increase to such an extent that the machine pulls out of step with the other machines. Hunting is also sometimes set up on account of a tendency of the engine governor to hunt. The cause of this is that the governor, when it acts, tends to go too far and admit too much steam or gas to the cylinder. The action is very similar to the hunting of the alternator, but should not be confused with it.

212. Prevention of Hunting.—The best way to avoid hunting, is to avoid, if possible, the causes which may induce it. Thus, if it is practicable to drive by means of steam or water turbines, there is little prospect of trouble from this source. If driving by means of reciprocating steam engines or gas engines is unavoidable, much may be done to remove or minimize the trouble. A change in the weight of the flywheel may have a very good effect. The change may involve the use of either a heavier or a lighter wheel. The reason for this will be plain if it is considered that if we pass continuous current through one section of the armature winding of an alternator, the field at the same time being excited by means of continuous current, there will be an attraction tending to cause the field to assume some definite position with respect to the armature. If the field and the armature are in some other position than this one of rest, and current is turned on to the two, the field will oscillate about its position of rest with a certain definite period. Similarly, if the field be in motion,

and some cause tends momentarily to make the machine run either faster or slower than before for an instant, the field will tend to oscillate with this same period in addition to its motion of rotation. Ordinarily, this oscillation will die out very quickly, but if the period of the impulses is the same as the natural period of vibration of the machine, it may readily occur that the natural vibration will be maintained and increased until the hunting becomes violent. The action is similar to that involved in the motion of the pendulum of a clock, which is kept in comparatively violent motion by the feeble impulses imparted to it in exact synchronism with its motion. A change in the weight of the flywheel will cause the period of vibration of the moving masses to differ from that of the impulses communicated to them, and

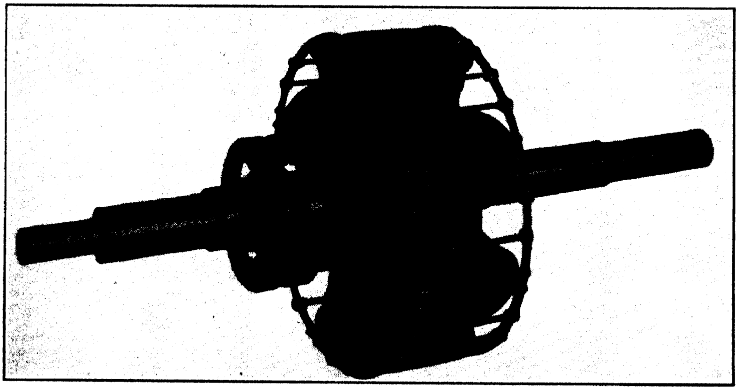

Fig. 177.

consequently the resonance will be destroyed, and the hunting either completely stopped or at least greatly diminished.

213. Damping Grids.—There is also another method of attacking the problem. If, in the case of the pendulum mentioned, the motion were to take place in some liquid such as water, the oscillation for the same applied force would be much weaker. A similar effect can be obtained in a synchronous machine by causing the production of eddy currents in the moving mass whenever it moves from the position of equilibrium. Thus, considering again the machine with continuous current in both the field and the armature, and displaced from the position of rest, it is evident that as the field oscillates, it will cut the lines of induction set up by the armature. If the poles are solid,

eddy currents will be produced. This requires the expenditure of energy, and results in strongly damping the motion. The oscillation will then quickly die out. The same effect can be produced by providing each pole with a copper grid as shown in Fig. 177. This is sometimes called an amortisseur winding. It is similar to the squirrel-cage winding of an induction motor, and like the latter has eddy currents induced in it whenever it cuts across the flux.

214. The Synchronous Motor.—The preceding discussion applies to the synchronous machine whether used as a generator or as a motor. When a machine is used exclusively as a motor, however, certain problems arise which are not present when it is operated as a generator. The principal difficulty is in regard to starting.

215. Methods of Starting.—*Synchronizing by Means of Lamps.*—If a single-phase synchronous machine is at rest, and current is applied to the armature, there will be no tendency at all to rotate. This holds whether the field is excited or not. To enable such a machine to operate, it is necessary that it be started by some external power, brought up in speed until its frequency is the same as that of the line, and then be connected to the line when it is in such a phase that its e.m.f. is directly equal and opposite to that of the line. The driving power may then be removed and the machine will continue to operate as a motor.

To be sure that the motor is at the proper frequency and phase with respect to the line, the simplest apparatus that can be used is an incandescent lamp, connected as shown in Fig. 178. If connected to the line only or the machine only, the lamp would light up and remain steadily in this condition. When connected to both machine and line, the e.m.f. applied to the lamp will be the vector sum of the e.m.fs. of the two. This is shown in Fig. 179 where one vector is the e.m.f. of the line and the other that of the machine about to be synchronized. For convenience, the e.m.f. of the line has been drawn as directed inwardly toward the center of the circle. With this construction, the resultant e.m.f. is given in magnitude by the line so marked. If the frequency of the machine to be synchronized and the line are not the same, the angle between the two vectors is constantly changing, or it may be considered that one of the vectors (say that representing the line e.m.f.) is stationary and that the other is revolving slowly

around the center point in a counter-clockwise direction if the machine to be synchronized is operating at too high a speed, and clockwise for too low a speed. In either event, it is evident that the resultant e.m.f. will vary in value from zero to double the value of either of the two main e.m.fs. The lamp will then alternately light up and go out, making a complete cycle every time the machine gains or loses a cycle with respect to the line. The lamp will then show two things: its rate of lighting up and going out will show the nearness of the machine to synchronism, and the moment of darkness will indicate the phase which gives zero resultant e.m.f., and consequently the proper moment for closing the switch.

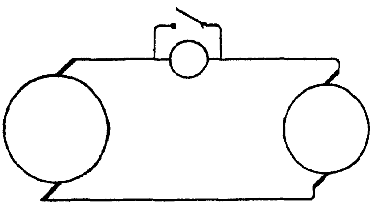

FIG. 178.

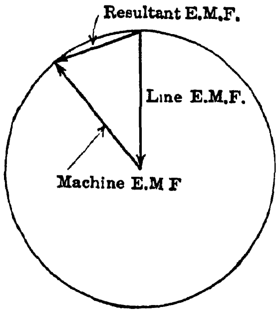

FIG. 179

In practice, it would not be desirable to use a switch on one side only of the circuit as indicated in Fig. 178. To make possible the use of a two-pole switch, two lamps are necessary, one connected across each side of the switch. With a three-phase machine, a three-pole switch, and with a two-phase, a four-pole switch would be needed. It would, however, be unnecessary to provide more than two lamps in either case, since when one phase is synchronized, the others must also be in the proper relative position, unless the connections have been changed, or the direction of rotation reversed since the machine was connected.

216. The Synchroscope.—With either of the foregoing arrangements, there is some difficulty in determining the moment of exact synchronism, since an incandescent lamp will not light up at all unless about 15 per cent. of its rated voltage is impressed on its terminals. A voltmeter may be used instead of the lamps to indicate more exactly the zero of the e.m.f. or the connection may be made as shown in Fig. 180. This diagram shows that

at the moment of phase opposition, the lamps will be fully lighted instead of being dark. This condition is more easily recognized than the preceding. After the machines are connected together, the lamps will in this case remain lighted.

For the synchronizing of large machines, where a slight mistake in synchronism might be disastrous, an instrument called a synchroscope is used. This indicates by means of a pointer revolving around a dial whether the incoming machine is "fast" or "slow" and also, the moment of exact phase opposition. These can be built of large size, so that their indications, can be read in any part of a large engine room. Automatic synchronizers have also been constructed which operate to close the main switch when, and only when, the two voltages are approximately equal, are opposite in phase, and the frequencies are nearly the same. It will be obvious that the preceding remarks apply equally well to the synchronizing of synchronous generators as well as to synchronous motors.

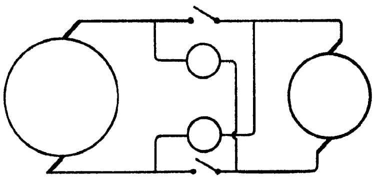

FIG. 180.

The method of starting described is practically always used in the case of synchronous generators, both single phase and polyphase, as a prime mover is always present to bring the machine up to speed. In some of the applications of synchronous motors it may also be used. Synchronous motors driving direct-current generators are frequently used to transform alternating current into continuous current. In this case, there is usually present a storage battery on the direct-current line, or if not, there may be other means of supplying direct current to the line. If this is the case, the obvious method of starting is to use the continuous-current machine as a shunt motor, to bring the synchronous machine up to speed, and such machines are normally started in this manner.

If the foregoing plan can not be used, it is sometimes practicable to install the synchronous motor with a friction clutch so that it may be started without load. A small induction motor may be mounted on the shaft, and be used to start the larger machine. The induction motor should have two poles less than the synchronous machine, so that it will be able to bring it somewhat

above synchronous speed. This method works well, but is somewhat costly.

217. Direct Starting of the Synchronous Motor.—A single-phase synchronous motor, has absolutely no starting torque. If, however, a polyphase machine be connected to the line, it will develop a certain torque depending in value upon the construction of the motor. This is in general sufficient for starting the motor without load.

In starting in this way, the torque is greatest if the field circuit is left open. The torque developed is due principally to induction motor action. When the polyphase current is applied to the armature, a revolving magnetic field is set up just as in the case of the induction motor, and this field, cutting across the solid pole faces, sets up currents which produce torque in the same manner as in a squirrel-cage induction motor.

If, as shown in Fig. 177, the poles are provided with grids to prevent hunting or with a squirrel-cage winding for the same purpose, the starting action will be stronger, and in fact, the starting torque may be made nearly equal to that of a squirrel-cage induction motor. To obtain the greatest possible starting torque, the squirrel-cage winding or the grids employed, must be made of high resistance. On the other hand, to secure the greatest damping effect so as to prevent hunting, the resistance must be kept low. The two requirements are therefore opposed to one another, and the designer must exercise his judgment as to the best average solution.

In starting in this way, the machine will accelerate until it is nearly in synchronism, or if the load is light may attain full synchronism. This latter is due to the fact that the "poles" of the armature attract strongly those of the field, and since the relative motion is small near synchronism, they are frequently able to exert enough pull to bring the field up to full synchronism. Having once attained this speed, the pull is easily sufficient to retain the field at full synchronism. In any event, as soon as the field current is applied, the attraction becomes sufficiently strong so that the field is pulled into full synchronism, and the machine continues to operate as a synchronous motor.

In many cases it is desirable to construct a synchronous motor with laminated poles instead of solid ones. Such a motor will also exert a moderate starting torque. This is due to hysteresis

and eddy current loss in the field poles. As is explained under induction motors (see Art. 251), any loss in the rotor causes a torque. Since the laminations are usually rather thick, causing a large eddy current loss, and since the flux densities when operating in this manner are large (giving a large hysteresis loss), the torque developed may be considerable. Of course, in most motors both of these actions are present to some extent.

In starting up a synchronous motor in this manner, the field is either left open or is closed through a resistor. As the flux set up by the current in the armature rotates rapidly around the stationary field, there is a change of flux through the field windings of the frequency of the applied current. Thus, at one instant a north "pole" of the armature will be opposite one of the field poles, and flux will be passing into the field pole. An instant later, a south "pole" will be opposite this field pole and the flux through it will be reversed. This rapid reversal of the flux induces an e.m.f. in the field winding, and since the number of turns in the field winding is large and all these turns are connected in series, the induced voltage will generally be high. In a moderate-sized machine, this may readily amount to several thousand volts; the use of a resistor connected across the field terminals greatly reduces this. As the field begins to rotate, this voltage becomes less, since the cutting is less rapid, and at exact synchronism the induced e.m.f. is zero, since the flux and the field are moving at the same speed.

In addition to the danger to life from this high voltage, there is also danger that the insulation of the field may be punctured. To guard against this, it is necessary that the insulation of the field be much thicker than would be necessary if the machine were not to be started in this manner. Synchronous converters (which operate as synchronous motors as long as no continuous current is taken from them) are also often started in this way. In this case, since the field is stationary, it is possible to avoid this high voltage to some extent by disconnecting the field coils from one another, thus reducing the induced voltage since the sections are no longer connected in series. A switch for this purpose is known as a field break-up switch.

218. Combination Methods of Starting.—In many cases in practice, a combination of some of the preceding methods of starting may be advisable. Thus it sometimes occurs in the operation of motor-generator sets or of synchronous converters

that starting from the continuous-current side is difficult on account of fluctuations in the supply voltage. This variation may introduce danger, since the voltage may change just as the operator is starting to throw the switch, or it may be that a great amount of time is necessary to secure proper conditions for synchronizing. This is very objectionable, particularly in the case of rotaries or motor-generator sets supplying current to electric railways. The difficulty may be avoided by running the machine up to a speed considerably above synchronism by means of the continuous current and then disconnecting the machine from the direct-current line. The operator then watches the synchronism indicator until the speed has fallen nearly to synchronism. The field of the synchronous machine is opened and the machine connected to the alternating-current line. The field circuit is immediately closed again and the machine drops into step. The disturbance to the line voltage is less in amount and lasts for a much shorter time than would be the case if the machine were started entirely by means of the alternating current. At the same time, the skill and attention required of the operator is much less than would be the case if the machines were synchronized, since the machine being started need be only approximately at synchronous speed. If thrown directly on the line, a synchronous motor takes from four to eight times full-load current. This is undesirable since the current, in addition to being large, also lags nearly 90° and generally seriously lowers the voltage of the line. Any of the starting devices used with induction motors may be used to reduce the starting current. A synchronous motor is, however, often supplied from its own transformers, and a rotary converter is almost always so supplied. In such cases the cheapest starting arrangement is the use of low-voltage taps from the transformers in connection with a double-throw switch. For further information in regard to starting devices, see the section on induction motors (Art. 262.)

219. Armature Reaction.—So far in the discussion of the synchronous machine the magnetizing effect of the armature current has been neglected. In Fig. 159 if the point marked *N* on the armature be considered, it will be seen that all of the currents to the right of this point, for a distance equal to the pole pitch, are in the one direction, while those on the other side flow in the opposite direction. This portion of the armature surface then has currents circulating around it, and a tendency will exist

to form a pole there. If the machine is operating as a generator, the polarity of the armature will be as shown. This is necessary if the machine is to develop power, as the poles of the armature may be regarded as attracting those of the field. This is perhaps not as exact a method of looking at the subject as that formerly used, but it is occasionally very useful, as in the present instance. If the machine is operating as a motor, the polarity of the armature will of course be reversed.

The diagram shown is that for unity power factor. Whether the action is that of a generator or a motor, the magnetizing action will be weak. This arises from the fact that the points of greatest magnetic action of the armature are situated half way between the field poles, and hence have comparatively little effect upon them. There is, however, a tendency to *distort* the flux, causing it to be strengthened in one pole tip and weakened in the other.

If the current is either leading or lagging by 90°, the conditions are the best for the armature to exert a strong magnetic action upon the field. At smaller angles of lag or lead the current will have a smaller magnetizing or demagnetizing effect. Thus in Fig. 163, in which the current of a generator is lagging by about 40°, it will be seen that the poles of the armature are partially over those of the field of the same polarity. Thus a leading current in a motor or a lagging current in a generator exerts a strong demagnetizing action. Conversely, a lagging current in a motor or a leading current in a generator tends to increase the magnetization.

220. Regulation.—It results from the foregoing facts that the regulation of an alternating-current generator is more dependent upon the nature of the load than is that of a continuous-current machine. In the latter the only variable is the current; in the former we have to consider both the current and its angle of lag or lead. In Fig. 181, the middle curve represents the variation of the voltage of an alternating-current generator with current output, the speed and field current being constant, for 100 per cent. power factor. The voltage drops off as the load is increased, the curve being approximately the same as would be obtained in the case of a separately excited continuous-current generator. If, however, the current is lagging, the voltage will drop off much more rapidly, as shown in the lower curve of the same figure. If, on the contrary, the current is leading, the

voltage will drop less or even rise as the current increases. This is indicated in the upper curve of Fig. 181.

This peculiar behavior of the synchronous generator is due to the magnetic action of the armature as just described. If the machine is furnishing a lagging current, there is, in addition to the drop in the armature due to resistance and the drop due to reactance, a large reduction of the useful magnetic flux of the machine due to the demagnetizing action of the armature. This also reduces the voltage. If, on the other hand, the current is leading, the armature tends to magnetize the field and the external voltage is increased.

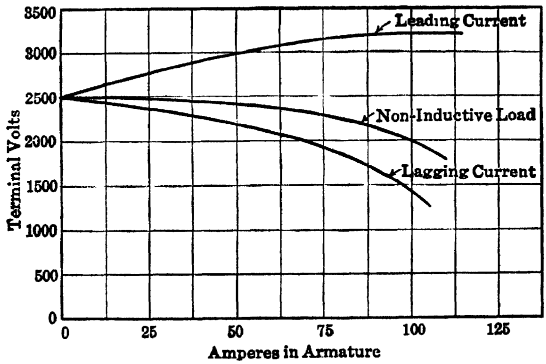

FIG. 181.

In giving the regulation of an alternator, it is necessary that the power factor be stated. Thus, a machine which has a regulation of 6 per cent. at 100 per cent. power factor will have a regulation of perhaps 20 per cent. at 80 per cent. power factor lagging current, and perhaps one of −5 per cent. at 80 per cent. power factor leading current.

221. Rating of Synchronous Machine.—We must also keep these facts in mind in stating the rating of a machine. An alternator is rated usually in kilovolt-amperes, *i.e.*, it is rated to deliver a certain voltage and a certain amperage. The power will be the same as the kv-a. in case the power factor is unity. At any other power factor the power will be reduced in the same proportion as the power factor. Moreover, with low power factor and lagging current it is necessary that the field be far stronger than normal. We could, for example, hardly expect an alternator to carry its full kv-a. rating at a power factor of zero lagging

current, since an unreasonably strong field would be required. In case the generator must cope with unusual conditions, the requirements should be clearly stated in the specifications.

This peculiarity of the regulation of the alternating-current generator makes it difficult to devise any system of compounding which will be suitable under all circumstances. Formerly, when alternators were used almost exclusively on lighting loads having a power factor of nearly 100 per cent., alternators were frequently compounded in nearly the same manner as continuous-current machines. The alternators were built with stationary fields and revolving armatures, and a commutator was added to rectify the alternating current as it passed through the series field. In many cases, instead of rectifying the actual current of the machine, a current transformer was used, and the current from the secondary of this was rectified. However, this compounding or compensating, as it was more commonly called, was ordinarily correct only for a non-inductive load. With a lagging current it helped a little, but with a leading current it was worse than useless. Moreover, it was necessary to set the brushes in such a position with respect to the commutator that the brush passed from one segment to the next, as the current passed through its zero value. Any change in the lag or lead of the current caused this point to change, and hence led to sparking unless the brushes were shifted. These facts have caused the abandonment of this device, and at the present time practically all synchronous machines are built with revolving fields and without compensating windings. Hand regulation is generally depended upon, or in the case of rapidly fluctuating loads, some one of the various forms of automatic regulators is employed.

222. Regulation in Large Machines.—To secure good regulation of a synchronous machine whether used as a generator or as a motor, it is necessary that the magnetic strength of the field be large compared with that of the armature. In order that the large number of ampere turns on the field should not force too much flux across the air gap and through the armature, it is in turn necessary that the air gap be large. Thus it sometimes happens that in the case of large turbo-alternators with few poles the air gap is as long as 2½ in. A small fraction of this would be sufficient as far as mechanical clearance is concerned.

Good regulation is almost always desirable in a synchronous generator in the smaller sizes. In machines of large size, very

good regulation is *not* always advisable. One reason for this is that such machines are used in power houses of large capacity, and on such systems the load is not liable to as violent fluctuations as is the case with smaller systems. Moreover, with large machines the chance of injury in case of a short-circuit is greater than in that of small alternators. A machine of poor regulation, particularly if the regulation is relatively poorer in the case of inductive loads, will have a smaller short-circuit current than would one of good regulation, and hence would be less liable to suffer damage itself or to injure the circuit-breakers in the event of a short-circuit.

223. Effect of Good Regulation in the Synchronous Motor.—With the synchronous motor the case is somewhat different. If a machine has good regulation, it is evident that if the adjustment of the field current is somewhat faulty, the machine will take a large leading or lagging current. Moreover, a drop in the line voltage will have the same effect as an increase in the field strength and *vice versa*, and hence will lead to the circulation of a large current through the machine. This increases the heating, thus cutting down the capacity of the machine, and at the same time causes it to operate less efficiently since the copper loss will be increased. Hence, from the standpoint of the *operator* of the machine, very good regulation in a synchronous motor is not desirable.

From the standpoint of the electricity supply company, however, the reverse is true. If for any reason the supply voltage drops, the machine takes a leading current, since now its voltage is higher than that of the line. A leading current through an inductive line has the effect of increasing the voltage at the end of the line. Hence the presence of the synchronous motor tends to prevent the fall of voltage. The reverse action takes place in the event of a rise in voltage. A simple way of looking at the matter is to regard the synchronous machine (whether it is acting as a generator or as a motor) as a sort of flexible prop, tending to prevent any change in the voltage at the point to which it is applied. This corrective action will be more effective the better the regulation of the synchronous machine, and hence good regulation in motors as well as in generators is desirable from the standpoint of the power company.

224. Synchronous Condensers.—So desirable is this action, that synchronous machines are sometimes installed by power

supply companies for the sake of their regulating action alone, or they may be utilized to carry a load either as motors or possibly as generators in addition. However, the action is different from that of a condenser. The latter takes a leading current, under all conditions. The synchronous machine, on the other hand, takes either a leading or a lagging current as the case may require, the current in each case being such as will nearly correct the departure of the line voltage from normal.

In other cases where the need of such correction is not pressing enough to warrant the installation of a synchronous machine for the sole purpose of keeping the voltage constant, it, nevertheless, is worth while for the power company to offer a better rate to a large customer if he will install a synchronous motor instead of an induction motor. In many cases, the contract specifies that the field current of the synchronous machine shall be adjusted in accordance with the instructions of the power company. If desirable, the field may be so strengthened that the machine takes a leading current, thus offsetting lagging current due to induction motors or other devices.

PROBLEMS

91. A certain large factory is equipped with induction motors. The input to the factory is 1000 kw. at a power factor of 0.80, the current being lagging. It is proposed to install a synchronous machine running light on the line to improve this power factor by taking a leading current. What will be the kv-a rating of the synchronous machine if the power factor is raised to 100 per cent ? If to 90 per cent.?

92. In the foregoing factory a motor is needed to develop 1000 hp. Assuming that it would operate as a motor at an efficiency of 90 per cent., what would be the kv-a. rating of a synchronous machine to raise the power factor to unity and at the same time carry 1000 hp. as a motor?

93. The cost of motors and generators increases approximately in proportion to the square root of the capacity, the speed, etc., being the same. What would be the relative costs of the two machines in the above problems?

94. A certain power house is equipped with ten turbine-driven alternators. The rated output of each turbine is 14,000 hp. What would be the correct size of each generator to utilize the full power of the turbines if it is assumed that the station will operate at 100 per cent. power factor? What at 85 per cent. power factor? The efficiency of the generators may be taken as being 95 per cent.

95. A certain station operates at a power factor of 65 per cent. and has a maximum output of 50,000 kw. What is the capacity of the transformers required? What is the capacity of the generators, assuming that the trans-

former efficiency is 98 per cent.? What is the horse-power rating of the turbines, assuming a generator efficiency of 95 per cent.?

96. A certain alternator is operating under full load at unity power factor and a terminal voltage of 4400. When the load is reduced to zero, the speed and the field excitation being kept the same, the voltage increases to 4730 volts. What is the regulation for 100 per cent. power factor?

CHAPTER XVII

THE ROTARY CONVERTER OR SYNCHRONOUS CONVERTER

225. General Description.—A rotary converter (frequently called a rotary) is intended primarily as a means of transforming alternating to continuous current or *vice versa*. It consists essentially of a continuous-current machine to which two or more slip rings have been added. These rings are mounted on the shaft and connected to proper points in the armature winding, or what is equivalent, to proper commutator bars. Suitable brushes are provided to carry the current from the rings to the external circuit.

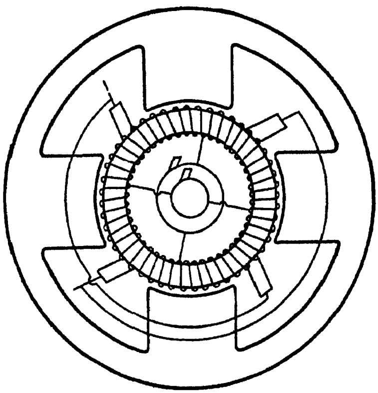

FIG. 182.

Figure 182 shows a two-ring or single-phase rotary. Without the slip rings, it would be a continuous-current generator or motor. The winding is shown as a ring winding merely for convenience. In practice, drum windings are used exclusively. Only the one winding is used on the armature. If the continuous-current brushes were removed, and the field were excited in some suitable manner, the machine would operate as a single-phase alternating-current generator. The voltage would be a maximum when the points connected to the slip rings were under the direct-current brushes, and at that time would be equal to the direct e.m.f. The alternating voltage would be zero when the tapping points were 90 electrical degrees from the above position. It is also evident that the machine would operate as a single-phase synchronous motor.

226. General Operation.—If the machine is driven by external power with the direct-current brushes on, it will be capable of

operating either as a continuous-current or as an alternating-current generator. It is also possible to combine these functions and use the machine to generate *both* kinds of current at the same time. The machine is then known as a double-current generator. Machines are occasionally used in this manner to furnish continuous current to a trolley line in the vicinity of the station. The current for the distant parts of the line is taken out as alternating current. This is then stepped up in transformers, transmitted over the high-tension lines, stepped down to the proper voltage, and transformed into continuous current by means of rotary converters.

It will also be apparent that if the machine is operated as a continuous-current motor, there will be present at the slip rings an alternating e.m.f. It is therefore possible to take alternating current from these rings. The machine is then operating to change continuous current into alternating. When used in this manner, it is called an inverted rotary. This use is comparatively infrequent.

The most common use of the machine is to convert alternating current into continuous current. It will be evident that the rotary will operate as a synchronous motor. When running in this manner, there is a constant e.m.f. at the direct-current brushes, and by connecting to a suitable receiving circuit, continuous current can be led off and utilized.

227. Field Winding.—A rotary converter may be excited by any of the methods used with continuous-current machines. Separate excitation is frequently used in the case of inverted rotaries for a reason which will be explained presently. Series excitation is rarely, if ever, employed. Shunt excitation is common with rotaries used on lighting circuits, while those employed in connection with railroad work are usually compounded.

228. Voltage Relations.—As was pointed out, in a two-ring rotary the *maximum* of the alternating-current voltage is the same as the continuous-current voltage. If the wave shape of the alternating-current voltage is sinusoidal, the *effective* value of the alternating voltage will be equal to the continuous voltage divided by $\sqrt{2}$ or equal to the continuous voltage multiplied by 0.707. Further, if four rings connected to four equidistant points, or six rings connected in the same manner were used, the voltage across any diameter would be given by the same relation.

The three-phase relation is somewhat different. Referring to Fig. 183, a two-ring or single-phase rotary would be connected to the points A and B. The circle is supposed to represent a ring winding of the kind shown in Fig. 182. A four-ring or so-called two-phase rotary would have taps at the points A and B, and C and D. For a three-phase winding the points A, E, and F would be used. The angle EAB is equal to 30°, and we have the relation

$$AE = AB \cos 30° = \frac{\sqrt{3}}{2}AB = 0.866AB$$

Also since AB is equal to 0.707 times the continuous voltage, we may conclude that the three-phase alternating voltage is equal to $0.707 \times 0.866 = 0.612$ times the continuous voltage.

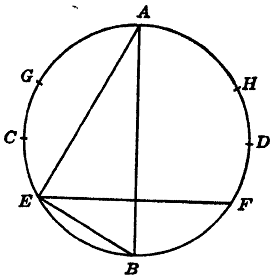

Fig. 183.

The six-phase or six-ring connection is sometimes used with large rotaries. This would require connection to the points $AGEBFH$. The voltage across any diameter is the same as with the single-phase or the two-phase connection.

229. Starting.—In general, the methods of starting rotaries are the same as those discussed under the head of synchronous motors. Any of the methods there described may be used. On account of the fact that rotaries are started without load, and since the machines are small for their rating, starting does not present as great difficulties as in the case of synchronous motors.

If continuous current at a steady voltage were always present, starting from the continuous-current end would be the preferable method. This is, however, rarely the case. Sometimes a small induction motor is provided, mounted upon the end of the rotary shaft. This motor should have two poles less than the rotary in order that the latter may be brought above synchronous speed.

If the rotary is to be started from the alternating-current end, it is customary to provide the fields with a field break-up switch, to avoid danger from the high potential generated in the field when the machine starts. This is readily done since the field is stationary.

The fact that the rotary converter has a commutator, leads to a little difficulty when it is started by applying current to the alternating-current end. At least one armature coil for each field pole is short-circuited by the brushes. As the rotating field revolves around the armature it cuts this coil and generates a large current in it. This sometimes causes flashing at the brushes. This is particularly true in the case of commutating pole rotaries. In these machines it is usually necessary to provide means for raising the direct-current brushes while the machine is being started.

230. Reversed Polarity at Start.—One point in which the operation is different from that of a motor-generator set (that is, an alternating-current motor driving a direct-current generator) deserves mention. In the latter, the direct-current machine acts like any other generator and will always build up with the same polarity, unless something out of the ordinary has happened to reverse it. In the rotary, however, it will be apparent that when the armature is at rest and an alternating current is passed into the slip rings, there will be an alternating potential at the continuous-current brushes. The frequency will be the same as that of the supply. This is due to the rotary magnetic field set up around the armature by the action of the alternating current. As the armature starts to rotate it moves in the opposite direction to this rotating magnetic field, and as the motion of the latter is relative to the armature surface, its speed of rotation becomes less. At full synchronism this rotating magnetic field is stationary with respect to the field and brushes. The frequency of the alternating e.m.f. at the direct-current brushes therefore becomes less as the armature approaches synchronism.

A direct-current voltmeter connected to the continuous-current brushes will at first give no indication when the machine is attached to the alternating-current mains, since the alternating potential is of too high a frequency to affect it. As the armature gains speed the needle will at first tremble, then begin to swing across the scale, first in one direction and then in the other. If the applied e.m.f. is sufficiently high, the machine may pull into complete synchronism before there is any current in the field. In this event, the deflection of the needle will become steady. This permanent deflection may be in either of the two directions. Whether the deflection is positive or negative

depends upon whether a north pole of the armature takes hold of what would normally be a north pole of the field, or the reverse. No matter what the potential of the brushes when the machine falls into synchronism, if the field circuit be closed the machine will excite itself and continue to operate. This is in consequence of the fact that a continuous-current machine can excite itself in either direction.

It is of course essential, in general, that the machine excite itself in the usual direction before it is thrown on the load. If the attendant watches carefully as the machine approaches synchronism, and closes the field switch just as the voltmeter is starting a positive swing, the machine will build up in the proper manner. If a failure should be made in this, the field switch and then the main switch may be opened for an instant to allow the machine to drop out of step, and a new attempt may be made.

This is sometimes objectionable since it causes a sudden rise in voltage followed by a fall. Moreover it is necessary to break the large alternating current, thus causing considerable burning of the switch. It is perhaps better to provide a switch which will reverse the connections of the field to the armature. This has the effect of quickly bringing the field magnetism down to zero, since the current in the field is in the wrong direction to magnetize it. The machine will then drop out of step and by quickly reversing the switch as the voltmeter passes through zero, the machine can be brought to the proper condition for operation. The voltage of the machine is then adjusted to the same value as that of the bus-bars, and the main switch thrown in the same manner as for a direct-current machine.

231. Voltage Control.—As already shown, there is a definite relation between the voltage of the alternating- and the continuous-current brushes of a rotary converter. This ratio is somewhat modified by the drop in the winding when the machine is delivering current, but remains practically constant. In order, then, to be able to vary the voltage at the continuous-current brushes, it is necessary to have some means of changing the voltage *at the alternating end.*

The first thought is that it should be possible to regulate the voltage by changing the field current in the same manner as in the case of a direct-current machine. If, however, the regulation of the supply line and the generators is such that the al-

ternating-current voltage is really constant, this would have no effect. The result would be that the machine would draw a large current, leading if the field were strong and lagging if it were weak, and this current would demagnetize or magnetize the field so as to produce substantially the same generated alternating voltage. This matter has been fully treated in connection with synchronous machines. (See Art. 206.) A study of this section will show that the back alternating e.m.f. of the rotary does increase somewhat as the field is strengthened, but only to a minor extent. An attempt to control the voltage in this way would therefore have little effect in cases where the regulation of the alternating supply voltage is good.

In practice, however, there is sure to be more or less resistance and reactance in all transmission lines. The line voltage therefore acts as though it were flexible instead of rigid, and allows the potential at the receiving end to be adjusted more or less independently of that at the sending end. This is treated in detail in Arts. 174–176. It is shown there that a leading current produced by a strong field raises the voltage, while a lagging current with weak field lowers it. In many cases of rotary converter practice, the natural reactance of the line is insufficient, and it becomes necessary to install reactors in the converter substation.

When this method of control is adopted, it is customary to make the action automatic by using a compound winding on the rotary. As the load increases the flux is increased by the action of the series field, the machine draws a leading current, and the potential at the alternating-current rings is therefore raised or held constant according to the adjustment of the compounding and the reactance of the line. This arrangement of the apparatus is in almost universal use in the substations of electric railway systems. The variations of load are so frequent that it would be impracticable to attempt to follow them with hand regulation.

Sight should not be lost of the fact that by operating in this way, with leading or lagging current, the heating of the rotary is increased; or what is equivalent, its capacity is decreased since operation much of the time is at low power factor. This is not always a serious objection, as in railway work the rotary is frequently called upon to carry only a moderate average current, and its capacity is limited more by considerations of its possible overload than by its heating. It is certain, however, that the

combined efficiency of both the line and of the rotary is lowered by the increased current they are called upon to carry.

232. Use of Voltage Regulators.—It is also apparent that the alternating- and, consequently, the continuous-current voltage of a rotary could be changed by providing a number of different taps giving several voltages from the transformers. There are two serious objections to this plan. Rather elaborate arrangements would have to be made to avoid breaking the circuit and consequently throwing the rotary out of step in passing from one voltage to another. Moreover, the number of steps would need to be very large, if sufficiently close adjustment of voltage were provided. This plan is consequently not in practical use.

The most common method of varying the alternating-voltage is by means of a polyphase potential regulator. Such a machine is shown in Fig. 184. The mechanical construction is similar to that of an induction motor, except that the "rotor" is incapable of rotation except through a comparatively small angle. This movement is effected by means of a worm and wheel, and frequently a small motor is provided to allow the action to be controlled from a distance. The primary is connected across the line (usually three phase) while the secondary is connected in series with it.

It is clear from the action of the induction motor (see Art. 248) that a rotating magnetic field will be set up in the primary, and since the applied voltage is approximately constant, the flux also will be constant. A constant voltage will then be induced in the secondary. It would appear that a constant voltage would then be added to the primary voltage and that there would be no possibility of regulation. It must be considered, however, that as the secondary is turned, the *phase angle* of the generated voltage with respect to the primary changes. We then have the sum of two constant voltages but the angle between them may be anything desired. The vector sum of the two may then be anything from the arithmetical sum to the arithmetical difference of the two. If, therefore, the secondary should give a maximum voltage of say 50 volts, it would be possible to raise or lower the primary voltage by 50 volts, giving a range of regulation of 100 volts.

This method possesses great advantages. As noted, the apparatus can readily be arranged for control from a distance.

The changes of voltage may be made as small as desirable, and there are no sudden jumps in e.m.f. With any given alternating voltage, the field current of the rotary may be so adjusted that the power factor is unity and consequently the rotary and line will be operating at their best efficiency. This method of control is commonly adopted in substations on large lighting and power circuits in cities. The changes of load are gradual, and there is usually no difficulty in taking care of them by hand regulation.

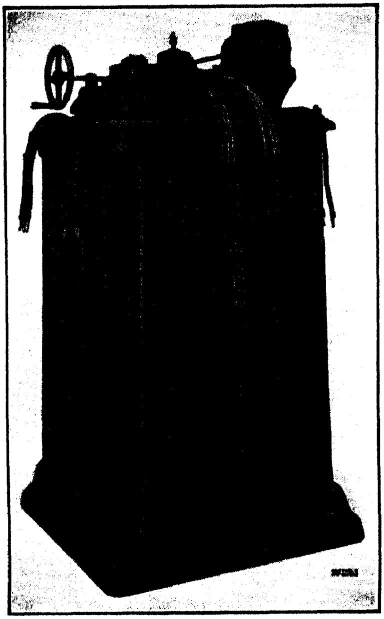

FIG. 184.

Another method that has been employed to some extent is to provide the rotary with a small synchronous machine mounted on the rotary shaft. The armature windings of this synchronous machine are connected in series with the leads of the rotary, and the field excitation is so arranged that any value of the field current from zero to a maximum in either direction can be readily obtained. The synchronous machine is mounted on the shaft in such an angular relation to the rotary armature that its e.m.f. is either in phase with or in phase opposition to the e.m.f. of the rotary. By proper manipulation of the

field rheostat of the synchronous machine, any desired voltage within the range of the auxiliary machine may be either added to or subtracted from the voltage of the rotary. Since the voltage of the line remains substantially constant, the voltage of the rotary will vary. The action may be made automatic if desired by compounding the field of the auxiliary machine with the continuous current delivered by the rotary. A machine of this type is shown in Fig. 185.

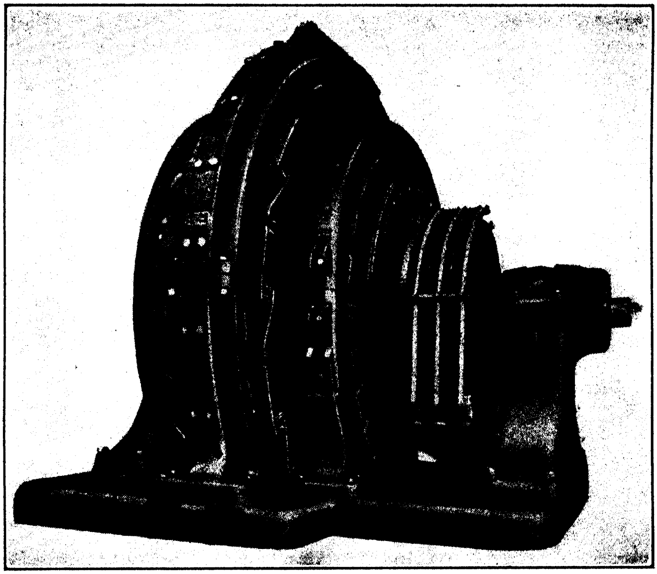

Fig. 185.

233. Split-pole Rotaries.—The type of machine known as the split-pole rotary offers many advantages over any other form of adjustable voltage rotary. In these, each of the usual field poles is split up into two or three parts, with separate windings. The two-part pole is the more common.

It has been pointed out that the maximum voltage at the alternating-current rings is the same as the steady value of the continuous voltage in the case of a two-ring or a four-ring rotary. This holds true, however, only when the continuous-current brushes are on the neutral point. If they are moved from this point, the continuous voltage becomes less than the maximum of the alternating voltage. It would, however,

not be practicable to attempt to vary the direct voltage by shifting the brushes since the sparking would be prohibitive. What is done is to keep the brushes stationary and shift the field, leaving a neutral space at the position of the conductors undergoing commutation.

An outline drawing of a two-pole rotary embodying these principles is shown in Fig. 186. Connections are made so that the current around the poles N' and S' may be varied in strength and may be passed in either direction. If N' and S' are of the polarity shown the direct voltage will be lowered. On the other hand, it will be raised if they are reversed. Changes in N' and S', however, have little effect upon the alternating voltage since the voltage induced by the flux from N' and S' is nearly at right angles to that due to the main flux. We may therefore obtain a wide range of direct voltage with but little change in the alternating voltage. The distribution of the flux changes somewhat as the voltage is varied, and the wave shape of the alternating voltage may be somewhat changed. This may, however, be largely avoided by proper shaping of the pole faces.

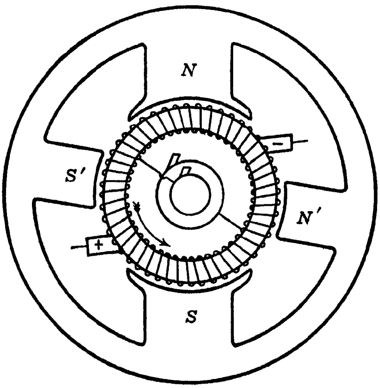

FIG. 185.

It is desirable that the wave shape be distorted as little as possible, since if the wave shape of the rotary is not the same as that of the circuit, they will fail to balance one another at all times, and idle currents will circulate in the windings. It will then be impossible to adjust the rotary to unity power factor, since additional current will always flow. The matter is not one of paramount importance, and some distortion can be introduced without serious effect. Some split-pole rotaries, in fact, operate upon this principle. Thus if we change from a flat top wave to a peaked one without changing the effective value of the alternating voltage, the direct voltage will in both cases be equal to the maximum value of the alternating voltage, and consequently it will be much greater with the peaked wave.

The split-pole rotary is a desirable piece of apparatus, since it combines in itself all the elements required for regulating as well as for converting the current. The cost of building a machine of this character should not be seriously higher than that of a plain rotary and presumably would be less than that of a rotary and potential regulator.

234. Heating of Rotary Converters.—In general, the heating of a rotary converter is much less than that of the same machine used as a continuous-current generator. Referring to Fig. 182, it will be seen that four times in each revolution an alternating-current brush is connected to a continuous-current brush and current passes directly into the continuous-current system *without passing through the armature winding.* If the taps are made on the back of the armature, the current passes through only one conductor at these times; and if the taps are connected to the commutator, it does not pass through any winding. If the rotary has three rings, there are six times in each cycle when the current can pass directly from one system to the other. With four rings there are eight, and with six rings, twelve such opportunities.

On the other hand, in the case of the two-ring rotary the alternating current is 41 per cent. larger than the direct current of a direct-current machine of the same rating. This tends to increase the heating and it can be shown that it is more than sufficient to offset the advantage just described, *particularly in the coils close to the tapping points.* With more than two rings (as is practically always the case in practice) the rotary heats less than it would as a direct-current generator operating at the same rating.

In accordance with these facts, rotaries are rated higher than the same machines would be as direct-current generators. The following table gives the relative ratings:

Direct-current generator	100 per cent.	
Two-ring rotary...	85 per cent.	(Practically never used.)
Three-ring rotary .	132 per cent.	
Four-ring rotary	162 per cent.	
Six-ring rotary. .	192 per cent.	

The foregoing are on the basis of unity power factors. If the adjustment of the rotary field is such that it takes a current at any other power factor, the alternating current will be larger and consequently the heating will be greater. The designer would therefore hesitate to take full advantage of the increased

rating, since during much of the time, the rotary will be operating at other than unity power factor.

235. Commutation of Rotaries.—The commutation of rotaries is usually good. One of the principal causes of commutation trouble with continuous-current generators is the fact that as the load on a machine is increased, the armature sets up a cross magnetizing effect, tending to distort the flux. The conditions for correct commutation are therefore interfered with and the machine tends to spark. This cross flux may be thought of as setting up poles on the armature at points approximately midway between the field poles.

If the rotary were used as a synchronous motor, there would likewise be a cross flux causing poles on the armature. These would be midway between the field poles if the machine were operating at unity power factor. The poles would, however, be of the opposite sign to those of a generator.

When the rotary is in normal operation converting alternating to continuous current or *vice versa*, it combines the functions of a continuous-current generator and a synchronous motor. It will therefore be apparent that the two armature fields will tend to oppose one another, and the net result will be a small field of the same polarity as would be present in a synchronous motor. There will therefore be practically no distortion of the magnetic field as the load comes on, and consequently there will be but little tendency to spark.

If a rotary hunts there is alternately a strong torque tending to accelerate the machine and to retard it. To produce these torques, requires a cross magnetic field, first in the one direction and then in the other. Under these conditions, there will of course be a decided tendency to spark, and rotaries sometimes flash over the commutator when hunting. The pole shoes, however, are frequently provided with damping grids as described in connection with synchronous machines, and these tend to prevent hunting. Moreover the generators supplying current to rotaries are usually driven by rotating prime movers such as steam or water turbines.

236. Frequency.—The frequency commonly adopted for the operation of rotary converters is 25 cycles. Sixty cycle rotaries are in use to a large extent but are much more difficult to design, and give somewhat less satisfactory results in operation. A simple calculation will suffice to show the difficulty. Suppose it

is desired to design a 60-cycle rotary for electric railway work. The rotary would be required to generate a voltage of about 600 volts as a maximum. It is found that in order to obtain satisfactory commutation, it is desirable that the average voltage between commutator bars should not exceed 15 volts. This requires forty commutator bars between neutral points on the commutator. This holds irrespective of the number of poles or the type of winding employed. It is likewise found undesirable that the commutator bars be made less than $\frac{1}{4}$ in. in width. The distance on the commutator, from one neutral point to the next, must then be at least $\frac{1}{4} \times 40 = 10$ in. Any given commutator bar must move the distance from one neutral point to the next in the time of one-half wave, in this case in $\frac{1}{120}$ sec. The surface speed of the commutator must then be $\frac{10}{12} \times 120 \times 60 = 6000$ ft. per minute. This is over a mile a minute, and is an undesirably high peripheral speed. The conditions for a lower voltage would be nearly as bad, as it would not be practicable to use so high an average value of the volts per commutator bar. The difficulty of building a machine for this frequency will therefore be apparent. These difficulties have, however, been largely overcome by careful design, and 60-cycle rotaries are now freely used.

237. Connections of Rotaries.—There are a great variety of possible connections of rotaries. In the following figures, the connections of the secondaries of the transformers only are shown. The primaries might be connected in any of the well-known ways. Thus, in Fig. 187, the primary connections to the three-phase line might be either in star or in delta, proper provision being made in the former case to hold the neutral at the proper potential.

In Fig. 187, the secondaries of the transformers are shown connected in *Y* and the three terminals led to the three slip rings of the rotary. In this figure the two continuous-current brushes are also shown. A dotted line is drawn from the neutral point of the transformers. This conductor may or may not be used. If it is employed, the voltage from either of the direct-current brushes to this neutral wire will be half of the generated continuous voltage. This wire may then be used as the neutral of a three-wire system.

Figure 188 shows a three-phase connection in which the secondaries are connected in delta. In this case, it is impossi-

ble to have a neutral wire as there is no neutral point on the transformers.

As previously explained, if the power factor is at or near unity, a considerable increase in the output or a decrease in the heating of the armature of a rotary results by employing six rings instead of three. The rotary is then known

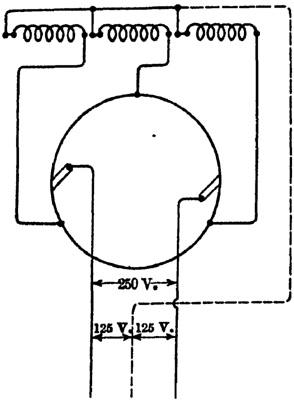

FIG. 187.

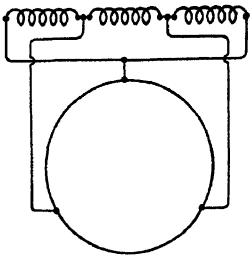

FIG. 188.

as a six-phase or six-ring rotary. The six-phase circuit to feed such a machine is readily derived from the secondaries of three transformers connected on a three-phase circuit. The primaries of the transformers may be connected either in Y or in delta. Instead of connecting the secondaries together, they

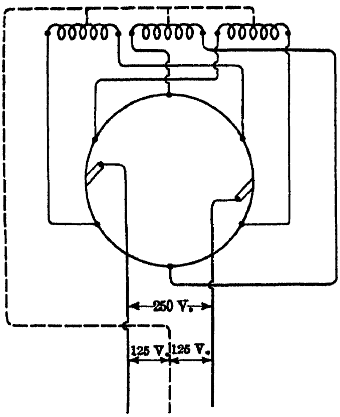

FIG. 189

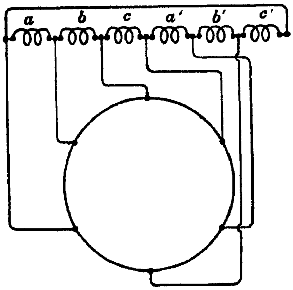

FIG. 190.

may be led to such rings that the ends of each secondary winding are connected to rings connected to electrically opposite points of the armature winding. This is known as the diametral connection and is shown in Fig. 189. The neutral points of the

three secondary windings may be connected together if desired as shown by the dotted line, to give a neutral point.

Figure 190 represents the six-phase delta connection. To use

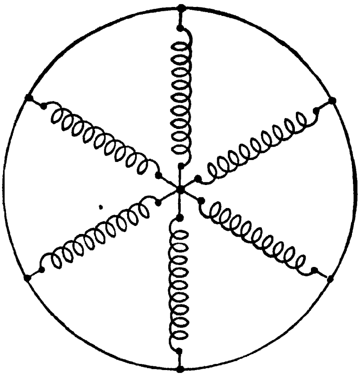

FIG. 191.

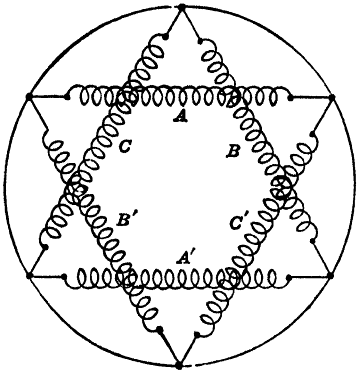

FIG. 192.

this connection, it is necessary that each transformer be provided with two secondary windings. Thus, the windings a and a' are on the same core. Likewise, b and b' and c and c' are on the same cores. The windings are connected as shown. The coils a and a', b and b' and c and c' are so connected that their e.m.fs. oppose one another. This might not be apparent from the dia-

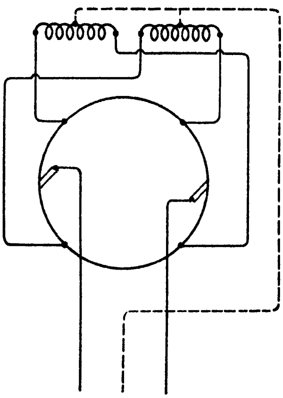

FIG. 193

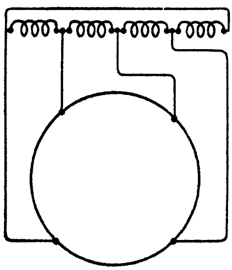

FIG. 194.

gram. To avoid crossings of the lines the connections here are as though they were helping one another.

Figure 191 shows a connection known as the double Y connection. This is the same as Fig. 189 provided the neutral points

in the latter are connected. Figure 192 is a double delta connection. As before, the coils a and a', b and b' and c and c' are on the same cores.

Two-phase or four-ring rotaries are not in common use. In Fig. 193, however, is, shown the diametral connection of a four-ring rotary, and Fig. 194 is a corresponding delta connection.

In these descriptions, it has been assumed that three transformers are used in the case of three phase, or two transformers for two phase. It will, however, be apparent that instead of the two or three transformers, one polyphase transformer might have been employed. This is often done, since as explained in the chapter on transformers, a polyphase transformer costs less than the corresponding number of single-phase transformers.

238. Rotary Converters Versus Motor-generator Sets.—Besides the method of converting from an alternating to a continuous current by means of a rotary converter as described in the preceding pages, it is also possible to accomplish the same result by means of a motor-generator set. The two machines are commonly mounted on the same shaft, and the set is frequently provided with only two bearings.

The alternating-current motor of such a set may be either an induction motor or a synchronous motor. The latter is the one usually employed. The two most troublesome points in connection with a synchronous motor are the matter of starting and the provision of direct current for the fields. With a set changing from alternating to direct current, both of these objections usually disappear. Direct current is almost always available, since most direct-current systems include a storage battery on the line, or if not, only one machine need be started from the alternating-current end, and the rest started from the continuous current of the first machine. In any event, the machine is started without load and starting is therefore a comparatively easy matter. Direct current for the field excitation of the synchronous motor is, of course, available as soon as the set is in motion and ready for it. The synchronous machine is slightly more efficient than the induction machine, and the ability to regulate the power factor is of great advantage. It is, however, common practice when there are several motor-generator sets in the same substation to provide one of them with an induction motor, and the remainder with synchronous motors. This insures one set which can be readily started at any time.

239. Cost.—In comparing the advantages of the rotary converter and the motor-generator, the first consideration is the relative costs of the two forms of apparatus. However, it is necessary to consider not only the rotary on the one hand and the motor-generator set on the other, but also the auxiliary apparatus. The first point is that the rotary almost invariably calls for the use of transformers to reduce the line e.m.f. to that required by the rotary. The synchronous motor of the motor-generator set, on the other hand, can in many cases be wound to operate at the line potential. The rotary, in addition to the transformers, requires in general some method of regulating the direct voltage. This, as previously explained, may take the form of reactors connected in the line, of an induction regulator, a special synchronous machine mounted on the rotary shaft or of a split-pole rotary. No matter what the method adopted, there will be some extra cost on account of the regulating arrangements. On the other hand, the motor-generator set consists of two machines, and each of these must be somewhat larger than the rotary which would replace both of them. Considering everything, the cost of the motor-generator set will be appreciably higher than that of the rotary.

240. Frequency.—As already pointed out, rotaries are not very well adapted to operation on 60-cycle circuits. The motor-generator set is entirely relieved of this difficulty as far as the direct-current generator is concerned, as the generator may have any convenient number of poles, irrespective of the number of poles of the synchronous machine. It is, moreover, practicable to design synchronous motors for either 60 or 25 cycles. For 60-cycle installations then, the motor-generator set is frequently to be preferred.

241. Efficiency.—In the matter of efficiency, the rotary has a distinct advantage as it has the losses of only one machine instead of two. The losses in the transformers and in the regulating apparatus will offset this advantage to a certain extent, but the efficiency of the rotary and the necessary auxiliary apparatus will average from 2 to 3 per cent. higher than that of the motor-generator set.

242. Regulation.—The matter of the voltage regulation in the two methods of transformation is of importance. There are two classes of variations to which the incoming power on the alternating-current line is subject, namely, variations in voltage and varia-

tions in frequency. The latter is the easier to take care of, since in the case of large stations, particularly if provided with turbine-driven units, the frequency is remarkably constant. This is, of course, to be expected, since on account of all of the machines operating in synchronism, small variations of the individual governors will have but little effect upon the frequency of the current generated by the station. It is not so easy to prevent fluctuations of the voltage at the receiving end, particularly when the power is supplied to a number of substations, each with a varying load.

The motor-generator set is *not affected at all by variations of voltage* unless the fluctuations are so violent that the machines will not remain in step. As long as the alternating voltage remains anywhere near constant, the synchronous motor will continue to operate at the same speed, and consequently the voltage of the continuous-current machine will not be affected by the variations in the voltage of the supply.

In the case of the rotary, on the other hand, the voltage at the continuous-current end bears a nearly constant relation to that of the alternating-current end. Any variation, therefore, in the voltage of the supply is immediately reproduced in the same proportion at the direct-current end. In this respect, the rotary is distinctly inferior to the motor-generator set.

If variations of frequency are considered, the case would be exactly reversed. The rotary would not be affected at all, while all fluctuations would be reproduced at the direct-current commutator in the case of the motor-generator set. However, variations of frequency are not of importance, to nearly the same extent as are those of voltage.

243. The Cascade Converter.—It will be shown in the chapter upon the induction motor, that it is possible to operate two induction motors in cascade or concatenation. One of the two motors is connected to the line in the usual manner. This motor must be provided with a wound rotor. The current for the primary of the second motor is supplied by the secondary of the first. The secondary or rotor of the second motor may be either of the wound-rotor type or of the squirrel-cage type. If the two machines have the same number of poles, the set will rotate at slightly less than half the synchronous speed of either machine. In any event, the speed will be that corresponding

to a machine having as many poles as the sum of the poles on the two machines.

There is no reason why we may not substitute a synchronous motor for the second induction motor. In this case, the set will rotate at the synchronous speed corresponding to a machine of the total number of poles. If the field of the synchronous machine is so adjusted that this machine operates at unity power factor, the power factor of the combined set will be somewhat less than unity, as lagging current will be required to produce the flux of the first machine. If the field of the synchronous machine is made stronger, the combined set may be caused to have a power factor of unity, or to take a leading current.

If in place of the synchronous motor a rotary converter is substituted, the operation will be essentially the same. Instead of taking mechanical power out of the machine, current may be taken from the commutator of the rotary. If the two machines have the same number of poles, the speed will be half the synchronous speed of either. The frequency of the current supplied to the rotary will be half the line frequency. Thus if the line frequency is 60 cycles, the frequency of the current applied to the rotary will be 30 cycles. Hence, a machine of this character removes most of the difficulties connected with 60-cycle converters.

In a cascade converter, if the two machines have the same number of poles, half of the power received by the induction motor is transformed to mechanical power and appears at the shaft. The other half is transformed to electrical power at half of the line frequency. The rotary converter, therefore, receives half of its power as mechanical power through the shaft, and the other half as electrical power. It therefore combines the functions of a rotary converter and those of a continuous-current generator.

It has been shown that both the induction motor and rotary converter are improved by increasing the number of phases. There are obvious objections to employing a great number of phases in primary circuits. The circuit between the secondary of the induction motor and the rotary, however, is never opened, and these objections lose their weight. It is therefore customary to wind the secondary of the motor for from six to twelve phases. There is no need of slip rings on either the induction motor or on the rotary, since by using only two bearings for the set,

it is possible to take the current directly from one to the other. The cross connections may therefore be made numerous without complication.

Such a set may be started by applying current to the primary of the induction motor in the usual way. The starting torque will, however, be small, since the losses in both rotors will be small, but it may be increased if necessary by opening up the secondary winding of the induction motor at the opposite ends from these connected to the rotary windings, and inserting

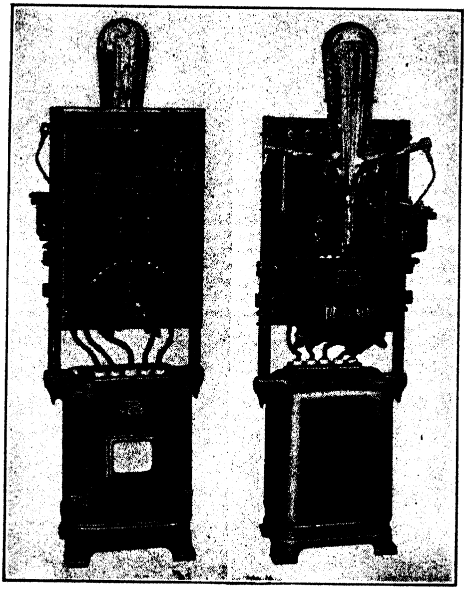

FIG. 195.

resistance for starting. If direct current is available, the set can be started from the direct-current end.

The principal advantage of such a set over a rotary converter is that it is well adapted to operation on 50 or 60 cycles. It is also easy to control the voltage if only a moderate variation is desired, since the primary and secondary of the induction motor will usually have sufficient reactance to allow considerable voltage regulation by changing the field current of the rotary.

244. The Mercury Arc Rectifier.—Practically all transformation of alternating to direct current on a large scale is accom-

plished by means of the rotary converter or the motor-generator set. For certain classes of work where only a small amount of power is to be transformed, the low first cost and the simplicity of operation of certain other devices may make them preferable. The mercury arc rectifier falls within this class.

The general appearance of a mercury arc rectifier is shown in Fig. 195. The principal element is a vessel, usually of glass, from which the air has been exhausted. The vessel has one electrode formed by a pool of mercury and two other electrodes of iron or graphite. With no current passing, the resistance between any two electrodes is very high. If, however, an arc is once started *the resistance for a current passing from the iron or graphite electrode to the mercury is low;* but the resistance to current passing in the opposite direction is very high. In fact, practically no current can pass even at a pressure of several thousand volts.

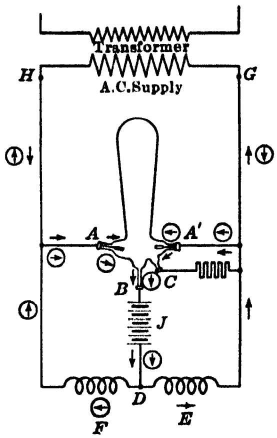

Fig. 196.

The connections are shown in Fig. 196. The coils *E* and *F* are constructed so as to have high reactance and low resistance. A slight tilting of the tube causes the mercury to bridge over from *B* to *C* and current flows through this bridge and the resistor connected to *C*. When the connection through the mercury bridge breaks, a flash is produced inside the tube and current can at once flow through the main circuit. The auxiliary circuit through *C* and the resistor is then opened.

If at any given instant the terminal *A* is positive, current will flow from it through the tube and out at the terminal *B*. It then passes through the battery *J*, or other load, the reactor *E* and back to the transformer. An instant later the transformer has reversed its polarity, the terminal *A'* becomes positive and the current passes from *A'* to *B*, the terminal *A* remaining inactive. Thus the direction of the current is always the same through the battery *J*. The function of the reactors is to prevent sudden changes of the current. Thus the times when the current would be zero may be bridged over and a nearer approach

to a steady continuous current produced. This also lessens the liability of the arc to go out while the current is passing through zero, thus stopping the action. Figure 197 shows an oscillogram of the applied alternating voltage and the resultant direct current. It will be seen that the latter is by no means steady, although it is all in the same direction.

The fact that the current delivered is not steady is a disadvantage if an attempt is made to use it to drive a continuous-current motor. Such a motor can be operated but with considerably increased losses due to hysteresis and eddy currents. A more serious objection is the fact that if the current drops below a certain percentage of its full-load value, the arc will go

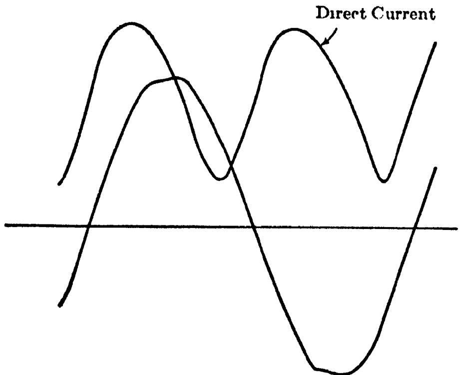

FIG. 197.

out and the action cease. If the load on a motor becomes very light it is therefore likely to stop.

Neither of these objections apply to the charging of small storage batteries and mercury arc rectifiers are extensively used for this purpose in places where continuous current is not available. The rectifying outfit is reasonable in first cost, requires little attention, and gives good efficiency.

At the present time, serious efforts are being made to develop the mercury arc rectifier in larger sizes, with particular reference to its use on electric locomotives. It would then be possible to transmit power to the locomotive by means of single-phase currents. The voltage would be stepped down by means of transformers on the locomotive, and the current changed to direct current by the rectifier.

PROBLEMS

97. An eight-pole rotary converter has 480 conductors on the armature connected to a 240-bar commutator. The winding is a simplex lap. One of the rings is connected to commutator bar No. 1. To what other commutator bars must this same ring be connected?

98. If the machine is a two-ring rotary, to what commutator bars must the other ring be connected?

99. In the case of a four-ring rotary, state the connections of the rings to the commutator bars.

100. What will be the connections for a three-ring rotary?

101. A four-ring rotary generates a d.-c voltage of 250. The a.-c. supply voltage is at 44,000 volts. What is the ratio of the turns on the transformers, making no allowance for the drop in the rotary? What is the correct ratio for a three-ring rotary?

102. A three-ring rotary has a d.-c. voltage of 600. The supply voltage is 22,000. Allowing a drop of 5 per cent. in the rotary windings, and assuming that the transformers are connected in delta on the primary side and in star on the secondary, what is the ratio of the primary to the secondary turns?

CHAPTER XVIII

THE INDUCTION MOTOR

245. General Description.—A view of a complete induction motor is shown in Fig. 198 and an "exploded view" giving an idea of the construction of the various parts is shown in Fig. 199. The stationary part is ordinarily known as the stator while the rotating member is called the rotor. An idea of the relative dimensions and arrangement of the stator laminations and slots may be gained from Fig. 200. The air gap of an induction motor

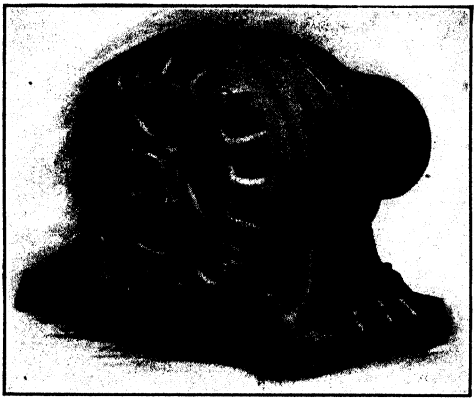

Fig. 198.

is always as short as it is safe to make it and in consequence the shaft and bearings are heavy.

246. The Stator.—The stator, both in its mechanical construction and in its winding is the same as the armature or stator of a synchronous machine. In fact, it is always possible to remove the rotor from an induction motor and by substituting for it a revolving field, to convert the machine into an alternator or synchronous motor.

247. The Rotor.—The rotor shown in Fig. 199 is of the type known as a squirrel-cage rotor. The "winding" consists of

bars of copper passed through the rotor slots and all connected together at the ends by rings of copper or brass. These latter are known as end rings. The insulation on the rotor bars is light and in many cases is omitted altogether.

FIG. 199.

Another form of rotor known as a wound rotor is shown in Fig. 201. The winding on this is identical with that of a rotating armature alternator and is therefore the same in principle as that of the stator. The ends are brought to three slip rings. This winding is always three phase although the stator may be wound for one, two or three phases. It is essential that the rotor winding be polyphase, and since the three phase is simpler than the two phase and at the same time is better, it is always used. A slight advantage could be gained by winding for a greater number of phases, but the gain is not great enough to offset the complication incident to using more than three rings.

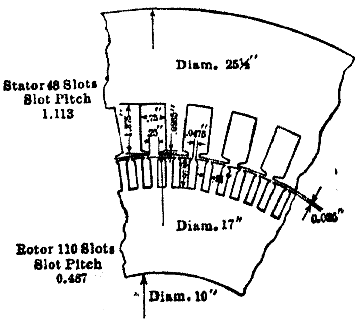

FIG. 200.

248. The Rotating Magnet Field.—On applying current to a polyphase stator, a rotating magnetic field is set up. This subject has been fully treated in Chaps. 14 and 16 in connection with the operation of synchronous machines. It is unnecessary

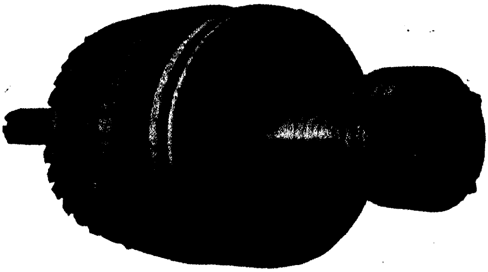

FIG. 201.

to repeat the discussion here. The facts, however, should be carefully reviewed before proceeding further.

249. The Production of Current in the Rotor.—Figure 202 shows a portion of the surface of a rotor. The shaded parts are supposed to represent the "poles" of the stator magnetism as they pass over the rotor winding. It is understood that the

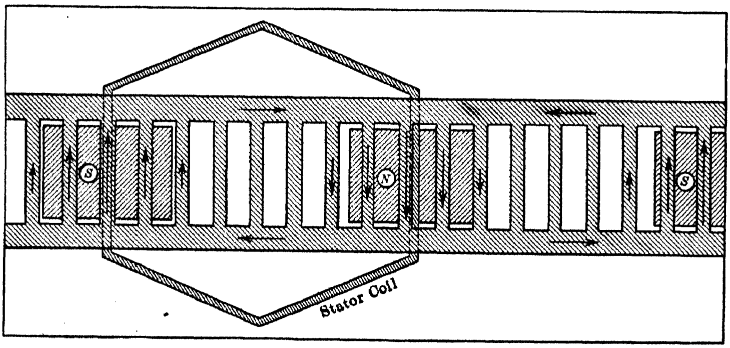

FIG. 202.

magnetism is not confined to definite rectangles as shown, but varies from a maximum at the middle of the rectangles to zero midway between them. A field magnet rotating outside the rotor would give essentially the same results.

Suppose the rotor to be at rest. As the poles pass over the surface they will induce e.m.fs. in the rotor bars. These e.m.fs.

will be *greatest at the points where the flux is greatest* and will, in fact, be proportional to the flux at any point. The e.m.fs. in the various bars are indicated in Fig. 202 by means of arrows. These are drawn longest directly under the center of the poles and gradually shorter as the bars midway between the poles are approached. The frequency of the e.m.fs. in the rotor when the latter is at rest, will be the *same as the line frequency.*

250. Rotor Current.—If the rotor were non-inductive the same arrows used in Fig. 202 to indicate the e.m.fs. of the various bars might also be considered as indicating the currents. This however is not the case. The current as it passes through the bars and through the end rings, sets up leakage fluxes. These fluxes set up an additional e.m.f. in the rotor bars and therefore cause the current to lag behind the e.m.f. As a consequence *the sheets of current will be displaced* some distance to one side or the other of the sheets of e.m.f. If the flux be considered as moving from left to right, the sheets of current will lie to the left of the sheets of e.m.f.

251. Production of Torque.—Whenever a current lies across a magnetic field a force is produced tending to force the current out of the field or to force the field away from the current. The induction motor has this condition and consequently the motor tends to start. *This starting torque would be a maximum if the sheets of current coincided with the sheets of flux* and consequently of e.m.f. Unfortunately, as just explained, this is not the case. The sheets of current are displaced to one side so that the maximum of the current sheet does not coincide with the maximum of the flux sheet. As the current sheets are displaced it will be evident that in any given flux sheet some conductors will be carrying current toward the observer and other conductors carrying current from him. The forces on these two sets of conductors will be in opposite directions and consequently the torque will be reduced. If the lag of the current were 90°, the net torque would be zero. An exactly similar condition was found in connection with the study of the synchronous machine, and it would be well to refer again to Figs. 159, 163, and 164 and the accompanying descriptions.

In the induction motor with squirrel-cage rotor the reactance of the rotor is usually greater than the resistance. The lag at starting is therefore great, usually about 60° and the starting torque for a given current and flux is consequently small. *To*

start a load requiring a torque equal to the full-load torque of the motor requires about five times full-load current.

252. Influence of the Resistance of the Rotor upon Starting Torque.—If the resistance of the rotor of an induction motor were zero, the lag of the current behind the e.m.f. would be 90° and the starting torque would be zero. The greater the resistance of the rotor, the greater the starting torque *for a given current, but the less the current.* There is a "happy mean" and it is reached when the resistance is equal to the reactance or the lag of the current is 45°. It would not be desirable to have the resistance of the rotor of an ordinary squirrel-cage induction motor so great as this as the efficiency of the motor would be very low. It is, however, necessary in most cases to make the resistance far greater than it need be, that is, ordinarily the current density in the end rings is made far higher than that in the bars, and moreover the end rings are made of high resistance material such as cast copper or brass. This added resistance is a detriment after the motor is up to full speed. It is a necessary evil, used only to improve the starting torque.

253. The Use of the Wound Rotor.—By using an induction motor with a wound rotor it becomes possible to adjust the resistance of the rotor circuit to the proper value to suit the conditions. Thus the resistance may be made such as to give the maximum starting torque (or may be made even greater so as to take as small a current as possible and still start the load) and this resistance may be cut out as the motor attains speed so that the machine is at all times operating under the best possible conditions. A wound-rotor induction motor costs approximately 35 per cent. more than a corresponding squirrel-cage machine, but the added cost is frequently justified in the case of motors which have to start frequently under heavy loads, or in the case of those whose speed must be adjustable.

254. Conditions at Normal Speed.—As just shown, when current is applied to the stator of a polyphase induction motor the rotor develops torque and the motor will start from rest. As the motor speeds up, since the rotor is moving in the same direction as the rotating magnetic field the rate of cutting of the magnetic lines becomes less. Consequently both the e.m.f. induced in the rotor bars and the frequency of the e.m.f. and current become less and finally reach zero at synchronism, that is,

when the rotor surface and the flux are moving at the same speed. Synchronous speed is then the *limiting speed* of the motor. Its speed will always fall short of synchronous speed by an amount such that *sufficient current will be induced in the rotor to develop the torque necessary to keep the motor in rotation.* If the external load is zero, only enough torque will be required to overcome the friction and the speed will drop below synchronism by only a fraction of 1 per cent. As the load is increased, the speed decreases and this can continue until the load becomes so great that the motor is unable to carry it and it will "pull out" and stop. When the motor is in operation it is said to have a certain amount of slip. Slip is the difference between the synchronous speed and the actual speed divided by the synchronous speed. Thus, if the synchronous speed were 1200 r.p.m. and the motor were operating at 1100 r.p.m. the slip would be 8.33 per cent. The slip at full load may be as low as 1 per cent. in the case of large motors or as great as 15 per cent. for very small ones.

Imagine a motor operating at full load with a slip of say 5 per cent. If the line frequency is 60 cycles per second, the frequency of the current in the rotor will be only 5 per cent. of this or 3 cycles per second. The resistance and the inductance of the rotor are the same as at standstill but the reactance (which is equal to 2π times the inductance times the frequency) will be only 5 per cent. as great as at standstill. Consequently, the lag of the rotor current behind the e.m.f. will be small and the torque per ampere in the rotor will be much larger than at standstill. In other words, the power factor of the rotor at load will be far higher than at standstill.

255. Speeds of Induction Motors.—As has been pointed out, the rotor of an induction motor tends to revolve at the same speed as the rotating magnetic field. In practice the actual speed under full load will be from 1 to 5 per cent. less than this. This fact limits the possible speeds of induction motors. Thus in a 60-cycle, two-pole motor the flux revolves sixty times a second or 3600 r.p.m. No higher speed than this is possible (with 60 cycles current) since we can not have less than two poles. The next possible number of poles is four giving a speed of 1800 r.p.m. The following table gives the common speeds:

Number of poles	R p.m. 60 v.	R.p.m. 25 v.
2	3600	1500
4	1800	750
6	1200	500
8	900	375
10	720	300
12	600	250

Two-pole motors are rarely used, since they are difficult to wind. The above speeds apply also to synchronous machines.

256. The Induction Generator.—As we have just shown, it is impossible for the induction motor to operate as a motor at a speed above synchronism. If, however, we apply power to the shaft by means of a steam engine or otherwise, it may be forced to rotate faster. The rotor bars will again be cutting the flux but in the opposite direction from that in the machine operating as a motor. The direction of the e.m.f. at any instant will therefore be reversed, the current will flow in the opposite direction and the torque will be reversed. As a consequence, instead of the torque being in the direction of the rotation as in a motor, it will be opposed to it, or the machine will operate as a generator.

The machine differs, however, from the ordinary generator in several respects. It will not generate alone, that is, it will operate only on a line already connected to a synchronous machine. Both the frequency and the voltage of the line will be determined *by the synchronous machine.* All regulation of the voltage must therefore be accomplished by means of the field rheostat of the synchronous machine.

On acount of these facts the field of application of the induction generator is rather limited. Some electric locomotives are operated by means of induction motors. When the train is descending a grade at a speed slightly above that corresponding to the synchronous speed of the motors, the latter automatically become generators, and return power to the line.

Induction generators are occasionally used in the development of small water powers. Thus a small fall may be available on or near one of the lines of a power-distributing system. The available power may be so small that it is not practicable to install a power house with attendants to regulate the voltage, etc., but it may be worth while to install a simple water-wheel, probably without a governor, and an induction generator. An overspeed device would be desirable to disconnect the induction generator

from the line and stop the water-wheel in case the power supply to the line should fail. The generator would be run up to speed, connected to the line and the gate of the waterwheel opened. The installation would then be left alone except for a daily inspection. The frequency and voltage would be controlled from the other stations operating on the line. The induction generator would continue to supply to the line an amount of power corresponding to the power developed by the fall.

Another use is in connection with the generation of continuous current by means of turbine-driven generators. The continuous-current, turbine-driven generator in large sizes presents great difficulties in design. They are consequently manufactured only in comparatively small sizes. A synchronous generator may be built and its output rectified by means of a rotary converter. A somewhat simpler outfit consists of an induction generator and a synchronous converter. The operation is particularly convenient since there is no field on the generator to be adjusted. The induction machine will not excite itself, and it is necessary that the rotary converter be driven at a moderate rate of speed by some outside source of power before it is connected to the induction machine. There is of course no necessity for synchronizing the two machines nor is it even necessary that the speeds be very nearly correct before they are connected together.

257. Vector Diagrams of the Induction Motor.—Consider a wound-rotor induction motor in which there are the same number of turns on the stator and on the rotor. If there is no connection between the slip rings, that is, no resistors connected across them, and current is applied to the stator the machine will not revolve. If the voltage across the slip rings be tested, we shall find that there is present a three-phase voltage equal to the primary voltage. The frequency will be the same as the frequency of the current applied to the stator. If the rotor is held so that it can not revolve, current could be taken from the slip rings and used to supply a load. The machine would then be acting as a transformer. Transformers are not built in this manner as the usual construction is cheaper and better.

If the motor is allowed to rotate at half speed, the rotor bars will be moving half as fast as the rotating flux, and half voltage and half frequency will be obtained at the slip rings.

Half the power supplied to the primary (neglecting losses) will appear as electric power in the secondary circuit, the other half will appear as mechanical power at the shaft.

The induction motor may be regarded as a general type of transformer, capable of transforming electric power either to mechanical power or to electirc power or to both at the same time. The ordinary transformer then is a special form of transformer, not capable of receiving or of giving out mechanical power.

These considerations lead one to expect that the vector diagrams of the induction motor will be very similar to those of the transformer, and in fact this is the case. Thus, Fig. 203 shows the conditions at no load. The flux is indicated by the vector marked Φ. The direction of the e.m.f. induced in both the stator and in the rotor is indicated by the vector marked E_r. At no load the motor operates practically in synchronism. The rotor bars therefore do not cut the flux and the rotor e.m.f. is practically zero. The rotor current is also practically zero and may be neglected. The current in the stator winding produces the flux and is therefore nearly in phase with it. It, however, leads the flux by a slight angle on account of the losses in the motor and is represented by I_n. This is called the no-load current. To overcome the back e.m.f. of the stator E_r, we must apply an equal and opposite e.m.f. This is represented by E_s. In addition to this, the applied e.m.f. would have a component in phase with the current to overcome the ohmic drop. The construction would be the same as in the case of the transformer (see Fig. 137), and it is unnecessary to complicate the diagram by introducing it.

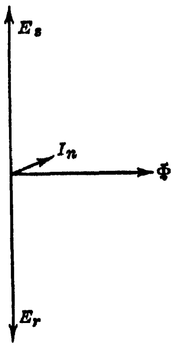

FIG. 203.

258. Full-load Diagrams.—As soon as we apply a load to the motor, the rotor slows down. The rotor bars therefore cut the revolving magnetic field inducing an e.m.f. in them and a current flows. This current will lag somewhat behind the rotor e.m.f. The lag, however, will not be great, since the frequency in the rotor will be low. The rotor current may be represented by the vector I_r (see Fig. 204). As in the case of the transformer, the *resultant* of the stator and the rotor current must

always be equal to the no-load current and this latter is very nearly constant. The magnetizing current may therefore be represented by I_n as before. The stator current will then be represented by I_s, whose magnitude and direction are such that the resultant of I_s and I_r is I_n. The stator current of the motor under load does not lag nearly so much as in the unloaded motor. In other words, the power factor is better.

259. Diagram Representing the Conditions at Start.—If full voltage be applied to the stator of an induction motor while the rotor is at rest, the e.m.f. induced in the rotor is of course large since the rotating magnetic field is moving at full speed while the rotor is at rest. The rotor current is therefore large. The frequency in the rotor is the same as the line frequency. Hence

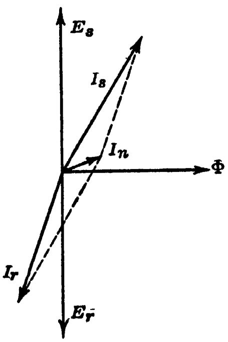

FIG. 204.

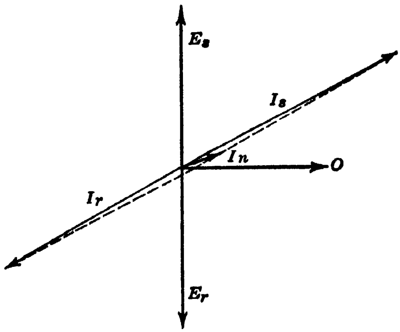

FIG. 205.

the lag of the rotor current will be large, and the current will be kept from becoming very great on account of the large reactance. The diagram of the motor during starting is shown in Fig. 205. The large lag of the rotor current results in a corresponding lag of the stator current and the power factor of the machine is low.

260. The Circle Diagram.—If a careful test is made of an induction motor under loads varying from zero to such a load that the motor is unable to start, the results may be embodied in a diagram similar to that of Fig. 206. The voltage is kept constant and the amperage and wattage measured for each load. The power factor, and the angle of lag can be readily computed from the above data. Thus the angle of lag at no load may be laid off as E_sOA and the value of the no-load current may be

represented by the length OA. Similarly at full-load the angle of lag may be E_sOB and the magnitude of the current may be OB. At start the lag is represented by the angle E_sOD and the current by OD. At no load we may take the rotor current as being zero. At full load (assuming that the motor is wound with a one to one ratio) it will be AB and at start AD.

If the work is carefully done it will be found that all points thus taken lie approximately upon a circle having as diameter the line AG parallel to OF. The proof of this fact while not difficult, lies outside the scope of this work. In determining the circle in practice, usually only two points are taken, those for no load and for the starting condition. The values for the starting condition are usually determined at a reduced voltage and the quantities multiplied by the proper factors to give full

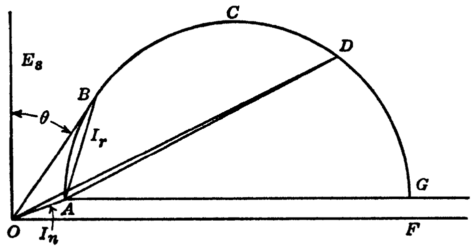

FIG. 206.

voltage values. Thus, but little power is required in the test of the motor.

Two important characteristics of the motor may be at once determined from the diagram. The maximum power factor of the motor will be attained at such a load that the current vector drawn from O is just tangent to the circle, since then the angle of lag will be a minimum. The maximum *input* to the motor will be at such a load that the current vector is represented by OC. The power component of the current is the projection of the current vector upon the voltage vector OE_s and this will obviously be a maximum for the point shown. The maximum output of the motor may be determined approximately if the efficiency is known.

By simple additions to the circle diagram one can readily determine the efficiency, slip, torque, starting torque and other characteristics of the motor. However, these will not be taken up here.

261. Starting Devices for Squirrel-cage Motors.—Squirrel-cage induction motors of 5 hp. and less are usually started by connecting them directly to the line. A double-throw switch should preferably be used with either no fuses at all or large fuses connected to the starting clips, and smaller fuses connected to the running clips. This is done in order that the great rush of current at the start may not blow the fuses. The switch should be provided with a spring so that it is impossible to leave it in the starting position.

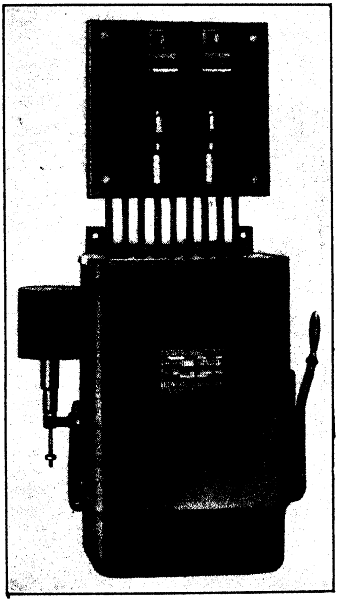

FIG. 207.

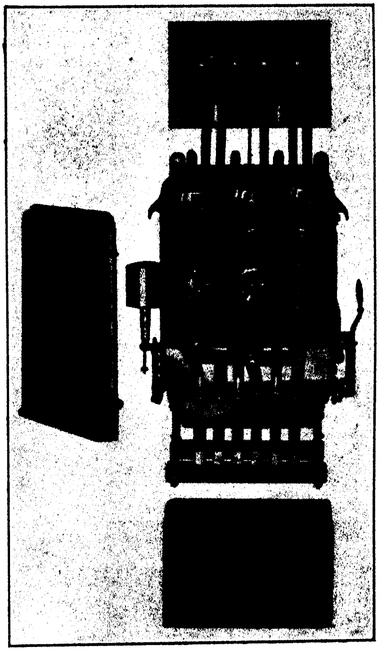

FIG. 208.

In the case of large motors, it is customary to provide some form of device to reduce the pressure applied to the terminals of the motor during the starting period. This is done more to protect the connected apparatus against the danger of too rapid acceleration, and to lower the line current rather than to protect the motor. The latter would not be at all injured by being directly connected to the line. The inductance of the motor is such that only from five to eight times normal current would flow,

and acceleration would be so rapid that no serious heating would take place during starting.

262. The Auto-starter.—Figure 207 shows the external appearance of an auto-starter or compensator and Fig. 208 illustrates the same starter with the cover and oil well removed. The connections are shown in Fig. 209. The apparatus contains a three-phase transformer, having three legs upon which the coils are wound. The three coils are star connected and each coil is provided with three taps so that three different voltages are available. The sliding contacts in the middle row are mounted on a drum. This can be rotated through a small angle so that contact can be established between the middle row and either the top or the bottom row. When the handle is thrown to the

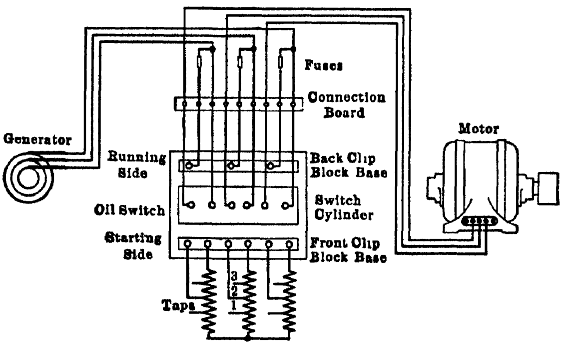

Fig. 209.

starting position the three terminals of the auto-transformer are connected to the line wires back of the fuses, and the wires leading to the motor are connected to the three taps on the transformer. When the motor has attained nearly full speed the handle is thrown to the running position. This disconnects the auto-transformer from the line, and connects the motor terminals directly to the line through the fuses.

Generally only one starting position is provided. This is in marked contrast to continuous-current practice in which provision is made for cutting out the starting resistance very gradually. Two things make this possible; the high reactance of the induction motor which prevents great rushes of current, and the fact that the speed depends primarily upon the frequency. The motor will therefore attain practically full speed upon the

reduced starting voltage and may then be thrown directly upon the line.

Three sets of starting taps are provided in order that the voltage applied at start may be made as low as possible and still start the load. The starting torque of an induction motor is proportional to the square of the voltage. The starting torque with full voltage applied is usually from 150 to 200 per cent. of full-load torque. Taking the larger figure as applying to a particular motor, it will be seen that if one-half of the line voltage is applied the torque will be reduced to one-fourth of its maximum value or 50 per cent. of full-load torque. A voltage of 70.7 per cent. of normal would give full-load torque. The three voltages supplied are about 60, 75, and 90 per cent. of line voltage.

Fig. 210.

Auto-starters or compensators as they are sometimes called, are frequently equipped with no-voltage release and occasionally with overload release as well. In starters for 2200 volts or more, the overload release is practically always incorporated with the starting switch.

263. Resistance Starters for Squirrel-cage Motors.—A lower voltage for starting an induction motor may also be obtained by inserting resistors in the three lines supplying the motor. A starter of this type is shown in Fig. 210. Such starters are somewhat cheaper to construct and are simpler to repair than the auto-starter type. They are not so efficient since there is a loss in the resistors, but this is a very small matter except in the case of large motors which are frequently started. Central stations frequently object to this form of starter on the plea that a large current is taken from the line, and that therefore the starting of the motor interferes with the regulation of the line.

Undoubtedly more current is taken although the difference is not great, particularly in the case of a heavily loaded motor where perhaps 90 per cent. of full voltage is required in any case. The amount of *wattless current,* however, is the same in the two types, and it is principally the wattless current which interferes with the line regulation. Tests show no appreciable difference in the line disturbance produced by the two types.

264. Star-delta Starters.—Some motors may be started by connecting the stator windings in star, to start; and in delta for running. This means that during starting we have impressed across each winding $1 \div \sqrt{3}$ or 57.7 per cent. of normal voltage. Since the starting torque varies as the square of the applied voltage, it will be seen that the starting torque will be only one-third of the maximum which the motor can develop. This in many cases will be too small, and since there is no possibility of an intermediate step, it constitutes an objection to this method. When the starting load is light or may be made so without great inconvenience, this is an excellent method of starting.

265. Starters for Motors with Wound Rotors.—In starting these motors full voltage is applied to the stator and a three-phase resistor is connected to the three slip rings. This cuts down the current in the rotor, and consequently that in the stator. At the same time, the current and the voltage are brought more nearly in phase and so the starting torque is improved. Such a motor will therefore develop more torque and at the same time take less current from the line than a corresponding squirrel-cage machine. It is therefore to be preferred when frequent stopping and starting are required. The efficiency is usually a little greater than that of a squirrel-cage machine, particularly in the larger sizes, but the power factor is materially lower and the motor is more costly. The squirrel-cage machine is to be preferred in most cases. More detailed information will be found under Art. 270.

266. Adjustable-speed Induction Motors.—The induction motor has many advantages over the continuous-current motor, such as simplicity and durability, but for adjustable-speed work it is at a serious disadvantage. This arises primarily from the fact that the speed of an induction motor is dependent upon the *frequency,* while that of a direct-current motor depends upon the *voltage* or what is equivalent, upon the flux passing through the armature. It is obviously easy to change the voltage or the flux, but to change the frequency presents great difficulties.

267. Changing the Number of Poles.—It is possible to make one change which is equivalent to varying the frequency, namely, changing the number of poles. Thus a certain stator may be provided with two entirely distinct windings, one arranged for say twelve poles and the other for eight. Operated on 60 cycles, the first would give a synchronous speed of 600 r.p.m. and the latter of 900. If the rotor were of the squirrel-cage type, it would serve equally well for both stator windings. If a wound rotor were used, it would be necessary to provide it as well with two windings and consequently with five slip rings, one ring being common to the two windings. This method gives a reasonably good motor with two speed changes, but it involves the use of a large stator to carry the two windings and the machine will have slightly poorer characteristics than a standard motor.

There is one special form of winding which can be applied in such a manner that it is possible to double the number of poles merely by changing the points of connection to the winding and at the same time connecting together three points on the winding. This gives us two speeds with a single winding, but the speeds must be in the ratio of two to one. The characteristics of the motor also suffer somewhat.

268. Connection in Cascade or Concatenation.—Imagine an induction motor in which the stator and the rotor have the same number of turns. If the motor is at rest with voltage applied to the stator, it will act as a transformer. The voltage at the slip rings will be the same as that of the line, and the frequency will also be the same. If resistors are connected to the three slip rings, and the load and the resistance are so adjusted that the motor is operating at half of synchronous speed, the rotor conductors will be moving half as fast as the flux and the voltage and frequency at the rotor slip rings will be half as great as when the motor was at rest. The power wasted in the resistors connected to the three slip rings will be nearly half of that supplied to the stator. By connecting another motor of the same capacity and number of poles both mechanically and electrically to the first one, this power may be used. The connection is shown in Fig. 211. The mechanical connection is best made if possible by mounting the rotors on the same shaft and the electrical connection is made by connecting the slip rings of the first motor to the stator of the second. The second motor may have either a squirrel cage or a wound rotor. As the power supplied from the

first motor is of half frequency and half voltage it will be correct to drive the second motor at the same speed as the first. The first machine will be acting partly as a motor and partly as a frequency changer. Either motor or both of them may be connected directly to the line in the usual way in which case they will run at full speed, *i.e.*, double the speed in cascade.

The motors need not, however, have the same number of poles. Thus the first may have four and the second six poles. The synchronous speed of the first one alone on 60 cycles would be 1800 r.p.m., and that of the second alone 1200 r.p.m. The two connected in cascade would have a speed corresponding to the sum of the two sets of poles, in this case ten poles or 720 r.p.m. Thus with the two motors three speeds would be available. The same principle may be applied to more than two motors, although it is rarely done. This method gives quite good results, but it is somewhat expensive.

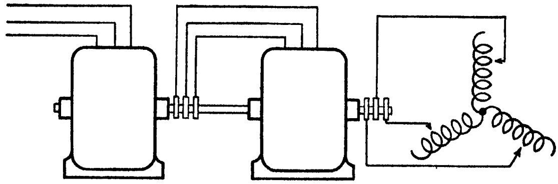

FIG. 211.

269. Induction Motors with Commutators.—A commutator is a frequency changer. Thus in a continuous-current motor it changes a continuous current (zero frequency) into an alternating current or *vice versa*. In a similar manner it may change an alternating current from one frequency to another. If one places, in a stator of the usual type, a rotor having a winding and a commutator identical with those of a direct-current machine, the frequency at the brushes will always be that of the line, no matter what the speed of the rotor. This being so, it is possible to impress different voltages upon the brushes by means of a variable ratio transformer and in this manner change the speed. The speed can therefore be adjusted through a considerable range both above and below synchronism without the use of resistors and consequently with high efficiency.

Such motors are expensive and present difficulties from the standpoint of commutation. They are occasionally built for special requirements, usually in large sizes.

270. The Wound-rotor Machine for Adjustable Speed Work.—A number of speed-torque curves of a wound-rotor induction motor are shown in Fig. 212. The torque plotted is that developed in the rotor. The torque actually applied to the load is slightly less on account of bearing friction. Curve *A* shows the torque at different speeds when the external resistance in the rotor is zero. As previously explained (see Art. 251), the torque at starting is small on account of the great lag of the current behind the e.m.f., and the consequent fact that the current in the rotor is displaced in position from the flux. As the motor speeds up, the current decreases but the torque increases to a maximum value of *OD*. This is attained when the motor has nearly reached synchronism. Beyond this point the torque rapidly decreases with increased speed until it reaches zero at synchronism. If driven above this speed the torque reverses and the machine becomes a generator.

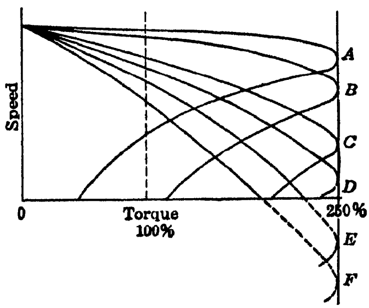

Fig. 212.

With a low resistance connected to the slip rings the speed-torque curve would be represented by such a curve as *B*. This would also correspond to an average speed-torque curve for a squirrel-cage motor, as such motors are purposely built with a moderate rotor resistance in order that the torque at start may be sufficient for average conditions. The insertion of still more resistance would give such curves as *C*, *D*, *E*, and *F*.

It will be noted that the maximum torque that can be obtained is the same no matter what the speed, or to state it in different words, the motor can carry this maximum torque at any speed between zero and a speed perhaps 10 per cent. below synchronous speed, provided the rotor resistance is properly adjusted. The same is true of any smaller torque. Thus, full-load torque could be obtained at any of the speeds given by the intersection of the speed curves with the line representing 100 per cent. torque. With a given torque, then, the speed may be adjusted through a considerable range.

The motor is, however, not an adjustable-speed motor in the

sense that the term is usually employed, since if the resistance were so adjusted that the motor operated at half speed with full-load torque it would run at nearly full speed with no load, at about three-fourths speed with 25 per cent. of full-load torque, etc. In other words, the speed regulation of the motor is poor when there is considerable resistance in the rotor circuit.

The efficiency is also low. Thus if the motor is operating at half speed, practically half of the power supplied to the stator is wasted in the resistance of the rotor and external resistors. The efficiency will therefore be less than 50 per cent. If the speed were reduced to 25 per cent. of normal, the efficiency would be less than 25 per cent., etc. On account of these facts, this method of speed regulation is not used to any great extent.

271. The Single-phase Induction Motor.—A complete treatment of the single-phase induction motor involves considerable difficulty and will not be attempted here. However, some of the principal facts in connection with it should be stated.

If a two-phase motor is operating without load and one of the phases is opened, the motor will continue to rotate and the only noticeable change will be a slight difference in the hum of the motor. If an ammeter is inserted in the phase that is left closed, it will be found that the current per phase operating single phase is about double that operating two phase. If the same experiment be made with a three-phase motor by opening one of the three leads, it will be found that the current changes in the ratio of 1 to $\sqrt{3}$, or it increases 73 per cent.

272. Rotating Magnetic Field.—If with the motor operating single phase at no load one investigates the magnetic field by suitable experiments, it will be found that there is a rotating magnetic field just as in the case of the polyphase motor. If the motor is loaded it will be found that the field is rotating but that it is stronger in the direction of the axis of the primary winding than it is in a direction at 90° to this axis. The difference will be in the neighborhood of 5 to 10 per cent. If the motor is at rest, the field does not rotate but merely pulsates.

We may gain a rough idea of the manner in which this rotating magnetic field is set up in the following way: The rotor is highly inductive, and like any inductive circuit it resists strongly any attempt to change the flux passing through it. Imagine that at a certain instant the stator current is a maximum and that flux is passing through the stator coil and through the rotor in line

with the axis of the stator coil (see Fig. 213). The rotation is in a clockwise direction. If there were no winding on the rotor the flux would drop to zero when the rotor had moved through an angle of 90 electrical degrees. The moment, however, that the flux begins to decrease *along this axis of the rotor*, a powerful current is induced in the rotor winding. This current, in accordance with Lentz's law, resists the change of flux, and is powerful enough to prevent much change. The result is that when the stator current drops to zero and the rotor has turned through an angle of 90 electrical degrees, as shown in Fig. 214, there is a current in the rotor bars at such a position on the rotor surface as to magnetize it along an axis 90°

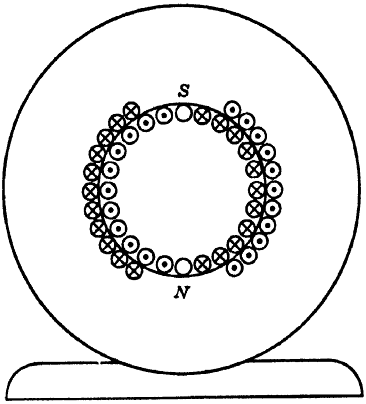

FIG. 213.

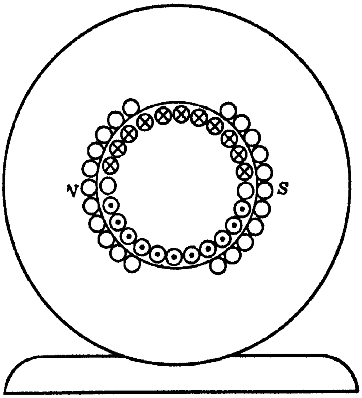

FIG. 214

from the axis of the stator coil. The flux through the rotor has changed slightly in order that this current may be generated, but the change is small. It may then be said roughly that the flux along the axis of the stator coil is provided by the current in the stator. That flux which is along an axis at right angles to the stator coil is due to current in the rotor.

Similar considerations will show that when the rotor turns further the flux through it will again tend to increase. The currents in the rotor bars will so arrange themselves as to oppose the slight change, and when the current in the stator winding is again a maximum, the rotor currents will be in such a position as to oppose the stator curent. The stator current will therefore have to be larger than if the rotor current were not present,

that is, larger than would be the case in a polyphase motor. As has been stated, in the case of a two-phase motor it would be doubled.

273. Starting Torque.—If single-phase current be applied to one phase of a polyphase motor when the latter is at rest, absolutely no starting torque will be developed. If the motor be given an initial start in *either direction* by pulling the belt or otherwise, the motor will develop a small torque and will accelerate to nearly synchronism, in the direction in which the impulse was given. That there would be no starting torque and that the motor would operate equally well in either direction might have been inferred from the fact that the motor is symmetrical. There is absolutely no reason why it should rotate in one direction rather than in the other. A mechanical structure exhibiting the same peculiarity is the familiar two-cycle gasoline engine with the spark set on dead center.

274. Split-phase Starters.—One method by which the single-phase induction motor may be given a small starting torque is

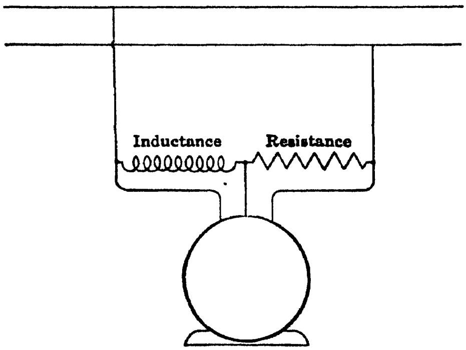

FIG. 215.

illustrated in Fig. 215. The winding is that of a standard three-phase motor. For starting, two leads are connected directly to the line. A reactor and a resistor connected in series are also connected across the line. The third lead is connected to the junction of the resistor and the reactor. The currents in the three circuits will differ in phase, and an imperfect rotating magnetic field will be set up. After the motor is up to speed, two of the leads will be connected to the two line wires, and the third lead will be left on open circuit. The resistor and reactor are, of course, disconnected.

The torque produced in this manner is feeble, and it is generally necessary to make provision so that the motor may be started without load. This may be done by means of a centrifugal clutch which takes hold of the pulley when the motor has nearly reached synchronous speed.

275. Starting as a Repulsion Motor.—A great many single-phase motors are built to start as repulsion motors but operate after starting as induction motors. The rotor resembles the armature of a continuous-current machine, and is provided with a commutator and brushes. The brushes are short-circuited. If the brushes are given a lead in either direction, a powerful torque will be developed tending to turn the rotor in that direction. When the motor has nearly reached synchronous speed a centrifugal device acts to remove the brushes from the commutator and at the same time to short-circuit all of the bars of the commutator. The motor then continues to run as a squirrel-cage induction motor. The action of the repulsion motor is described in more detail in the next chapter. (See Art. 300.)

276. Synchronous Motors versus Polyphase Induction Motors.—The following is a brief comparison of the points to be considered in making a choice between the above two types of motors.

Efficiency.—The synchronous motor should run a little higher in efficiency since the power factor may be made better and, therefore, the I^2R losses in the stator are less. The loss in the field should also be less than that in the rotor of a squirrel-cage induction motor since the resistance of the latter is necessarily high to get the requisite starting torque. This advantage is partially offset by the fact that the field current of the synchronous motor is usually generated in a small and comparatively inefficient machine.

277. Power Factor.—The synchronous machine has here a great advantage. If the wave shapes of the line voltage and the machine voltage are the same, the power factor can always be made unity. It is also always possible to make the current lead the e.m.f. by overexciting the field. This leading current may be of great value as a means of offsetting lagging current due to the presence of induction motors or other causes. The synchronous machine also greatly improves the regulation of the line by taking a lagging or leading current as may be required to correct the voltage variations.

278. Speed Regulation.—Under full load a small induction motor may slow down as much as 10 per cent. and a large one perhaps as little as 1 per cent. This amount of variation in speed is usually not objectionable. In the case of the synchronous machine, however, the speed regulation is perfect, that is, the motor does not slow down at all under load. Under certain conditions this may be advantageous.

279. Overload Capacity.—It must not be supposed that a synchronous motor is readily pulled out of step and that consequently they are incapable of carrying heavy overloads. As a matter of fact, almost any synchronous motor will carry from two to three times its rated load without difficulty. Approximately the same figures apply to the induction motor and there is, therefore, little to choose between the two in this respect.

280. Hunting.—The induction motor causes no trouble by hunting. The synchronous motor may do so. However, if provided with a squirrel-cage winding in the pole faces or with special grids on the poles there should be no serious hunting under usual commercial conditions.

281. Starting Torque.—A wound-rotor induction motor of fair size will develop 100 per cent. starting torque with about 110 per cent. of full-load current, or a maximum of about 225 per cent. of full-load torque with about 270 per cent. of full-load current. The squirrel-cage motor will develop a maximum of 200 per cent. torque with about 600 per cent. of full-load current. A synchronous motor will develop a maximum torque of perhaps 125 per cent. with 500 per cent. of full-load current. These figures vary greatly, however, in different motors.

It will be seen that the synchronous motor is not far behind the squirrel-cage motor in starting torque. However, some difficulty may be experienced with heavy loads at the instant when the current is passed through the fields and the motor should drop into step. Up to this time, it has been acting as an induction motor and it is necessary that it accelerate quickly so that it may fall into step as a synchronous motor. Whether or not it will be able to do this depends more upon the *inertia* of the connected load than upon the torque required to maintain rotation.

282. Air-gap Clearance.—The synchronous motor has a considerable advantage in the fact that the air-gap clearance is several times as great as in the induction motor. There is,

therefore, less danger that the bearings may wear to such an extent that the rotor will strike upon the stator.

283. Attention Required.—About all the care required by the induction motor is keeping the machine clean and replenishing the oil in the bearings occasionally. In the case of the synchronous motor a new factor is added—the adjustment of the power factor by means of the field current. To check this adjustment, instruments are usually used with the motor. The operation of the motor therefore requires a slightly higher grade of labor, and much damage could be done by a careless or ignorant attendant.

284. Slow-speed Motors.—An induction motor of moderate size, say 200 hp. operating at such a speed as 240 r.p.m. on 60-cycle current, is a very poor machine. The power factor is low and in consequence the efficiency suffers. The starting torque is also likely to be low. These difficulties can be largely avoided in the synchronous motor and it is therefore generally preferred for such service.

PROBLEMS

103. An induction motor is to operate on 60-cycle current and have a synchronous speed of 150 r.p.m. How many poles must it have?

104. How many poles must it have when operating on 25-cycle current in order that the speed may be the same?

105. What is the maximum speed that a 60-cycle induction motor may have? A 25-cycle motor?

106. A six-pole 25-cycle generator is driven by a 60-cycle induction motor. How many poles must the latter have, allowing a small amount for slip?

107. A 60-cycle, eight-pole induction motor is operating under load at a speed of 856 r.p.m. What is the slip? What is the frequency of the current in the rotor?

108. What would be the slip and the rotor frequency at a speed of 450 r.p.m.?

109. An induction motor when connected directly across a 440-volt circuit develops a starting torque of 200 per cent. What torque will it develop when started by means of a compensator if the terminal voltage of the latter is 220? What if 340 volts?

110. A 60-cycle induction motor has two windings upon the stator giving respectively ten and sixteen poles. What will be the respective speeds with the two windings?

111. Two induction motors are wound with ten and six poles respectively and are arranged on the same shaft for operation in cascade on a 60-cycle circuit. What are the possible synchronous speeds?

112. In the above the supply voltage is 440 volts, three-phase. The ten-pole motor is the one connected to the line and is provided with a three-phase wound rotor. The ratio of turns in the stator and rotor is 1 to 1. What

is the voltage across the slip rings of the first motor, the motor being at rest and the rotor circuit open? When the two motors are connected in cascade as above, what is the voltage at the slip rings? What is the frequency? Are the voltage and frequency correct for the second motor at the speed attained?

113. A six-pole induction motor operating upon a 60-cycle, three-phase circuit gave the following results upon test: Input, 22 kw., current 64 amp., voltage 220, frequency 60 cycles. What was the power factor? What was the power component of the current? The wattless current?

114. At the same time that the above readings were taken, the readings on a Prony brake were as follows: Length of arm 3 ft., net weight 38 lb., speed 1150 r.p.m. What was the horse-power output, the kilowatt output, the efficiency and the slip? What was the loss in the motor?

115. Assuming the same efficiency, power factor and output as in the above problem, what would be the current taken by a two-phase motor operating on a 440-volt circuit?

CHAPTER XIX

THE SINGLE-PHASE COMMUTATOR TYPE MOTOR

285. Methods of Operating Electric Locomotives.—Withi the last few years there has come into use a type of single-phas motor designed somewhat along the lines of a continuous-curren motor. This motor has been developed with the primary objec of adapting it to the requirements of railway service. As ha been previously explained, in practically all large installation the power is generated as alternating current, usually three phase For railway operation, this is frequently converted to direc current by rotary converters or by other suitable means. Thi involves a double conversion, since the pressure must first b reduced before supplying the current to the rotaries, and it mus then be changed to direct current. The direct current is usuall supplied at pressures approximating 600 volts, and since it i impracticable to transmit current at this potential efficiently t great distances, it becomes necessary to provide rotary converte sub-stations at distances of about 10 miles apart. Even with thi spacing the cost of copper for efficient operation is large, and sinc the rotating machinery must be under constant supervision, th operating cost is high. This latter charge is not so burdensom in the case of interurban roads, since the converter stations ca usually be placed at points at which it would be necessary in an event to have a ticket or freight agent. In the case of trunk-lin electrification, this solution is usually impracticable. By usin higher voltages on the trolley wire it becomes possible to spac the sub-stations farther apart, thus reducing the cost of attend ance and the amount of copper installed. Direct-current poten tials as high as 2400 to 3000 volts are now being successfull used.

Another solution is to mount the converting machinery on th locomotive, thus avoiding the loss in transmission and part of th cost of attendance. The current in this case should preferabl be supplied as high-voltage single-phase current, and may be con verted either to continuous current or to three-phase current

This involves considerable complication, and the added weight on the locomotive is usually a serious handicap, unless the weight is needed for traction.

Three-phase induction motors have been successfully used on electric locomotives. In the opinion of most engineers, however, their field of usefulness is limited on account of the constant-speed characteristics of this type of motor. It is true that by adding somewhat to the complication of the apparatus two, three or more speeds can be obtained. Even then the motor still lacks the flexibility of the series-wound direct-current motor. It is the influence of these facts that has led to the development of the single-phase system.

286. The Single-phase System.—The continuous-current motor is entirely satisfactory for railway work. The difficulty is to get the direct current to it without great loss and complication. The three-phase induction motor is exceedingly simple, but it is handicapped by its constant-speed characteristics. To get the three-phase current to the locomotive requires the use of three conductors—the track and two overhead wires. This leads to some complication, particularly in terminal yards.

In the single-phase system the power may be generated either as single- or as three-phase current. It is transmitted at high voltage (usually 40,000 or over), and is stepped down by means of stationary transformers to a lower voltage (say 6600 volts) to supply the overhead conductor. On the locomotive or car the pressure is again reduced by means of a transformer to about 200 volts. There is no moving machinery between the generating station and the motors, and the current is transmitted at high pressure all the way to the locomotive. This should therefore lead to a cheap and efficient distributing system. Unfortunately, as will appear from the following, the single-phase motor is by no means as good as the direct-current motor. To get a just comparison we must consider each case on its individual merits and must take care to consider the systems as a whole and not any particular element alone.

287. Series-wound, Commutator Type, Single-phase Motor.—Consider first a direct-current, series-wound motor. If the current through such a motor is reversed, the polarity of the fields will be changed at the same time that the current is reversed through the armature. The torque will, therefore, be exerted in the same direction as before. If it is assumed that

the reversals are very slow, say one per second, the principal point of difference from direct-current operation will be the fact that the torque, instead of being constant, will vary with the time, being zero at the instant when the current is zero and a maximum when the current is at its highest value. It will be apparent that if the *average* torque is to be the same as it would be if a direct current of, say, 100 amp. were used, the effective value of the slowly alternating current must also be 100 amp. The *maximum* value must, however, be greater than this in the ratio of $\sqrt{2}$ to 1, or 141 amp. The I^2R loss will be the same in the two cases. The commutation will be more difficult in the case of the motor on the slowly alternating current since this is largely determined by the maximum value of the current.

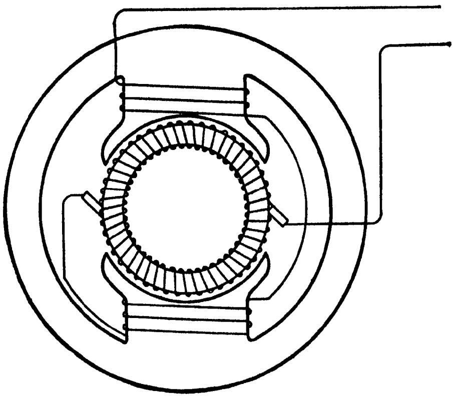

FIG. 216.

288. Heating.—If an attempt were made to operate a direct-current series motor on the ordinary commercial frequency of 25 cycles, three serious difficulties would be encountered. In the first place the motor would heat excessively. This would be due to the eddy currents induced in the solid poles and yoke of the motor. The solid core would act like the short-circuited secondary of a transformer having one secondary turn. Operation under these circumstances would be impossible. This difficulty can, however, be rather easily overcome by building the entire magnetic circuit of laminated iron.

289. Power Factor.—The second difficulty encountered would be that the power factor would be found to be very low. To understand this fully, it will be necessary to construct the vector

diagram of the motor. Consider first the diagrammatic representation of a series motor in Fig. 216. For simplicity this is shown as a two-pole machine. In practice, the motors, except in the smallest sizes, would be multipolar and would, in general, have more poles than a corresponding direct-current machine. The armature is shown as of the ring-wound type. This is frequently assumed for simplicity in drawing diagrams of such machines. In practice, the ring-wound armature is not used for this purpose.

290. Generated E.M.F.—Consider, now, that the armature is rotating in a *constant* field. A difference of potential will exist between the points on the commutator at which the brushes are placed. If the brushes are moved from this position the difference of potential would become less, reaching zero at a position 90° from the one indicated.

If, instead of supplying the field with a continuous current, it is excited by means of an alternating current, the e.m.f. at the brushes will also be alternating. The frequency will be the same as the frequency of the current in the fields. With the field flux zero, the generated e.m.f. will also be zero, and with the flux at its maximum, the e.m.f. will also be maximum. This e.m.f. will therefore be in phase with the flux, and since the flux is nearly in phase with the current, the induced e.m.f. will be nearly in phase with the current in the field. If it were desirable, an alternating-current generator could be constructed in this way. The ordinary construction is, however, much cheaper, except possibly in the case of machines designed for frequencies of 10 cycles per second or less.

291. Induced E.M.F.—The conditions are different in the field. The only induced e.m.f. present will be that due to the cutting of the field conductors by the alternating flux. This induced e.m.f. will be 90° behind the current, and there will also be a component of applied e.m.f. in phase with the current to overcome the resistance of the windings.

When the field and armature are connected in series and the machine operated as a motor, the conditions in the field will be the same as before. The induced e.m.f. in the armature now becomes the back e.m.f. of the motor. Moreover, since the armature has itself reactance and resistance, it will act as an inductive circuit, in the same manner as the field.

292. Vector Diagram of Motor.—Figure 217 shows the vector diagram of the motor. If the vector representing the current is drawn as a horizontal line, the flux will be nearly in the same phase, and may be represented without serious error as being exactly in the same phase. The line marked $L_fI\omega$, is drawn to represent the e.m.f. required to overcome the reactance of the field. The line R_fI, is the e.m.f. required to overcome the resistance of the field. Similarly $L_aI\omega$ and R_aI represent the e.m.fs. required to overcome the reactance and resistance of the armature. In addition there is an e.m.f. required to overcome the back induced e.m.f. of the armature. This is marked E_a in the diagram. The total e.m.f. applied to the motor is the vector sum of these five e.m.fs., and is marked E. The angle of lag of the current behind the e.m.f. is the angle θ as shown.

Of the five e.m.fs., the two due to the inductance of the

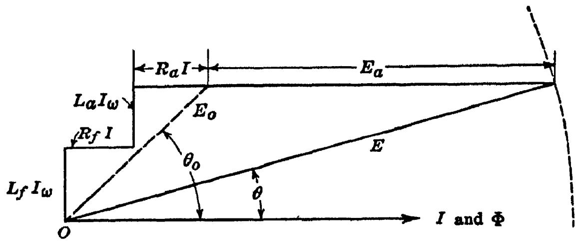

FIG. 217

armature and the field, and the two due to the resistance of the same elements are proportional to the current flowing in the motor. The induced e.m.f. in the armature E_a is directly proportional to the flux and nearly proportional to the current. It is, however, also directly proportional to the speed of the armature. If the armature is at rest as at the moment of starting, this latter e.m.f. will be zero. The resultant applied e.m.f. will then be represented by the dotted line marked E_0, and the power factor will be the cosine of the angle θ_0.

293. Changes to Improve Power Factor.—To secure a good power factor with a motor of this type, two conditions are necessary; the back induced e.m.f. E_a must be made as great, relatively, as possible, and the induced e.m.fs. in the field and armature must be reduced as much as possible. It might seem that the power factor could be improved by increasing R_aI and R_fI. This is, of course, the case, but it would be only at the

expense of the efficiency, since these two multiplied by the current represent the loss in the resistance of the machine.

The value of E_a can be increased by increasing the flux, the number of conductors on the armature or by increasing the speed. It will be found that the average speed of single-phase railway motors is higher than that of the corresponding continuous-current motors. Weakening of the flux reduces the e.m.f. induced by the flux $L_f I \omega$, but to do this and still have the motor adapted to operate at the same voltage and speed, it is necessary to increase the number of armature conductors at the same time that the flux is reduced. Consequently, while the inductance of the field is reduced, that of the armature is increased. Moreover, on account of the strong armature reaction and the weak field, serious distortion of the flux would result, and the motor would spark badly.

294. Compensating Winding.—Fortunately, however, it is possible to provide a winding known as a compensating winding to render the armature nearly non-inductive. This can not be done in the case of the field as the field flux is essential to the operation of the machine. The flux due to the armature on the other hand may be regarded as a stray flux which is not essential to operation, and which in the present case is detrimental on account of the induced e.m.f. generated. Figure 218 shows the arrangement of coils and slots in a compensating winding. If, in addition to the armature conductors, an equal number of conductors could be provided in the same position as the armature conductors, each conductor carrying a current equal and opposite to the armature current, the inductance of the armature would be zero. In addition, it would be necessary that the compensating winding be stationary as otherwise it would generate an e.m.f. equal and opposite to that of the armature. The nearest one can come to the ideal construction is to place the compensating conductors on the field structure as near to the armature as is practicable, and connect them in series with the armature and the field. Moreover, it is impracticable to cover all of the armature surface as a certain amount of room must be left for the main field coils. The compensation will therefore not be perfect. In the figure the different coils are marked, and the construction will be readily understood. The series connection gives assurance that the compensating current will at all times be equal to the main current. Compensating

windings are not confined to alternating-current machines, but are frequently used for continuous-current motors and generators. It is possible, in this way, to build a continuous-current machine much lighter than would otherwise be possible. The interpole construction that is used in continuous-current machines may be considered a special case of the foregoing in which only a portion of the armature surface is compensated, or rather over-compensated.

With the construction above indicated, it becomes possible to use a very weak field and a correspondingly strong armature, *i.e.*, one having many turns of wire on it. Both the vectors

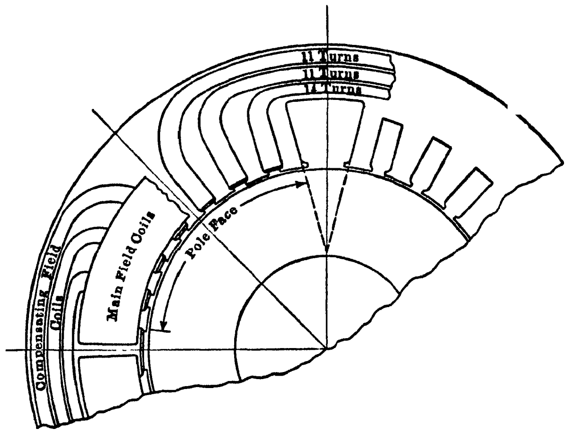

FIG. 218.

$L_f I \omega$ and $L_a I \omega$, can therefore be made short, compared with the induced e.m.f., and the power factor consequently large.

295. Variation of Power Factor with the Load.—In practice a motor of this type is usually operated at a constant potential. If this is the case, the length of the line E of Fig. 217 will be constant. We may, therefore, draw a circle of radius E about the point O as a center, and the extremity of the vector E will always fall upon this circle. The vector, E, will be more nearly parallel to the line representing the current, *the smaller the current.* In fact, with zero current the two would become parallel, and the power factor would be unity. If, therefore, a curve of current and power factor be plotted, the power factor will start from unity, with zero current, and will become less as the current is increased. The condition of zero current at full voltage is,

however, not attainable in such a motor, since with zero field, it would be necessary for the armature to rotate at an infinite speed in order to generate the required back e.m.f.

The other characteristics of such a motor, which are usually plotted with currents as abscissæ such as speed, torque (or tractive effort, if the motor is used in railway work), horse-power output, temperature rise, etc., will in general, resemble those of a series-wound direct-current motor. The torque will, however, be nearly proportional to the square of the current since the magnetic circuit is relatively unsaturated, and the flux is thus nearly proportional to the current. The speed at any given current will be less than that of the same motor operating on direct current, since in the case of alternating-current operation, the motor has only to generate the back e.m.f. corresponding to E_a while in the direct-current machine, the back e.m.f. would be equal to E minus the drop in the windings, or R_aI, plus R_fI.

296. Operation on Direct Current.—As before indicated, a motor of this type will operate even better on direct current than on alternating. In the matter of control, there are advantages in the use of the alternating current. It has been pointed out that with direct current, the speed will be higher and consequently the output greater for the same current. The efficiency will be better also since the hysteresis and eddy-current loss in the iron of the field will be eliminated. The armature iron loss will be less since for the same effective current the flux with direct current will be only about 70.7 per cent. as great as on alternating. Moreover, with alternating current the flux at any point of the armature goes through a somewhat complicated cycle, usually causing an increased loss.

Since the losses are greater with the alternating current, the heating will be more than with direct current, or looking at the matter from the other standpoint, the machine may be rated higher when operated on direct current. As has been pointed out, in order to get a good power factor, it is necessary to construct a motor of this type with a very weak field. If the machine were to be operated on direct current, it would be possible to work with a far stronger field, thus increasing the torque and consequently the power of the motor in the same proportion.

The foregoing will be sufficient to indicate that the single-phase alternating-current commutator motor, in its present form, is in every respect inferior to a similar direct-current motor. It

should not be taken from this that the motor is to be condemned. The motor is only one element of a system, and it may readily happen that the disadvantage under which the motor itself operates is more than offset by advantages elsewhere in the system.

297. Commutation.—The problem of commutation is perhaps the most serious one with which the designers of single-phase commutator motors have had to deal. Since the current is alternating it is necessary, at the peak of the wave, to commutate 41 per cent. more current in the alternating-current motor than in the direct-current machine. This is further increased by the fact that both the efficiency and the power factor are lower than in a direct-current motor. This is, however, not the worst phase of the situation. As was pointed out in considering the subject of direct-current commutation (see page 296), the coil under commutation is in a neutral field, that is, it has no e.m.f. induced in it due to the rotation of the armature. This is also true in the case of the alternating motor, as far as the e.m.f. due to the rotation, is concerned. An examination of Fig. 216 will show, however, that the coils short-circuited by the brushes and in the position to develop no e.m.f. due to their *rotation*, are in the best position to have induced in them an e.m.f. due to the *alternation* of the flux. They are, therefore, the seat of an e.m.f. which is short-circuited through the brushes, and this e.m.f. is present even though the armature is at rest. In fact, it is independent of the rate of rotation, and is dependent only upon the frequency of the supply and upon the flux passing through the coil.

In practice, every endeavor is made to decrease the value of the e.m.f. induced under the short-circuited brush. The number of turns per coil is usually reduced to one, and a lap winding adopted. The e.m.f. is then that due to one coil only. The strength of the poles is reduced to the lowest possible value. This would be done in any event in order to improve the power factor. Moreover, by increasing the number of poles, the flux per pole may be decreased. On this account, the number of poles employed in these motors is far in excess of that common in continuous-current motors.

Even with all of these modifications, the e.m.f. short-circuited under the brush is far too great to allow satisfactory commutation, except in the smallest machines. The most common

method of reducing the short-circuit current is the use of resistance leads. These consist of strips of resistance material connected between the commutator bars and the winding. The connection is shown in Fig. 219. In practice, the leads are frequently doubled and laid in the bottom of the slots. The current from the brushes passes through only those leads which are connected to the commutator bars actually in contact with the brushes at any given instant. Hence, these leads need not be so heavy as would be necessary if the current were passing through them all the time. Care must, however, be taken that they are not so light that they will be liable to be burned out in case the motor should fail to start at once when the current is applied. Under these circumstances, the current will be confined to a few leads, and these will, of course, become hotter than normal.

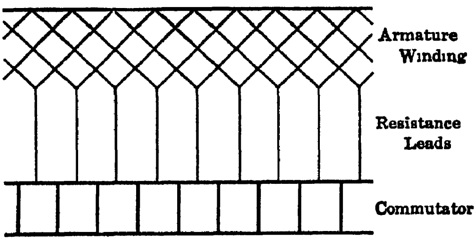

FIG. 219.

The difficulty due to the generation of an e.m.f. in the coils undergoing commutation, can be decreased by the use of a lower frequency. This results in an improvement of the power factor, since the vectors $L_a I\omega$ and $L_f I\omega$, in Fig. 217 are proportional to the frequency. On the other hand, advantage may be taken of these facts to increase the strength of the magnetic field employed, and thus increase the rating of the motor. This undoubted advantage of the lower frequency motor has led to the serious proposal to standardize another frequency for railway work. The frequency most frequently mentioned in this connection is 15 cycles. This would result in an increase in the weight and a decrease in the efficiency of the generators and transformers. Moreover, this frequency is not well adapted to the operation of small and medium-sized induction motors.

298. Control of Single-phase Motors.—When continuous-current motors are used on electric cars or locomotives, the con-

trol of the car is accomplished by means of resistors in connection with different arrangements of the motors. In the case of the alternating-current commutator motors, a more efficient and perhaps simpler method is used. The current is always supplied to the car or locomotive at a high voltage, varying usually from 4400 volts to 11,000 volts. This necessitates the employment of a transformer on the car to reduce the voltage to that suitable for use on the motors. It is a comparatively easy matter to provide this transformer with a number of taps so that any suitable voltage can be impressed on the motor terminals. The necessity for using resistors is removed. Thus the loss due to the use of resistance is avoided and the efficiency is improved. This will be especially important if frequent stops and starts are to be made, or if a great deal of slow-speed running is necessary.

Another advantage of this method of control is that it gives a method of making up time in emergencies. It is merely necessary to provide an extra high-voltage tap to be used only in case a speed above normal is necessary. This may also be useful in case of low voltage on the line.

As has been pointed out, motors of this type are capable of operation on continuous-current circuits, in fact, their operation is better on the latter than on alternating current. This is a very valuable property, particularly in the case of cars used in interurban service. Such cars almost invariably use the tracks of city cars, and these are always operated by continuous current. If the motors of the interurban cars were incapable of operation on direct current, it would be necessary to transfer passengers, or provide special locomotives to haul the interurban cars through the city. The same situation sometimes arises even in the case of trunk-line electrifications. The control apparatus used on continuous-current cars can be used with alternating-current motors. The reverse is, however, not true. It is therefore customary to sacrifice the advantage in point of efficiency that the alternating-current control possesses rather than go to the complication of two sets of control apparatus.

The voltage used on single-phase commutator motors is about 200 volts. This is undesirably low compared with direct-current practice, as it necessitates a commutator about twice as large as would be required for the same power at 550 volts. This

low voltage is, however, a necessity on account of the fact previously mentioned that the armature coils are usually wound with only one turn per coil in order to reduce the difficulty of commutation. This in turn involves the use of one commutator bar per coil, and consequently the number of bars is great. It is impracticable to crowd more than a limited number of bars within the limit of size of the commutator. Hence the voltage must be kept low.

299. Other Types of Single-phase Commutator Motors.— In addition to the series type of motor just described, numerous modifications have been introduced. In general, it may be said that these modified forms depend upon induction to produce the current in one or more of the elements, instead of employing conduction as in the series motor. When this is the

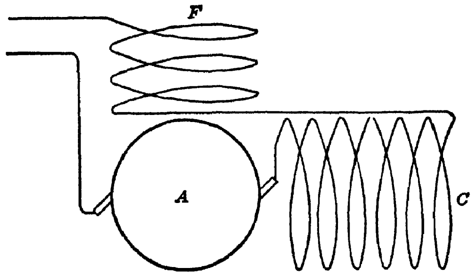

FIG. 220.

case, they are obviously incapable of operation on continuous current.

Figure 220 shows in a diagrammatic form the connection of the three elements of a single-phase series motor. The coil marked F is the field coil, C is the compensating coil, and A is the armature. The latter, for the purposes of the diagram, is supposed to be so wound that its "poles" are opposite the brushes. The simplest modification of these connections is illustrated in Fig. 221, in which the compensating coil, instead of being connected in series, is short-circuited on itself. It will then act like the short-circuited secondary of a transformer, of which the armature is the primary. Such a current will flow that the ampere turns of the coil will be approximately equal to the ampere turns of the armature. The coil may be wound with any convenient size of wire and number of turns, sufficient copper being provided to carry the current required. The opera-

tion on alternating current will not be markedly different from that of the series motor. On continuous current, no current would flow in the compensating coil, and on account of the great strength of the armature compared with the field, there would be such serious sparking that operation would be impracticable.

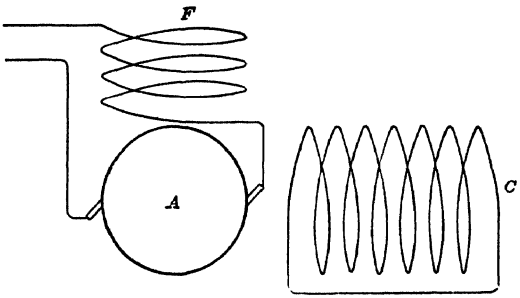

FIG. 221.

300. Repulsion Motor.—The motor illustrated in Fig. 222 is known as the repulsion motor. The armature is not connected in series with the main circuit, but is short-circuited and receives its current by induction. One very important advantage of this method of connection is that the field and compensating

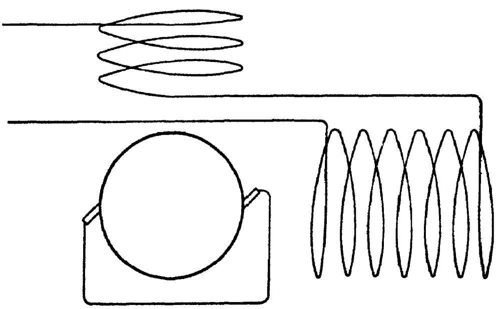

FIG. 222.

coil may be wound for high voltage, while the armature is wound for whatever voltage is most suitable for the design in hand. With this type of motor the use of a transformer on the car can, therefore, in many instances be avoided. This method of connection, however, leads to quite radical changes in the characteristics of the motor, particularly as regards commutation.

Nevertheless, the series characteristic is retained. In practice, repulsion motors are usually wound with a distributed winding for both the compensating coil and the field coil. With this carried to the limit, it will be apparent that the two coils become really one, with the axis of the brushes displaced from the axis of the coil.

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